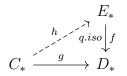
MATH 635 WINTER 2024 HOMEWORK 5 DUE FEBRUARY 16, 2024

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(This was formerly named "Homework 4".)

Required problems:

- Use the Universal Coefficient Theorems to confirm your computations in parts (b)–(e) in Problem (7) on Homework 1.
- (2) Hatcher 3.1.2 (p. 204).
- (3) Hatcher 3.1.3 (p. 204).
- (4) Let C_* be a bounded-below chain complex of free modules, and D_* any boundedbelow chain complex, and $f: E_* \to D_*$ a free resolution of D_* . Let $g: C_* \to D_*$ be a chain map. Prove there is a lift $h: C_* \to E_*$ so that the diagram



commutes up to homotopy. Prove, moreover, that h is unique up to homotopy.

- (5) Hatcher 3.1.9 (p. 205). Use the Universal Coefficient Theorem.
- (6) Hatcher 2.2.7 (p. 155)
- (7) Let K be a knot in S^3 , that is, a smoothly embedded circle. It follows from the implicit function theorem that K has a neighborhood U homeomorphic to $S^1 \times D^2$, so that K is identified with $S^1 \times 0$. Use the Mayer-Vietoris sequence to compute $H_1(S^3 \setminus K)$. (Hint: cover S^3 by U and $S^3 \setminus K$.)
- (8) Hatcher 2.2.33 (p. 158)

Optional problems:

- (9) The optional homological algebra problems from last week that I didn't solve in class.
- (10) More homological algebra: Hatcher 3.1.1, 3.A.2, 3.A.3, 3.A.4, 3.A.5, 3.A.6.
- (11) More degree problems; these make good qual problems: Hatcher 2.2.2, 2.2.3, 2.2.4.
- (12) More Mayer-Vietoris practice; these also make qual problems: Hatcher 2.2.28, 2.2.29.
- (13) The solid torus is the space $S^1 \times D^2$; this is a donut. Use the Mayer-Vietoris sequence to compute the homology of the solid torus.

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