

1. Find the indicated limit.

- (a)  $\lim_{x \rightarrow 1^+} \frac{e^{x^2} - 1}{1 - x}$
- (b)  $\lim_{x \rightarrow \pi^-} \ln(\sin(x))$
- (c)  $\lim_{t \rightarrow 2} \frac{t^2 - 4}{t^3 - 8}$
- (d)  $\lim_{v \rightarrow 4^+} \frac{4 - v}{|4 - v|}$
- (e)  $\lim_{x \rightarrow 3} \frac{\sqrt{x + 6} - x}{x^3 - 3x^2}$
- (f)  $\lim_{x \rightarrow -\infty} \frac{4 - 3x^3 - x^4}{5 + x - 7x^4}$
- (g)  $\lim_{x \rightarrow \infty} \frac{1 + 2x^3}{3 - x - 7x^4}$
- (h)  $\lim_{x \rightarrow 1^-} \left( \frac{1}{x - 1} - \frac{1}{x^2 - 3x + 2} \right)$
- (i)  $\lim_{x \rightarrow \infty} e^{x^2 - x}$
- (j)  $\lim_{x \rightarrow 0^+} (1 + \sin(x))^{\cot(x)}$

2. Let  $g(t) = |t - 2|$ .

- (a) For what values of  $t$  is  $g(t)$  continuous?
- (b) For what values of  $t$  is  $g(t)$  differentiable?

3. Find the indicated derivative.

- (a)  $f'(x)$ , where  $f(x) = (x^4 - 2x^2 + 3)^3$
- (b)  $g'(t)$ , where  $g(t) = \sqrt{t} + \frac{t - 1}{\ln(t)}$
- (c)  $y''$ , where  $y = e^{\sin(3\theta)}$
- (d)  $h'(\theta)$ , where  $h(\theta) = \cos^2(\tan(\theta))$
- (e)  $\frac{d^2y}{dx^2}$ , where  $y = e^{2x} \ln(\tan(x))$
- (f)  $\frac{dy}{dx}$ , where  $xy^4 + x^2y = x + 3y$
- (g)  $\frac{dx}{dt}$ , where  $t^2 \cos(x) + \sin(3t) = tx$

4. Find an equation of the tangent line to the curve at the given value of  $x$ .

- (a)  $y = (2 + x)e^{-x}$ ;  $x = 0$
- (b)  $y = \frac{x^2 - 1}{x^2 + 1}$ ;  $x = 0$
- (c)  $x^3y - 5xy = 4$ ;  $x = -1$

5. At what value(s) of  $x$  is the tangent line to the curve  $f(x) = 4 + 3e^x(1 + x^2)$  horizontal?
6. For each function below:
- Find all intercepts.
  - Find the vertical and horizontal asymptotes, if any.
  - Find the intervals of increase or decrease.
  - Find the local maximum and minimum values.
  - Find the intervals of concavity and the inflection points.

Then use this information to sketch a graph of the function.

- (a)  $f(x) = x^4 + 4x^3$
- (b)  $f(x) = \frac{x}{x^2 - 4}$
7. Find all local extrema of the function. Then find the absolute extrema of the function on the given interval. [You should include both **where** the extrema occur, as well as the values of the extrema.]
- (a)  $f(x) = x\sqrt{1-x}$  on  $[-1, 1]$
- (b)  $f(x) = \frac{\ln(x)}{x}$  on  $[1, 3]$
- (c)  $f(x) = \frac{3x-4}{x^2+1}$  on  $[-2, 2]$
8. A local hot sauce manufacturing company's employees on the morning shift each produce  $Q$  hundred bottles of hot sauce  $t$  hours after 8:00 a.m. where

$$Q(t) = -t^3 + 9t^2 + 12t.$$

Suppose the morning shift is 8:00 a.m. until 1:00 p.m. At what time during the morning shift should you expect the employees' rate of production to reach it's maximum?

9. Dan, the owner of the Dan's Pizza Pi company, is reviewing his daily sales figures and pricing structure for his specialty pizza. He has found that in order to sell  $x$  pizzas he needs to charge  $p(x)$  dollars for each pizza where  $p(x) = 40e^{-0.01x}$  for values of  $x$  in the interval  $[0, 150]$ .
- Find the function  $R(x)$  that gives Dan's revenue from the sales of his specialty pizza. [Hint: Revenue is price multiplied by quantity sold.]
  - It is in Dan's best interest to maximize the amount of revenue he makes each day. How many pizzas must Dan sell to maximize revenue?
  - What price does Dan need to charge for each pizza in order to sell the number of pizzas that will maximize his revenue?
10. Adam and Justin leave a cafe at 2pm. Adam walks directly west at a rate of 3 miles per hour. Justin walks directly south at a rate of 4 miles per hour.
- How fast is the distance between Adam and Justin changing at 3pm?
  - Gavin is standing 0.75 miles east of the cafe making a phone call. From where Gavin is standing, he can measure the angle between the street and Justin, as Justin walks south. How fast is this angle changing at 2:15pm?

- (c) At 2pm, Deb is 5 miles north of the cafe and begins to ride her bike. She rides directly east at a pace of 9 miles per hour. How is the distance between Adam and Deb changing at 3pm?
11. A conical water tank with vertex down has a radius of 4 m at the top and is 16 m high. Assume water is being pumped into the tank at a rate of  $2 \text{ m}^3/\text{min}$ .
- (a) Find the rate at which the water level is rising when the water is 3 m deep.
- (b) Find the rate at which the water level is rising after  $\frac{2}{3}\pi$  minutes.
- (c) What is the depth of the water at the moment when the water level is rising at a rate of 4 m/min?
12. A 5-year projection of population trends suggests that  $t$  years from now, the population of a certain community will be  $P(t) = -t^3 + 9t^2 + 48t + 50$  thousand. At what time during the 5-year period will the population be growing most rapidly?
13. A cylindrical can (with a top and bottom) is to be made with  $600 \pi \text{ cm}^2$  of aluminum. Find the dimensions (in cm) that will maximize the volume of the can. [The volume of a cylinder is  $\pi r^2 h$  and the surface area of the sides of a cylinder equals  $2\pi r h$ .]
14. If  $4800 \text{ cm}^2$  of material is available to make a box with a square base and an open top, find the largest possible volume of the box.
15. A manufacturing company is asked to create special bricks for a new retaining wall to be built in Eugene. The bricks are to be rectangular with the length equal to three times the width. The top and bottom of the bricks is to be coated with a reinforced material that costs 10 cents per  $\text{in}^2$ . The other sides of the bricks will be coated with a standard coating that costs 6 cents per  $\text{in}^2$ . Find the dimensions of the bricks that will minimize the cost of production if the bricks must have volume  $50 \text{ in}^3$ .
16. Use the function  $f(x) = \sqrt[3]{1+3x}$  at  $a = 0$  to estimate the value of  $\sqrt[3]{1.03}$  by using a linear approximation formula.
17. Give an approximation for  $\ln(0.9)$  using a linear approximation formula.