Due Wednesday November 19th.

5.9, 5.16, 5.18, 6.1
And the following

Consider \((0, 1)\) as a 1-manifold. Suppose I define a measure on \((0, 1)\) on an open set \(A \subset (0, 1)\) by

\[
\mu_1(A) = \int_A f(x)\,dx
\]

for a fixed smooth function \(f\) (for now, you can assume that \(A\) is a collection of open intervals - don’t worry about \(\sigma\)-algebras). Now consider a diffeomorphism

\[
F : (0, 1) \to (0, 1)
\]

and define a measure on \((0, 1)\) by

\[
\mu_2(B) = \mu_1 (F^{-1}(B)).
\]

There is a function \(f_2\) (you can assume this fact) such that

\[
d\mu_2(B) = \int_B f_2(x)\,dx.
\]

Show by counterexample that the following statement fails:

\[
f_1 \in L^\infty(dx) \implies f_2 \in L^\infty(dx)
\]