Some International Evidence on the Exogeneity of the Ex-Ante Real Rate of Interest and the Rationality of Expectations*

Litterman and Weiss (1985) report evidence in support of the joint hypothesis that the ex-ante real rate of interest is exogenous and expectations are rational using postwar U.S. data. This paper explores the robustness of their findings across countries, sample periods, data definitions, and to the inclusion of supply shocks. The joint hypothesis of exogeneity of the real interest rate and the rationality of expectations is supported in all cases when supply shocks are omitted from the model. However, when supply shocks are included in the model, the hypothesis does not fare as well. Another implication of their results, that the ex-ante real rate follows a stationary AR(1), is also supported, but there is some sensitivity to the choice of sample period.

1. Introduction

In models in which money matters for real activity, the real interest rate is typically the channel through which money reaches the real sector. Changes in the money supply cause changes in the ex ante real interest rate, which in turn cause changes in real activity. The important role played by the real interest rate in these models, as well as the need for an assumption regarding the evolution of the real rate over time in tests of the market efficiency hypothesis, has led to the search for the causes of movements in the real rate of interest. Fama (1975) argues that over the time period 1953–1971 the data are consistent with the assumption that the ex ante interest rate is a constant. He then tests the hypothesis of market efficiency under the maintained hypothesis of a constant ex ante real interest rate. In later work, Nelson and Schwert (1977), Garbade and Wachtel (1978), Fama and Gibbons (1982), and Mishkin (1984) show that if the sample is extended beyond 1971, the ex...

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ante real rate appears to follow a random walk. Accordingly, Fama and Gibbons' test of the market efficiency hypothesis for the extended sample 1953–1977 is conducted under the maintained hypothesis that the ex ante real rate follows a random walk.

Other researchers, such as Hess and Bicksler (1975), Levi and Makin (1978), Melvin (1982), Hoffman and Schlenkenthal (1982), Wilcox (1983), Makin (1982, 1983), and Mishkin (1984) argue that the real interest rate is determined by its own past, as in random walk models, as well as some additional variables. For example, Mishkin (1984) examines the relationship between the real rate and anticipated inflation in a multi-country study. He finds that anticipated inflation helps to predict real rates in many countries, but not in the United States. Levi and Makin (1978) add inflation uncertainty as a determinant of real rates. Makin (1982) also looks at the role of inflation uncertainty in explaining real rates as well as the role of fiscal policy. Wilcox (1983) shows that supply shocks are important determinants of real rates and resolves some longstanding puzzles concerning coefficient instability in real interest rate equations. Thus, unlike earlier work where the real rate is a constant, or later work where the real rate is assumed to follow a random walk, this line of research argues that the real rate is determined by demand-side factors such as anticipated or unanticipated money, inflation uncertainty, and government deficits as well as supply-side factors, such as the price of oil or other primary inputs to production.

In contrast to this large body of work is the widely cited paper by Litterman and Weiss (1985), who find that the ex ante real interest is exogenous with respect to money, inflation, and real output. In their work, the ex ante real rate is found to depend upon a constant and its own past. They soundly reject the hypotheses that the real rate is a constant and that the real rate follows a random walk. Instead, they find that the real rate follows a stationary AR(1) process. Thus, their work does not support the view that the real rate is a constant or a random walk, nor does it support the view that the real rate depends upon demand-side factors such as money and changes in the inflation rate.

The Litterman and Weiss results are viewed by many economists as evidence in favor of a real business cycle interpretation of aggregate fluctuations. However, the fundamental driving force in real business cycle models, supply shocks, are omitted from their model. This paper explores the robustness of the Litterman and Weiss results to the inclusion of supply shocks. Additionally, this
paper explores the robustness of the Litterman and Weiss results across countries, sample periods, data definitions, and estimation technique. These are all factors which have been shown to affect estimates of real interest rate equations.

The exogeneity of the real interest rate and the rationality of expectations is supported in all cases when supply shocks are omitted from the model. However, when supply shocks are included, the exogeneity and rationality hypothesis is overturned in three of the six countries examined, including the U.S. The failure to reject exogeneity of the ex ante real rate with respect to demand-side factors reported in this paper for Canada, Germany, Italy, Japan, the United Kingdom, and the United States suggests generality of the conclusion that monetary instability is not a central cause of macroeconomic fluctuations, at least in so far as monetary instability operates through the ex ante real interest rate. The results indicate that supply shocks are a more likely cause of macroeconomic fluctuations driven by movements in the real rate of interest.

The Litterman and Weiss finding that the real rate follows a stationary AR(1) is also examined. The results indicate that this conclusion may be due to choice of sample period.

2. The Econometric Methodology

Consider the following five-equation VAR model:

\[ R_t = a_{10} + \sum_{i=1}^{4} (a_{11} R_{t-i} + a_{12} Y_{t-i} + a_{13} M_{t-i} + a_{14} \pi_{t-i} + a_{15} S_{t-i}) + e_{1t}; \]

\[ Y_t = a_{20} + \sum_{i=1}^{4} (a_{21} R_{t-i} + a_{22} Y_{t-i} + a_{23} M_{t-i} + a_{24} \pi_{t-i} + a_{25} S_{t-i}) + e_{2t}; \]

\[ M_t = a_{30} + \sum_{i=1}^{4} (a_{31} R_{t-i} + a_{32} Y_{t-i} + a_{33} M_{t-i} + a_{34} \pi_{t-i} + a_{35} S_{t-i}) + e_{3t}; \]

\[ \pi_t = a_{40} + \sum_{i=1}^{4} (a_{41} R_{t-i} + a_{42} Y_{t-i} + a_{43} M_{t-i} + a_{44} \pi_{t-i} + a_{45} S_{t-i}) + e_{4t}; \]

\[ S_t = a_{50} + \sum_{i=1}^{4} (a_{51} R_{t-i} + a_{52} Y_{t-i} + a_{53} M_{t-i} + a_{54} \pi_{t-i} + a_{55} S_{t-i}) + e_{5t}; \]

where

\[ R_t = r_t + E_t \pi_{t+1} \] is the nominal interest rate,
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\( r_t = \) the ex ante real rate of interest,
\( Y_t = \) the change in the log of real output,
\( M_t = \) the deviation of the change in the log of the nominal money stock from trend, as in Stock and Watson (1989),
\( \pi_t = \) the inflation rate,
\( S_t = \) a supply shock.

The inclusion of the variables \( R_t, Y_t, M_t, \) and \( \pi_t \) follows Litterman and Weiss (1985), except that output and money are measured in growth rates rather than levels.\(^1\) Supply shocks are proxied by changes in the world supply of materials, measured as the ratio of the implicit price deflator for imports to the GNP deflator multiplied by the effective exchange rate. See Wilcox (1983) for details.

The hypothesis that the ex ante real rate is exogenous, or Granger-causally prior, is expressed as

\[
    r_t = b_0 + b_1 r_{t-1} + u_t. \tag{2}
\]

The assumption that \( r_t \) follows an AR(1) process, as opposed to a more general AR(p) process, follows Litterman and Weiss.

Adding and subtracting equivalent terms to both sides of Equation (2) produces

\[
    R_t = b_0 + b_1 R_{t-1} + E_t \pi_{t+1} - b_1 E_t \pi_t + u_t, \tag{3}
\]

where \( E_t \pi_{t+1} \) is the mathematical expectation of \( \pi_{t+1} \) conditional on time \( t \) information. Equation (3), along with the assumption that market participants form rational forecasts of future inflation rates, imposes testable restrictions on the five-variable VAR model given by Equation (1). Since there are 21 free parameters in the expression for \( R_t \) given in Equation (1) and only 2 free parameters in Equation (3), the hypothesis that the ex ante real rate of interest follows an AR(1) process imposes 19 testable restrictions on the (unrestriced) VAR model given by (1).\(^2\)

\(^1\)The use of deviations of the money growth from trend and output growth follows Stock and Watson (1989). Levels of \( R_t \) and \( \pi_t \) are used because specifying these variables as growth rates forces a unit root into the ex ante real interest rate equation. This would be inconsistent with the evidence reported below. Additionally, it is the omission of the trend term in the money growth rate which is responsible for the bias in the F-statistics in previous work reported by Stock and Watson, a problem not present here.

\(^2\)A step-by-step explanation of how real rates are constructed is available upon request.
The test of the restrictions is conducted as follows. First, the restricted version of the model is estimated using iterative nonlinear generalized least squares and the sum of squared residuals from the restricted model (SSR_r) is obtained. Next, the unrestricted model is estimated (weighted by the variance-covariance matrix from the constrained model) to obtain the unrestricted sum of squared residuals (SSR_u). The test statistic is then

$$2T[\ln SSR_r - \ln SSR_u],$$

which is distributed Chi-squared with degrees of freedom equal to the number of testable restrictions (19 in the five-equation model used here). The resulting test statistics are asymptotically equivalent to maximum likelihood.

3. The Data

Litterman and Weiss conduct their tests using quarterly U.S. data for the period 1949–1983. Neither their data definitions nor sample period can be matched for Canada, Germany, Italy, Japan, and the United Kingdom, the OECD countries used in this study. For this reason, estimates are reported for several alternative sample periods and data definitions. The estimates labeled A in the tables below are for the Litterman and Weiss sample period, 1949:ii to 1983:ii. The estimates labeled B are for the shorter period 1955:ii to 1983:ii, dictated by data availability for the OECD countries. Two additional sample periods are shown in the tables. Sample period C is 1949:ii to 1983:ii with 50:ii to 51:ii removed, and sample period D is 1949:ii to 1983:ii with 50:ii to 51:ii and 79:iv to 82:i.

3The SSRs are defined as the trace of the variance-covariance matrix. In the actual estimation conducted in the next section, the estimates were iterated until the change in the variance-covariance matrix was less than 0.001. For further details see Mishkin (1983).

4Weighting the unconstrained estimates by the variance-covariance matrix from the constrained model imposes a desirable degrees of freedom correction. For details see Mishkin (1983).

5Litterman and Weiss obtain their test statistics from direct maximization of the likelihood function, which is equivalent to minimizing the determinant of the variance-covariance matrix. It is shown below that the asymptotic equivalent to maximum likelihood used here produces nearly identical results.

6The simultaneous equation estimation technique used here avoids the problems associated with the use of generated regressors present in two-step estimators. See Mishkin (1983), Fagan (1984), and Hoffman, Low, and Schlagenauf (1985).
removed. The period 50:ii–51:ii is the preaccom period, and 79:ii–82:ii is the period of the recent shift to the nonborrowed reserve operating procedure. Omitting these subperiods excludes episodes where the conduct of monetary policy has changed and different interactions of the key variables may be present. The sample periods C and D, along with the corresponding estimates and test statistics, are taken from Litterman and Weiss for purposes of comparison. Turning to the data definitions, the label LW indicates data definitions identical to those used by Litterman and Weiss for the United States. The label OECD, which is attached to all of the OECD countries, indicates the use of quarterly data from the International Financial Statistics Yearbook and the OECD Financial Statistics Yearbooks: 1955–1971 and 1963–1983. The two sets of definitions differ most significantly for measures of aggregate price and real activity. Litterman and Weiss use the CPI less shelter and a measure of industrial production, while the measures for the OECD countries are the CPI with shelter included and a measure of real GDP.

4. The Results

The test statistics for the joint hypothesis of exogeneity and rationality are presented in Table 1. This hypothesis is not rejected for any of the countries, sample periods, or data definitions at the 95% level of significance when supply shocks are omitted from the model. These results strongly support the Litterman and Weiss conclusion that the ex ante real rate of interest is exogenous.

When supply shocks are included in the model, the conclusion that the ex ante real rate is exogenous does not hold with the same generality. In three countries, Canada, Italy, and the United States, the hypothesis is rejected. In the other three countries, Germany, Japan, and the United Kingdom, the addition of supply shocks has very little effect on the test statistics and the hypothesis is not rejected. These results give some credence to the real business cycle interpretation of the Litterman and Weiss results for the U.S. The estimation technique employed in this paper differs from that used by Litterman and Weiss. The results in rows 1 through 8 in Table 1 are obtained using the asymptotic equivalent to maximum likelihood described above, while the results shown in rows 9 through 11 are reproduced from Litterman and Weiss where direct maximum likelihood estimation is used. The estimates shown
TABLE 1. Joint Tests of the Exogeneity of the Ex Ante Real Rate of Interest and the Rationality of Expectations

<table>
<thead>
<tr>
<th>Country</th>
<th>Sample Period</th>
<th>Data</th>
<th>Without Supply</th>
<th>With Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>$X^2(15)^c$</td>
<td>$X^2(19)^d$</td>
</tr>
<tr>
<td>1. Canada</td>
<td>55:i–83:iv</td>
<td>OECD</td>
<td>24.41</td>
<td>32.08</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.06)</td>
<td>(0.03)</td>
</tr>
<tr>
<td>2. Germany</td>
<td>55:i–83:iv</td>
<td>OECD</td>
<td>11.78</td>
<td>12.08</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.70)</td>
<td>(0.88)</td>
</tr>
<tr>
<td>3. Italy</td>
<td>55:i–83:iv</td>
<td>OECD</td>
<td>8.49</td>
<td>45.16</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.90)</td>
<td>(0.00)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.85)</td>
<td>(0.93)</td>
</tr>
<tr>
<td>5. United Kingdom</td>
<td>55:i–83:iv</td>
<td>OECD</td>
<td>23.82</td>
<td>24.70</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.07)</td>
<td>(0.17)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.58)</td>
<td>(0.01)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.92)</td>
<td>(0.03)</td>
</tr>
<tr>
<td>8. United States</td>
<td>49:i–83:ii</td>
<td>CITI</td>
<td>10.36</td>
<td>35.44</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.80)</td>
<td>(0.01)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.86)</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.12)</td>
<td>—</td>
</tr>
<tr>
<td>11. United States</td>
<td>49:i–83:ii</td>
<td>CITI</td>
<td>17.65</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.28)</td>
<td>—</td>
</tr>
</tbody>
</table>

NOTES: Marginal significance levels appear in parentheses. Lines 9–11 are reprinted from Litterman and Weiss (1985).

*The sample periods in lines 10 and 11 are 49:i–83:ii, with 50:i–51:ii and 79:i–82:i removed, respectively.

**The data is described in the text.

***The critical values at the 5% and 10% levels are 25.00 and 22.31.

****The critical values at the 5% and 10% levels are 30.14 and 27.21.

in rows 8 and 9 for the model without supply shocks provide a direct comparison of the two methods. The test statistics corresponding to rows 8 and 9 are very similar and yield identical outcomes for the joint hypothesis of exogeneity and rationality. Thus, in this case, the two estimation techniques produce very similar results.
Table 2 presents estimates of the ex ante real interest rate equation for the same data definitions and sample periods appearing in Table 1. The coefficient on the lagged value of the ex ante real interest rate, $b_1$, is significant at the 95% level for every country, sample period, and data definition, and in both versions of the model. Additionally, in the model with supply shocks included, the estimates are larger in every case except Canada. The constant term is significant at the 95% level in Germany and in the United States when supply shocks are excluded. When supply shocks are included, the constant term is insignificant in the United States as well. Only Germany exhibits a significant constant term. Furthermore, when supply shocks are included, the estimates of the constant term are closer to zero in every case except the United Kingdom. Thus the results presented in Table 2 are generally supportive of the Litterman and Weiss result that the ex ante real rate depends upon its own immediate past, but not a constant term.

The Litterman and Weiss conclusion concerning the stationary AR(1) specification for the ex ante real rate fared well across countries, but there is some variation in the estimated value of $b_1$ across countries and across sample periods in the United States. The cross-country results are shown in rows 1 through 6. The coefficient on the lagged value of the ex ante real rate varies from 0.600 for the United Kingdom to a value of 0.905 for the United States when supply shocks are omitted and from 0.615 to 0.923 when supply shocks are included. In every case, a standard $t$-test rejects the hypothesis of a random walk. Therefore, neither of the standard assumptions used in tests of market efficiency, that the real rate is a constant and that the real rate is a random walk, is supported.

The results for the United States are shown in rows 5 through 9. The results indicate that the coefficient on the lagged real rate differs according to the time period examined. For the longer time

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7Sample periods C and D are listed in Table 1 but not in Table 2. This is because Litterman and Weiss do not report the estimates of the real interest equations in these cases; they only present the outcomes of the exogeneity tests.

8Dickey-Fuller tests performed on the estimated real interest rate series result in the same conclusions as the t-tests reported in the text. However, this evidence is only suggestive. To my knowledge, no formal test of the unit root hypothesis exists in this case. This is because no adequate distribution theory in the spirit of Dickey-Fuller tests exists for the case of systems of equations with non-linear cross-equation parameter restrictions. Even if such a test did exist, it would likely suffer from the lack of power that plagues standard unit root tests. See Nelson and Plosser (1982) and Blough (1990).
TABLE 2. Restricted Estimates of the Real Interest Rate Equations \( r_t = b_0 + b_1 r_{t-1} + u_t \)

<table>
<thead>
<tr>
<th>Country</th>
<th>Sample Period</th>
<th>Data*</th>
<th>Without Supply</th>
<th>With Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>( b_0 )</td>
<td>( b_1 )</td>
</tr>
<tr>
<td>1. Canada</td>
<td>55:i–83:iv</td>
<td>OECD</td>
<td>0.199</td>
<td>0.850</td>
</tr>
<tr>
<td>2. Germany</td>
<td>55:i–83:iv</td>
<td>OECD</td>
<td>0.396</td>
<td>0.829</td>
</tr>
<tr>
<td>3. Italy</td>
<td>55:i–83:iv</td>
<td>OECD</td>
<td>-0.098</td>
<td>0.838</td>
</tr>
<tr>
<td>4. Japan</td>
<td>55:i–83:iv</td>
<td>OECD</td>
<td>0.033</td>
<td>0.831</td>
</tr>
<tr>
<td>5. United Kingdom</td>
<td>55:i–83:iv</td>
<td>OECD</td>
<td>0.019</td>
<td>0.800</td>
</tr>
<tr>
<td>6. United States</td>
<td>55:i–83:iv</td>
<td>OECD</td>
<td>0.138</td>
<td>0.905</td>
</tr>
<tr>
<td>7. United States</td>
<td>55:i–83:iv</td>
<td>CITI</td>
<td>0.144</td>
<td>0.898</td>
</tr>
<tr>
<td>8. United States</td>
<td>49:i–83:ii</td>
<td>CITI</td>
<td>0.170</td>
<td>0.724</td>
</tr>
<tr>
<td>9. United States</td>
<td>49:i–83:ii</td>
<td>CITI</td>
<td>0.156</td>
<td>0.760</td>
</tr>
</tbody>
</table>


*The data is described in the text.
period A, the estimates of the coefficient are 0.724 and 0.760 when supply shocks are omitted, and 0.772 when supply shocks are included. For the shorter time period B, the estimates are 0.898 and 0.905 without supply shocks and 0.914 and 0.923 when they are included, estimates which are not immune to the suspicion that the ex ante real rate follows a random walk, even though a standard t-test refutes this. Therefore, the conclusion that the ex ante real interest rate follows a stationary AR(1) process is generally supported, but the choice of sample period does seem to matter with respect to the strength of the conclusion.  

Finally, estimation technique appears to make little difference in the estimates of the ex ante real interest rate equation. Comparison of rows 8 and 9 for the model without supply shocks reveals that the estimates derived from direct maximization of the likelihood function shown in Equation 9 are very similar to those obtained from the asymptotic equivalent used here. There does appear to be some difference in the estimates of the intercept terms and their standard errors, but the difference is fairly small. The slope coefficients and their standard errors are nearly identical.

5. Conclusions
This paper examines the joint hypotheses of the exogeneity of the ex ante real interest rate and the rationality of expectations across six countries for equivalent data definitions and sample periods and within the United States for various sample periods and data definitions. The results support the Litterman and Weiss finding that the ex ante real rate is exogenous when supply shocks are omitted from the model, and this result is robust to variations in the sample period, data definition, and country used to carry out the tests. However, when supply shocks are included, the exogeneity and rationality hypothesis is rejected in three of the six countries in the study, including the United States. Thus, the results support the conclusion that demand-side factors such as money growth do not

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9Results for the sample period 1964:4-1983:4 were also estimated for all six countries using the OECD data in the model without supply shocks. The outcomes of the joint hypothesis of exogeneity and rationality were the same as in the first column of lines 1-6 in Table 1; the test is not rejected for any country. The estimated real interest equations were also similar to those shown in the first column of lines 1-6 in Table 2. These estimates, omitted for brevity, are available upon request.
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were also estimated for all six del without supply shocks. The outcomes rationality were the same as in the first is not rejected for any country. The estimates shown in the first column omitted for brevity, are available upon cause the real interest rate to vary, but supply-side factors do affect the real rate in half the countries examined.

In models where money affects the real sector, the real interest rate is typically the channel through which money affects the real sector. In real business cycle models, the real interest rate changes in response to variations in the demand for capital brought about by supply shocks. The exogeneity of the real rate of interest with respect to money reported in this paper suggests that monetary instability does not operate through the real interest rate channel. The rejection of the exogeneity of the real rate in half the countries when supply shocks are included in the model suggests that fluctuations in the real interest rate are more likely the result of supply shocks.

The results presented here are supportive of the stationary AR(1) specification for the ex ante real rate estimated by Litterman and Weiss. Litterman and Weiss (1985, 152) state that the hypotheses that “the Markov parameter equals zero and one, respectively—are soundly rejected on all samples,” implying that the Fama and Gibbons hypothesis that the real rate is a random walk used in their tests of market efficiency is incorrect. The results obtained here also support this conclusion, though in the results for the United States, the Markov parameter is above 0.90 in the results using the shorter sample period. The results of this paper suggest that tests of the market efficiency hypothesis should allow for the possibility of a stationary representation of the real rate instead of the random walk model typically assumed, and also include supply shocks as an explanatory variable.

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References


An examination of the impulse response functions and the variance decompositions for the ex ante real interest rate (see Litterman and Weiss 1985, 142-45) for both the model without supply shocks and the model with supply shocks confirms that demand-side factors do not affect real rates in either model, and that supply shocks are responsible for the rejection of exogeneity.
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