Math 251
Final Exam - Spring 2013

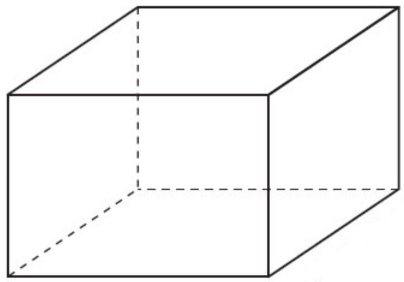
Your name:____________________________ **Student ID #:** ____________________________

Instructions:

- To receive credit, you must neatly and clearly show work that justifies your answer. All notation must be correct.
- No electronic devices may be used. (No calculators, phones, electronic dictionaries, headphones, etc.)
- Please silence your cell phone. If your phone is a distraction to others, you may lose points on your exam.
- Pace yourself. You have approximately 10 minutes per page to complete the exam. The total number of points is 100.
- Check your work once you have worked through all of the problems.

1	(10)	
2	(5)	
3	(5)	
4	(4)	
5	(8)	
6	(8)	
7	(10)	
8	(10)	
9	(8)	
10	(12)	
11	(10)	
12	(10)	
Total	(100)	

(1) (10 pts) Wire with a total length of 300 inches will be used to construct the edges of a rectangular box, and thus provide the frame for the box. The bottom of the box must have a width that is twice the length. Find the maximum volume that such a box can have.



(2) (5 pts) Evaluate each limit, justifying your answers.

(a) $\lim_{x \to 0^+} e^{1/x}$

(b) $\lim_{x \to 0^-} e^{1/x}$

(c) $\lim_{x \to 0} e^{1/x}$

(3) (5 pts) Find the absolute maximum and absolute minimum values of the function

$$f(x) = x^3 - 12x + |x|$$

on the interval $[0, 4]$.

(4) (4 pts) Let $a(t)$ be a function satisfying $a'(t) = 3a(t) - 3$. Let $f(t) = \ln(a(t))$. Give $f'(t)$. You may need to write this in terms of $a(t)$.

(5) (8 pts) The cost to produce x units of a certain product is given by $C(x) = 10,000 + 8x + \frac{1}{16}x^2$. Find the value of x that gives the minimum average cost. Average cost is given by $\frac{C(x)}{x}$. Be sure to justify that the value of x does give a minimum.

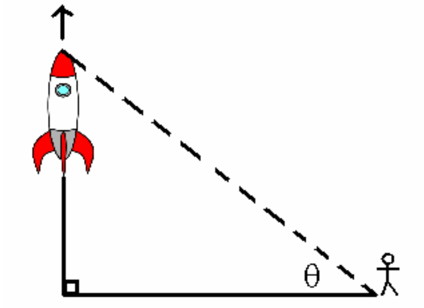
(6) (8 pts) Compute the following limits exactly (no rounding). You must show your work. In some cases the correct answer may be ∞, $-\infty$ or the limit may not exist.

(a) $\displaystyle\lim_{\theta\to 0} \frac{\theta}{\sin(\pi\theta)}$

(b) $\displaystyle\lim_{x\to\infty} xe^{-3x}$

(c) $\displaystyle\lim_{t\to 0} \frac{\cos(t)}{t^2}$

(7) (10 pts) A rocket is launched at time $t = 0$ and after launch it is moving upward at a constant speed. Let $\theta(t)$ denote the angle from a stationary observer up to the base of the rocket after t minutes.



Do the following:

(a) Find $\theta(0)$ and $\lim_{t\to\infty} \theta(t)$.

(b) Determine whether $\theta(t)$ is increasing or decreasing and whether $\theta(t)$ is concave up or concave down.

(c) Use the above information to sketch a graph of $\theta(t)$ for $t \geq 0$.

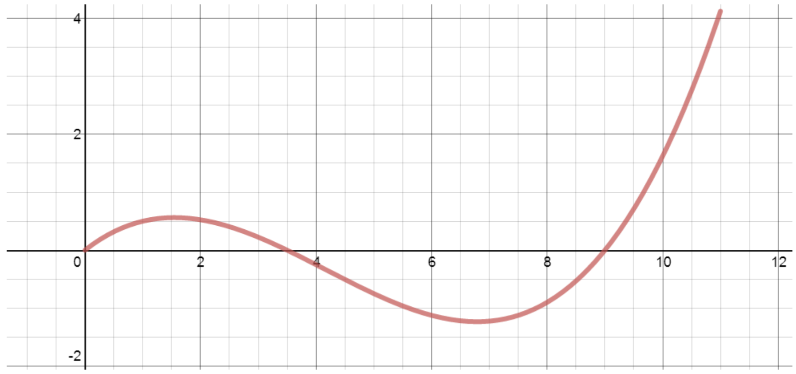
(8) (10 pts) For a certain pain medication, the size of the dose D depends on the weight of the patient W. We can write $D = f(W)$, where D is measured in milligrams and W is measured in pounds.

(a) Interpret $f(150) = 125$ and $f'(150) = 3$ in terms of this pain medication.

(b) Use the tangent line to $y = f(x)$ at $x = 150$ to estimate $f(155)$.

(c) Given that $f''(W) < 0$, is your estimation in (b) too big or too small? Justify your answer. (Hint: draw a picture.)

(9) (8 pts) Below is the graph of the derivative $g'(x)$ on the interval $[0, 11]$



Be careful, the questions below are about $g(x)$, but the graph is for $g'(x)$.

(a) For what intervals is $g(x)$ increasing?

(b) For what x is the graph of $g(x)$ concave down? Give your answer in interval notation.

(c) Give a reasonable sketch for $g(x)$ satisfying $g(0) = 0$. Make sure to label the x-axis.

(10) (12 pts) Find the indicated derivatives. You do not need to simplify.

(a) For $g(t) = 1 + \tan\left(\frac{t-3}{2}\right)$, find $g'(t)$.

(b) For $f(r) = (r^3 - 3r)e^{2r} + e^2$, find $f'(r)$.

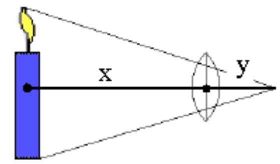
(c) Find $\frac{dy}{dx}$ for $y = \sqrt{\frac{ax+1}{bx+1}}$ and constants a, b.

(d) Find $\frac{dy}{du}$ for $y = \arcsin(u^2)$.

(11) (10 pts) A water tank has the shape of an inverted cone (point down) with base radius 250 cm and height of 900 cm. Initially, the tank is full of water. If the water level is falling at the rate of 7 cm/min, how fast is the tank losing water when the water is 520 cm deep? Include units in your answer.

(12) (10 pts) A candle is placed x cm from a convex lens. If the distance of the focused image is y cm from the lens (see diagram), then x and y are related by the equation

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{L}$$



where the constant L denotes the focal length of the lens.

(a) Find $\dfrac{dy}{dx}$.

(b) Using your answer from (a), justify that $\dfrac{dy}{dx} < 0$.

(c) If the candle is moved farther away from the lens, will the focused image move towards or away from the lens? Justify your answer.