MATH 635 WINTER 2024
HOMEWORK 5
DUE FEBRUARY 16, 2024

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(This was formerly named “Homework 4”.)

Required problems:

(1) Use the Universal Coefficient Theorems to confirm your computations in parts (b)–(e) in Problem (7) on Homework 1.
(2) Hatcher 3.1.2 (p. 204).
(3) Hatcher 3.1.3 (p. 204).
(4) Let C_* be a bounded-below chain complex of free modules, and D_* any bounded-below chain complex, and $f\colon E_* \to D_*$ a free resolution of D_*. Let $g\colon C_* \to D_*$ be a chain map. Prove there is a lift $h\colon C_* \to E_*$ so that the diagram

$$\begin{array}{ccc} & & E_* \\ & \overset{h}{\nearrow} & \big\downarrow f \ (q.iso) \\ C_* & \xrightarrow{\ g\ } & D_* \end{array}$$

commutes up to homotopy. Prove, moreover, that h is unique up to homotopy.
(5) Hatcher 3.1.9 (p. 205). Use the Universal Coefficient Theorem.
(6) Hatcher 2.2.7 (p. 155)
(7) Let K be a knot in S^3, that is, a smoothly embedded circle. It follows from the implicit function theorem that K has a neighborhood U homeomorphic to $S^1 \times D^2$, so that K is identified with $S^1 \times 0$. Use the Mayer-Vietoris sequence to compute $H_1(S^3 \setminus K)$. (Hint: cover S^3 by U and $S^3 \setminus K$.)
(8) Hatcher 2.2.33 (p. 158)

Optional problems:

(9) The optional homological algebra problems from last week that I didn’t solve in class.
(10) More homological algebra: Hatcher 3.1.1, 3.A.2, 3.A.3, 3.A.4, 3.A.5, 3.A.6.
(11) More degree problems; these make good qual problems: Hatcher 2.2.2, 2.2.3, 2.2.4.
(12) More Mayer-Vietoris practice; these also make qual problems: Hatcher 2.2.28, 2.2.29.
(13) The *solid torus* is the space $S^1 \times D^2$; this is a donut. Use the Mayer-Vietoris sequence to compute the homology of the solid torus.

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