MATH 432/532 SPRING 2017
HOMEWORK 1
DUE JANUARY 13, 2017.

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Reminder: homework is due at the *beginning* of class, handed to me or in my mailbox on the second floor of Fenton.

Note. This homework assignment is intended partly to make sure you are ready for the course, and is about a third the length, and somewhat easier, than a typical homework assignment.

Math 532 students. This week only it is okay to turn in your homework neatly written, instead of typed in LaTeX.

`V2: dimensions in 2(b) were swapped.`

Problems:

(1) Let V be the vector space of $n \times n$ real matrices (with operations addition of matrices and multiplication of matrices by real numbers). Let $F\colon V \to V$ be $F(A) = A^2$. Compute $DF(A)$. (*Note:* your answer should be a linear transformation from V to V!)

(2) Define $F\colon \mathbb{R}^6 = \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{R}^3$ to be $F(v, w) = v \times w$, the cross product of v and w.

  (a) Give a succinct description of $DF(v, w)(x, y)$ (where $u, v, x, y \in \mathbb{R}^3$). That is, say what the linear transformation $DF(v, w)$ does to a vector $(x, y) \in \mathbb{R}^3 \times \mathbb{R}^3 = \mathbb{R}^6$.

  (b) Write down the total derivative matrix for $DF(v, w)$. This should be a 3×6 matrix.

Challenge problems (required for Math 532, optional for 432):

(3) Given an $n \times n$ matrix A, let

$$\exp(A) = I + A + \frac{1}{2}A^2 + \frac{1}{6}A^3 + \cdots = \sum_{n=0}^{\infty} \frac{1}{n!}A^n.$$

  (a) Prove that this sum converges for any matrix A (with respect to the metric induced by any norm on the space of $n \times n$ matrices; since all norms are equivalent, choose your favorite).

  (b) Compute $\exp\begin{pmatrix} 2 & 0 \\ 0 & -1 \end{pmatrix}$, $\exp\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$, and $\exp\begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}$.

  (c) Suppose that A and B commute, i.e., $AB = BA$. Show that the directional derivative $D\exp(A)(B) = D_B \exp(A)$ is $\exp(A)B$. (Note: you probably want to avoid computing the total derivative $D\exp(A)$. You may restrict to the case $\|B\| = 1$, if you prefer to only define the directional derivative in the direction of a unit vector.)

  (d) Is the previous statement for general A and B? Justify your answer.

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