MATH 635 WINTER 2024
HOMEWORK 7
DUE MARCH 4, 2024

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Updated: made two problems optional.

Required problems:

(1) Hatcher 2.2.35 (p. 158).
(2) Hatcher 2.3.1 (p. 165)
(3) Hatcher 3.1.7 (p. 205)
(4) Hatcher 2.C.4 (p. 184) (Convince yourself that the fixed set is a subcomplex of the first barycentric subdivision, but you don't have to write down a proof. If you do write down a proof, you might want to use the following fact: there is one n-simplex in the first barycentric subdivision of Δ^n for each permutation $\sigma \in S_{n+1}$. The n-simplex in the barycentric subdivision of $\Delta^n = \{(x_0, \dots, x_n) \in \mathbb{R}^{n+1} \mid x_i \geq 0,\ x_0 + \cdots + x_n = 1\}$ corresponding to σ is $\{(x_0, \dots, x_n) \mid x_{\sigma(0)} \geq x_{\sigma(1)} \geq \cdots \geq x_{\sigma(n)}\}$. So, to specify a k-simplex in the barycentric subdivision one specifies a partial ordering of the coordinates *and* that k pairs of consecutive elements in the partial ordering are actually equal.)

Optional problems:

(5) Hatcher 2.3.3 (p. 165)
(6) Any matrix $A \in SL_2(\mathbb{Z})$ induces a self-homeomorphism of the torus T^2 as follows. Recall that T^2 is the quotient of $\mathbb{R}^2$ by the action by $\mathbb{Z}^2$. If $v, w \in \mathbb{R}^2$ differ by an element in $\mathbb{Z}^2$ then Av and Aw also differ by an element of $\mathbb{Z}^2$. So, multiplication by A, which is a continuous map $\mathbb{R}^2 \to \mathbb{R}^2$, descends to a continuous map $T^2 \to T^2$.
    Suppose that the map $T^2 \to T^2$ induced by $A \in SL_2(\mathbb{Z})$ is homotopic to a fixed-point free map. Show that the only eigenvalue of A is 1.
(7) Hatcher 2.2.36.
(8) Hatcher 2.C.2, 2.C.3, 2.C.5, 2.C.9.

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