MATH 635 WINTER 2024
HOMEWORK 4
DUE FEBRUARY 9, 2024

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Required problems:

(1) Use properties of homology (the long exact sequence for a pair, excision, etc.) to compute the homology of $S^1 \times S^4$ and $S^2 \times S^4$. (Hint: start from a cell structure. The case of $S^2 \times S^4$ is the easier one.)

(2) Describe, as explicitly as you can, the connecting homomorphism in the long exact sequence for a triple.

(3) Hatcher 2.1.26 (p. 133).

(4) Viewing S^n as $\Delta^n/\partial\Delta^n$, the quotient map $\Delta^n \to \Delta^n/\partial\Delta^n$ is an element $\alpha \in C_n(S^n)$. Show that this element is a cycle if n is odd but not if n is even (and positive); but is always a cycle in $C_n(S^n, \{pt\})$.

Alternatively, we can view S^n as the boundary of Δ^{n+1}. Let $v_0, \dots, v_{n+1}$ be the vertices of Δ^{n+1}. Then the sum

$$\beta = \sum_{i=0}^{n+1} (-1)^i [v_0, \dots, \widehat{v_i}, \dots, v_{n+1}]$$

is a cycle in $C_n(S^n)$.

Trace through the computation of $H_n(S^n)$ to show that the cycles α and β represent generators of $\widetilde{H}_n(S^n) \cong H_n(S^n, \{pt\})$.

(5) Compute the singular homology of the Klein bottle, and exhibit circles which generate H_1 of the Klein bottle (and prove they give generators).

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