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A quantum dot ratchet: Experiment and theory

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Abstract. – Particles in a ratchet, that is, a potential without spatial inversion symmetry, can move in one direction even in the absence of macroscopic forces, provided that there is a source of energy. In this paper, a quantum ratchet, based on an asymmetric (triangular) quantum dot, is investigated experimentally and theoretically. We find that coherent electron transport through such a device depends on the sign of the applied voltage. In this way a net current can be obtained even when the applied ac voltage is zero on average. Strikingly, the direction of the current depends on the amplitude at which the quantum dot ratchet is rocked.

Particles in an asymmetric potential can on average drift in one direction even when the time and space average of all macroscopic forces or gradients is zero (force-free motion). For directional net motion in such a device, called a ratchet, only a source of energy is required, for instance external, time-correlated fluctuations [1]. This phenomenon, which is believed to be relevant to biological systems [2], has been demonstrated experimentally by subjecting Brownian particles [3] and mercury drops [4] to asymmetric, periodic potentials. While these ratchets relied on classical effects, recently also ratchet devices involving quantum processes have been proposed. In particular, in theoretical studies of an asymmetric SQUID [5], of an incoherent tunneling ratchet [6], and of a Josephson junction close to a source of asymmetric, dichotomic noise [7], quantum ratchet effects were predicted.

Here we present an experimental and theoretical study of a novel quantum ratchet: Using nanolithography techniques, it is today possible to confine electrons in semiconductors in two-dimensional potentials of almost any desired shape, for instance a triangular quantum dot coupled to the environment via point contacts, as shown in fig. 1(a). Transport through such a cavity, also called an electron billiard, is at low temperatures coherent and determined by the coupling of the wave modes in the point contacts to the electron states inside the dot [8,9]. Since the density of states of the dot is nonmonotonic due to energy quantization, and also

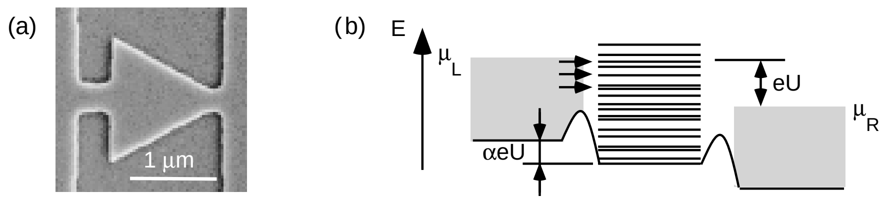

Fig. 1. – (a) Scanning electron micrograph of a triangular quantum dot of the type used in this work. An additional top gate allowed to tune the Fermi energy. (b) Schematic energy diagram for a biased quantum dot. At the point contacts barriers are formed at which a large part of the voltage drop occurs (αeU at the source contact). The density of states inside the dot has a nonmonotonic shell structure due to the energy quantization.

because it depends on the applied field, the response to an applied voltage is nonlinear already at small voltages. We will show in the following, experimentally and theoretically, that these nonlinear effects are in general asymmetric in voltage due to the broken spatial symmetry of the triangular dot [10]. This effect can be used to partially rectify an ac voltage, *i.e.* to generate a net current in one direction even when the bias is zero on average. Strikingly, we find that the direction of the dc current induced in such a quantum-dot ratchet depends on the amplitude of the applied ac voltage.

In the experiments, we used electron billiards defined by electron beam lithography and shallow wet etching in modulation-doped GaAs/AlGaAs two-dimensional electron gas material. In the present letter we will present one particular device shaped as an equilateral triangle (fig. 1(a)). The effective, inner side length of the potential as determined by classical magnetoresistance measurements [11] was about 1.7 μm, much less than the electron mean free path with respect to impurity scattering of about 15 μm. Using a top gate, the Fermi energy (E_{F}, determined in an area outside the billiard) was tunable in the range 7–9 meV, corresponding to a Fermi wavelength (λ_{F}) of 0.05–0.06 μm. Typically, three to four channels were open in the point contacts. All resistance measurements were carried out at $T = 0.3$ K in a current-controlled, four-terminal geometry with an excitation voltage $u_{\mathrm{ac}} < k_{\mathrm{B}}T \approx 25$ μeV.

To study the symmetry of electron transport through the billiard we measured the differential resistance $R = \partial U(I)/\partial I$ as a function of a dc bias current (I) which was added to the ac component used for lock-in detection. In fig. 2(a), we show a number of such measurements, recorded at different values of the Fermi energy, as a function of the bias voltage U. The most important observation is that the nonlinear resistance, which exhibits a rich, nonmonotonic behaviour, depends in general on the direction of the current. A point that should be noted is that the details of the signal depend on the exact value of the Fermi energy. In the following we will discuss these experimental observations.

To understand the details of the nonlinear resistance it is useful to study the variation of the linear-response resistance (no bias voltage applied) with Fermi energy as shown in fig. 2(b). A monotonic background (related to the gradual depletion of the point contacts) has been subtracted from the raw data (shown in the inset) such that only the resistance fluctuations as a function of E_{F} remain. The physical reason for these fluctuations is the nonmonotonic shell structure of the density of states inside the dot, formed by overlapping, quantized electron states [9, 12]. At small voltages (linear response) only states within $k_{\mathrm{B}}T$ around the Fermi energy contribute to the transport [13], and, consequently, the resistance fluctuates as the Fermi energy is tuned [8]. From this linear-response behaviour, one can also understand the basic features of the nonlinear resistance at small voltages. When a bias voltage U is applied, also the states within an energy window of size $e \mid U \mid$ around the Fermi energy contribute to the transport (fig. 1(b)). Consider, for instance, curve D in fig. 2(a), which was recorded at the top gate voltage $V_{\mathrm{tg}} = -3.08$ V ($E_{\mathrm{F}} \approx 8.34$ meV) where the linear-response resistance has a local minimum (point D in fig. 2(b)). In the energy range in the close vicinity of this Fermi energy the linear-response resistance is higher than exactly at this Fermi energy, such that the nonlinear differential resistance initially increases. The opposite behaviour is observed for curve B in fig. 2(a), which was recorded at a value of E_{F}, where the linear-response resistance is close to a local maximum. In this case, $R(U)$ decreases rapidly for low voltages because the vicinity of the Fermi energy yields a lower resistance. This behaviour at low voltages is superposed on an overall trend of $R(U)$ to decrease at higher voltage ($|U| > 1$ meV), which may be related to heating effects [14] or to the point contacts [15].

This first, qualitative analysis indicates that the nonlinear resistance, which contains the asymmetric behaviour, is related to quantum interference inside the dot. This conclusion is also strongly supported by several other observations: Firstly, weak magnetic fields of the order of one flux quantum through the area of the device, enough to alter the interference of different electron paths due to the Aharanov-Bohm phase shift (but too small to change the classical electron trajectories [9, 11]), change the nonsymmetric resistance (NSR) significantly [10]. Secondly, the NSR observed at small voltages ($U < 1$ mV) disappears at temperatures of a few kelvins [16], where variations of the density of states are smeared out and phase coherence is destroyed by electron-electron (e-e) interaction [13]. Further, the asymmetric resistance fluctuations are not observed at applied dc voltages larger than a few mV, where phase breaking occurs by e-e scattering among the nonequilibrium electrons injected into the cavity [14].

Having established the relation between the NSR and resistance fluctuations experimentally, we now turn to a theoretical analysis to understand the origin of the symmetry breaking. Consider an electron billiard connected to two reservoirs at temperature T and with a Fermi-Dirac distribution $f(\varepsilon, T)$ (local thermal equilibrium). The current through the billiard can then be written as

$$I(U) = \frac{e}{h} \int_0^{\infty} \mathrm{d}\varepsilon t(\varepsilon, U)[f(\varepsilon - (\mu_{\mathrm{F}} + eU), T) - f(\varepsilon - \mu_{\mathrm{F}}, T)], \quad (1)$$

where $t(\varepsilon, U)$ is the quantum-mechanical transmission probability for an electron injected at

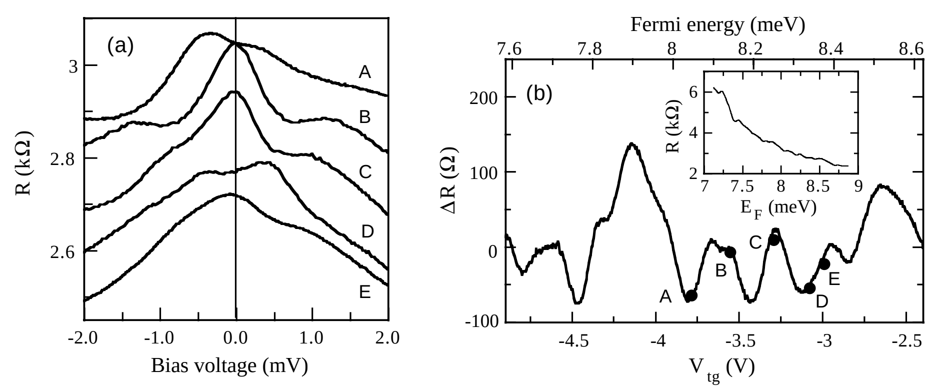

Fig. 2. – (a) Measurements of the differential resistance *vs.* bias voltage recorded at different Fermi energies as indicated by capital letters. In general, the resistance depends on the sign of the voltage. (b) Resistance fluctuations in linear response (zero bias voltage) as a function of the top gate voltage V_{tg}. The letters refer to the values of V_{tg} (corresponding to E_{F}), where the curves shown in (a) where recorded. Inset: Raw data of $R(E_{\mathrm{F}})$ before subtraction of a monotonic background.

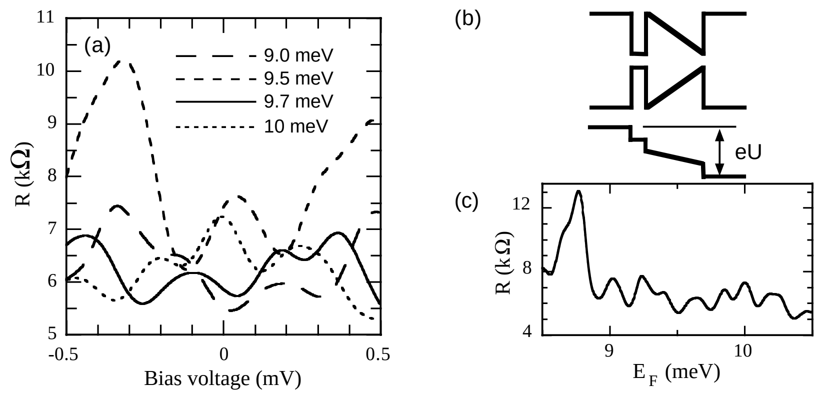

Fig. 3. – (a) Quantum-mechanically calculated nonlinear resistance at different Fermi energies for a triangular dot at 0.3 K. Qualitatively the same asymmetric and nonmonotonic behaviour as in the experimental case (fig. 2(a)) is apparent. (b) The hard-wall potential and the voltage drop distribution used in the calculation. For the quantum-mechanical calculation, the device can be viewed as a waveguide, and a potential drop is equivalent to a probability to wave reflection. Therefore, potential steps are assumed at each discontinuity of the waveguide, and a linear potential slope inside the cavity. (c) Calculated $R(E_\mathrm{F})$ in linear response.

energy ε. The differential resistance is given by $R(U) = [\partial I(U)/\partial U]^{-1}$, which yields, in the limit of very small voltages, the linear-response resistance.

From eq. (1) one can identify two types of nonlinear effects that may, in asymmetric potentials, depend on the sign of the voltage. Firstly, the transmission function depends explicitly on the voltage U, because a finite electric field modifies the potential landscape. When the billiard potential is not mirror symmetric, then the effective scattering potential at finite voltage is not the same upon voltage reversal. Consequently, $t(U)$ depends in general on the sign of the voltage, causing an NSR. The second mechanism is related to the energy dependence $t(\varepsilon)$. The states inside the billiard that contribute to the current are those within an energy window $[E_\mathrm{F} - (1-\alpha)eU; E_\mathrm{F} + \alpha eU]$, where αeU is the voltage drop at the source contact (fig. 1(b)). When the two contacts are not identical, then $\alpha \neq 1/2$ and different states contribute to the current when source and drain contacts are interchanged [17]. Again, this will result in an NSR.

In the following quantum-mechanical calculation of the nonlinear resistance we will use a simple model that considers both these effects. Specifically, we assume that the voltage drop at the base contact is twice as large as that at the tip contact and that a fourth of the voltage drop occurs inside the cavity. The real electric-field distribution may be complicated due to charging effects and has to be determined for the potential as a whole by a self-consistent calculation. For simplicity we assume a constant electric field inside the dot (fig. 3(b)). We then calculate the transmission function $t(\varepsilon, U)$ using a scattering matrix method [18] and determine the differential resistance using eq. (1). For computational reasons, a triangle with side length 1 μm was used, *i.e.* smaller than in the experiment, while the size of the point contacts was chosen to be 0.1 μm, comparable to the real device. For all calculations shown here a temperature $T = 0.3$ K was assumed.

In fig. 3(a) we show the calculated nonlinear resistance as a function of U for different Fermi energies. In fig. 3(c) also the corresponding linear-response resistance $R(E_\mathrm{F})$ is shown. Clearly, the experimentally observed asymmetric and nonmonotonic behaviour of $R(U)$ as well as the strong dependence of $R(U)$ on E_F is qualitatively reproduced by the calculations. We

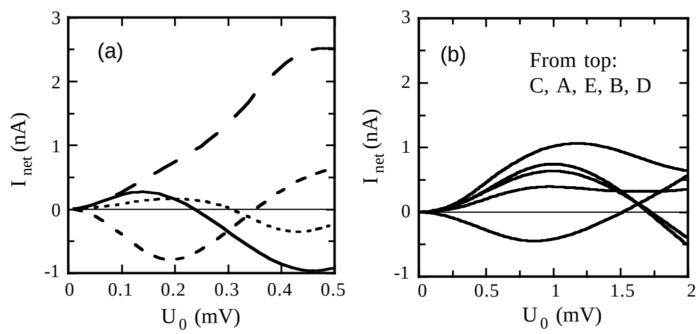

Fig. 4. – Calculated net current as a function of the amplitude of an applied ac voltage based (a) on the theoretical data from fig. 3(a) and, (b) on the experimental data from fig. 2(a). Both the theoretical and the experimental data indicate that the direction of the net current depends on the amplitude at which the dot ratchet is rocked.

have checked that, like in the experiment, the asymmetric fluctuations of $R(U)$ are smeared out when a temperature of a few kelvin is used. Different to the experiment is that the fluctuations of $R(U)$ persist to high voltages (not shown here) while in the experiment their amplitude decreases around $|U| \approx 1$ mV. This difference is presumably due to phase breaking among the injected nonequilibrium electrons in the experiment, which is not included in the calculation.

Since the real potential in the dot is not known, we have also used several potential distributions different to the one used above, and have verified that an NSR is observed in all cases. It is particularly interesting that also a flat potential inside a triangular billiard and symmetric voltage drops at the point contacts yield in calculations a weak NSR, due to the asymmetric confinement energy [16]. This is expected because, according to the considerations above, any breaking of the inversion symmetry of the two-dimensional scattering potential should give rise to a NSR. In contrast, in a rectangular (reflection symmetric) potential the quantum-mechanically calculated nonlinear resistance is found to be perfectly symmetric [16], as is required by the symmetry of the problem.

We have thus found a voltage rectification mechanism related to quantum-interference in asymmetric electron billiards. Similar effects have previously been found in devices in which the symmetry of the scattering potential was broken due to the random distribution of impurities [19]. The novel aspect of the concept used here [10] is that the symmetry of the scattering potential is controlled in the processing of the device, an idea that also was recently employed to study rectification in the classical transport regime [20]. The possibility to control the symmetry of the scattering potential is particularly interesting for ratchet applications.

The key parameter for an application of asymmetric billiards as quantum ratchets is the net current that is generated by applying an, on average, zero ac voltage $U_0 \sin(\omega t)$. The net current is given by

$$I_{\mathrm{net}}(U_0) = \frac{1}{2\pi} \int_0^{2\pi} \mathrm{d}(\omega t)(U_0 \sin(\omega t))U_0 \sin(\omega t)\,, \tag{2}$$

where

$$G(U) = \frac{1}{U} \int_0^U \mathrm{d}U R^{-1}(U) \tag{3}$$

is the conventional conductance, which can be obtained either from the measured data of the differential, nonlinear resistance $R(U)$ (fig. 2(a)) or, theoretically, from eq. (1). In fact, both based on the theoretical data and on the experimental data, one finds that for ac voltages of the order of $U_0 \approx 1$ mV, a dc current of the order of nA can be obtained (figs. 4(a) and (b), respectively). Interestingly, the direction of the net current depends not only on the Fermi energy, as is expected from figs. 2(a) and 3(a), but for some values of E_F also on the excitation amplitude U_0. In other words, asymmetric billiards can be used as rocking quantum ratchets in which the direction of the net current is reversed when the rocking amplitude is varied!

Extensions of the work presented here are to study the net current in the presence of different types of time-correlated noise, instead of the regular ac voltage, and to study electron ratchet effects in a periodic asymmetric potential created, for instance, by coupling many asymmetric billiards in series [21]. The behaviour of such a periodic quantum ratchet will depend on the type of coupling: In the case of incoherent coupling one would expect the nonsymmetric interference effects of the individual cells to average zero, because in the experimental case the cells are different on the scale of the Fermi wavelength. At the opposite extreme of fully coherent coupling, however, the entire device needs to be considered as a whole, and new, interesting behaviour can be expected.

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