Assignment 5; Due Friday, October 28

The first midterm will be on Monday, October 31. I will have extensive review sheets next week.

For this assignment, read section seven on compact spaces. Then do the following problems:

- 6.6ad
- (Graduate students only) 6.6l
- 7.13 ab
- (Graduate students only) 7.13cg
- 7.13h
- Find an open cover of $\mathbb{R}$ which does not contain a finite subcover. Repeat this problem for $[0,1)$.
- Consider the three examples which follow. Which of these spaces are compact? If the space is not compact, then find an open cover which does not contain a finite subcover. If the space is compact, prove it.
  1. $Q$, the set of rational numbers, with the topology induced from $Q \subseteq \mathbb{R}$
  2. $S^2 - \{ \text{north pole} \}$
  3. the Klein bottle, obtained as a quotient space of $[0,1]^2$ by the usual gluing operation along the boundary
- (Graduate students only): A topological space $X$ is a Hausdorff space if whenever $x \neq y$ in $X$, we can find open neighborhoods $U$ and $V$ of $x$ and $y$ such that $U \cap V = \emptyset$. If $A$ and $B$ are nonintersecting closed subsets of a compact Hausdorff space $X$, prove that there exist nonintersecting open sets $U$ and $V$ with $A \subseteq U$ and $B \subseteq V$.
- Prove that the open ball $\{ x : \|x\| < 1 \}$ and the closed ball $\{ x : \|x\| \leq 1 \}$ in $\mathbb{R}^n$ are not homeomorphic.