Physics 662
Problem Set 3

Due: February 27, 2014, 9:30 am

Work the following problems:

1. The neutral kaon decays from the states K_1^0 and K_2^0 with CP eigenvalues +1 and -1 respectively.

   (a.) If $p\overline{p}$ annihilation at rest takes place from an S-state only, show that $p\overline{p} \to K_1^0 + K_2^0$ occurs but that $p\overline{p} \to 2K_1^0$ and $p\overline{p} \to 2K_2^0$ do not.

   (b.) Consider the $p\overline{p}$ annihilation at a center-of-mass energy just above the threshold for $K^-\pi^+K^0$ and $K^+\pi^-\overline{K}^0$. This is the CPLEAR experiment at CERN.

   (i.) Calculate the center-of-mass energy required for this reaction and the minimum beam momentum of the equal momenta colliding proton and antiproton.

   (ii.) Describe how these reactions can be used to select events which contain purely K^0's and $\overline{K}^0$'s from the interaction.

2. The unit of radiation dosage is the rad, corresponding to an energy liberation in ionization of 100 erg g^{-1}. The annual permissible body dose for a human is cited as 5 rad. Assuming that 100 times this dose would lead to the extinction of advanced life forms, what limit does this set on the proton lifetime, assuming that in proton decay a substantial fraction of the total energy released is deposited in body tissue?

3. Supersymmetric SU(5) grand unification predicts proton decay, in analogy with muon decay, with a lifetime of

$$\tau_P = \frac{AM_X^4}{\alpha_{GUT}^2 M_P^5}$$

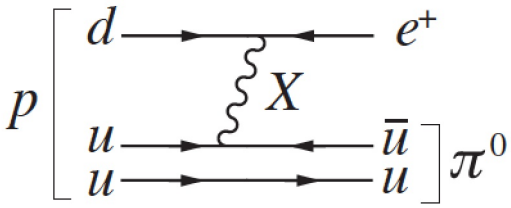
   where A is a dimensionless quantity of order unity and α_{GUT} is the coupling (analogous with the fine structure constant) at the GUT scale. Estimate the proton lifetime, if $M_X = 3 \times 10^{14}$ GeV, $\alpha_{GUT} = \frac{1}{42}$ and A = 1.

4. (a.) It is possible to assign values of B-L, where B is the baryon number and L is the lepton number, to the X and Y bosons of grand unified theories. Consider the following reactions and determine the values of B-L for the X and Y bosons in each case:

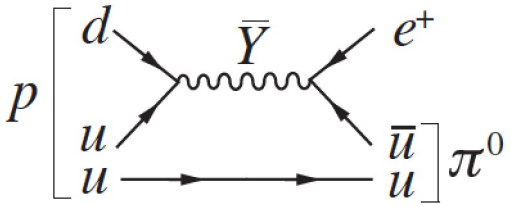
   - $d \rightarrow e^+ X$
   - $d \rightarrow \overline{\nu}_e Y$
   - $\overline{u} \rightarrow uX$
   - $\overline{d} \rightarrow uY$
   - $u \rightarrow e^+ Y$

   (b.) What are the charges of X and Y in this model?

5. The following diagrams could contribute to the proton decay reaction $p \rightarrow \pi^0 + e^+$.



(a)



   (a.)Relabel the lines to find two possible mechanisms for the decay $p \rightarrow \pi^+ \overline{\nu}_e$.

   (b.)Show that the electric charge and B-L values are conserved at the vertices based on the results from problem 4 above.

6. Consider a star moving with velocity v in a circular orbit of radius R about the center of a spiral galaxy.

   (a.)Calculate the dependence of v on R for the following extreme cases:

   - The total mass of the galaxy, like the luminous mass, is concentrated almost entirely at the centre of the galaxy.
   - The mass of the galaxy is distributed almost entirely in a spherical halo of dark matter, assumed to be of constant density and radius $R_h > R$.

   (b.)Compare your results with the observation that v is almost independent of R for stars observed in the outer arms of spiral galaxies.

   (c.)What conclusion might you draw regarding the population of dark matter from this observation?

7. Consider a collision between a cosmic ray proton and a photon in the cosmic microwave background whose energy corresponds to the ambient temperature of 2.7 K. Estimate the minimum proton energy required to produce

   (a.) an e^+e^- pair,

   (b.) a proton-antiproton pair,

   (c.) the Δ^+ resonance.