Math 251
Final Exam - Fall 2013

Name:____________________________ **Student ID #:** ____________________________

You will be asked to present your student ID when you turn in your final and may be asked to show your ID at other times during the exam.

Instructions:

- You must <u>show work</u> that justifies your answer to receive credit; your answers will be graded based on whether the work shown is correct. This means credit may be deducted for incorrect work, even if the answer is correct. Also, all notation must be correct.
- <u>No electronic devices</u> may be used, and any such devices must be put away.
  This includes calculators, phones, electronic dictionaries, headphones, iPods, etc.
- Please silence your cell phone.
  If your phone is a distraction to others, you may lose points on your exam.
- <u>Check your work</u> once you have worked through all of the problems.

1	(12)	
2	(12)	
3	(10)	
4	(10)	
5	(12)	
6	(14)	
7	(10)	
8	(12)	
9	(8)	
Total	(100)	

(1) (12 pts) Compute the following limits. In some cases the correct answer may be $\infty, -\infty$ or the limit may not exist.

You must justify your answer: no credit will be given if you just state the limit.

(a) $\displaystyle\lim_{\phi \to 0} \frac{\sec(2\phi)}{\phi}$

(b) $\displaystyle\lim_{x \to \pi/2} \frac{\cos(x)}{\pi/2 - x}$

(c) $\displaystyle\lim_{t \to \infty} \frac{t^3 - 2t^2 - 3t}{2t^3 + \cos t}$

(d) $\displaystyle\lim_{t \to 0^+} t \ln(t)$

(2) (12 pts) Find the derivative of each function.

(a) $f(x) = \tan(\ln(x))$

(b) $g(x) = \sin(u(x) - \pi x)$
Here $u(x)$ satisfies $u'(x) = 3u(x) + \pi$ and your answer may involve the function $u(x)$.

(c) $h(t) = [\ln(2(t-1))]^2$

(d) $A(u) = \sqrt{\dfrac{3u+1}{2u-1}} + \pi^2$

(3) (10 pts) A tub is 10 ft long and its ends have the shape of isosceles triangles that are 3 ft across at the top and have a height of 1 ft. When the water is 6 inches deep, the water is being poured in at a rate of 12ft^3 per minute. How fast is the water level rising when the water is 6 inches deep?

(4) (10 pts) You have just removed a pizza from a hot oven. Let $T(t)$ be the temperature in degrees Fahrenheit of the pizza at time t minutes after removing it from the oven.

(a) Interpret $T(5) = 400$ and $T'(5) = -6$.

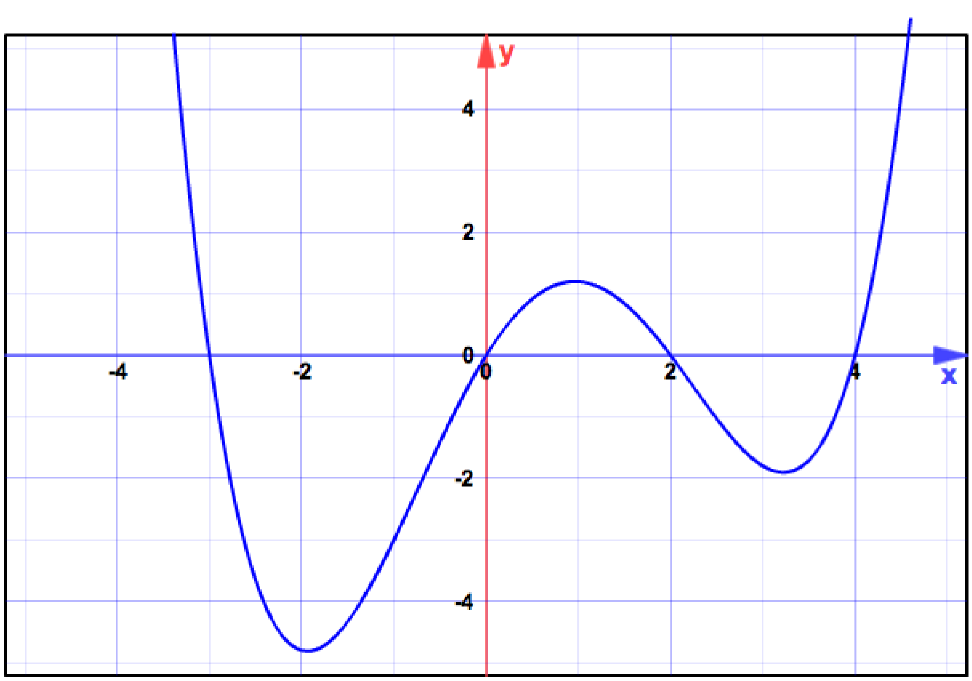
(b) Use the tangent line to $y = T(t)$ at $t = 5$ to estimate $T(10)$.

(c) Given that $T''(t) > 0$, is your estimate from part (b) too high or too low. (Hint: draw a picture.)

(5) (12 pts) Below is the graph of $f'(t)$. Use this to draw a plausible graph of $f(t)$ (on the same set of axes) including the information below. Assume that the domain of f is $(-4, 5)$.

(a) Indicate on which intervals the function $f(t)$ (not $f'(t)$) is increasing and on which intervals it is decreasing. Give the x-values where f has local maxima and minima.

(b) Indicate on which intervals the function $f(t)$ is concave up and on which intervals it is concave down. Give any points of inflection.



(6) (14 pts) A cylindrical container with no top is to be constructed to hold 300 cm^3 of liquid. The material for the bottom costs 3 cents per square cm, and the material for the sides costs 2 cents per square cm. What is the height and radius that minimize the cost of materials for the can? Be sure to verify that your answer actually minimizes the cost.

(7) (10 pts) Consider

$$f(x) = \frac{x^2 - 3x + 6}{x - 2}.$$

Sketch the graph $y = f(x)$ by finding all asymptotes, critical points, local minima and maxima, intervals on which f is increasing and those on which f is decreasing, inflection points, and intervals of concavity up and down.

(8) (12 pts) A sports team plays in a stadium whose maximum capacity is 51,000. With tickets priced at \$10, attendance is $24,000$. For each dollar reduction in ticket prices, attendance goes up 3000.

Revenue is the attendance times the ticket price. What ticket price maximizes revenue? Be sure to verify that your answer actually does maximize revenue.

(9) (8 pts) Suppose $x^2 - 3xy + y^2 = -1$. Find the equation of the tangent line at $(2, 1)$.