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Structural change and lag length in VAR models

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Abstract

This paper investigates the relationship between changes in policy rules and changes in the estimated lag length in empirical models. The paper finds that there is a close association between changes in the parameter on the output gap term in the monetary policy rule investigated here and changes in estimated lag length for an associated VAR model.

JEL classification: E52

Keywords: Lag length; Policy changes; Structural change

1. Introduction

To what extent do the dynamics of empirical macroeconomic models change when there is a change in monetary policy? This paper examines this question by looking at one aspect of macroeconomic dynamics, lag length in empirical models, and its relationship to changes in the parameters of a monetary policy rule for the federal funds rate.

Motivation for looking for changes in dynamics comes from the large literature on changes in the monetary policy rule over time. Theoretical and empirical investigations of shifts in monetary policy rules have a long history, but the recent literature is motivated largely by the Clarida et al. (2000) finding that policy changed from values consistent with indeterminacy in the 1970s to policy consistent with determinacy in the 1980s.

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This generated considerable debate concerning whether policy prior to the 1980s actually led to indeterminacy and whether structural breaks in policy parameters have occurred at all. Papers such as Clarida et al. (2000), Cogley and Sargent (2002), Boivin and Giannoni (2003) and Boivin (2004) support the idea that there have been important shifts in monetary policy parameters. Some papers, such as Sims and Zha (2004), argue that allowing for changes in the variance of structural disturbances over time can potentially overturn the result that there have been monetary policy shifts, but they also allow for parameter shifts over time and find support for such changes.[1] Thus, though work in this area is still developing, strong empirical evidence for changes in the parameters of the monetary policy rule over time exists, and theoretical work shows that such changes can have important effects on how variables in the model respond dynamically to shocks.[2]

There are several reasons to expect that a change in policy can affect estimated lag lengths in empirical VAR models: (i) policy parameters consistent with indeterminacy for some time periods, (ii) changes in policy that change the number of lags in the policy equation, (iii) agent's use of learning equations can bring variable numbers of lagged values into the solution causing estimated lag lengths to change, (iv) changes in the policy parameters causing structural breaks that, if unaccounted for in the econometric model, can cause changes in the estimated lags, and (v) changes in the parameters of the money rule causing changes in the degree of real rigidity and changes in the amount of persistence displayed in response to shocks. Finally, there are also reasons to think causality might run in the other direction, that is, that (vi) changes in the dynamic structure of the economy can bring about an altered policy response.

2. Empirical model

To estimate the monetary policy reaction function described below, data are needed on the federal funds rate, inflation, and the output gap. Quarterly data for the federal funds rate, the inflation rate, and the log of GDP from 1960 through the second quarter of 2006 are used in the estimation.[3] To estimate the output gap, estimates of real potential output from the Congressional Budget Office's are used.

The next step is to use rolling regressions to obtain two data series, one containing changes in the values of the policy parameters over time, and the other containing changes in the estimated optimal lag length according to likelihood ratio (LR) tests.[4] To obtain changes in the parameter values in the policy equation, the following regression, a Taylor rule augmented by interest rate smoothing, is estimated using rolling window regression

$$i_t = b_0 + b_1 i_{t-1} + b_\pi \pi_t + b_{\tilde{y}} \tilde{y}_t + w_t \tag{1}$$

[1] Sims and Zha (2004) discuss recent empirical work in this area and note that Bernanke and Mihov (1998) conclude there is little evidence for major shifts in policy regimes over time.

[2] Several papers examine the role of lag structure in assessments of monetary policy and how the effects of monetary policy change over time. For example, Kim and McMillin (2003) show that lag structure is important in assessing monetary policy effects, and Swanson (1998) assesses how the relationship between money and output changes over time using rolling window regression. Lee (1997) uses rolling window regression combined with optimal lag length selection to examine the dynamic relationship between money and income over both subsamples and lag specifications.

[3] The data are obtained from the FRED II data set available on the St Louis Fed web site.

[4] The use of a rolling window and the optimal lag selection resemble the techniques used in Lee (1997).

In this equation, the right-hand side variables are inflation, the output gap, and lags of the federal funds rate.[5] The window is assumed to be 60 observations, i.e. 15 years. The parameter values at each step are saved and indexed by the last observation in the window of data used in the estimation. This gives a series of parameter values, one for each parameter in Eq. (1).

The next step is to obtain changes in the estimated lag length for the associated VAR model involving inflation, output, and the federal funds rate. To obtain the optimal lag length for each sample period, again use rolling regressions on a VAR model containing the three variables output (not the output gap since it is much more common for output to be used in VAR models), inflation, and the federal funds rate.[6] For each window of 60 observations, estimate the lag length using the LR test, and save the result. The estimated lag lengths are indexed by the last time period in the window. This gives a time-series of values for the optimal lag length that can be matched to the changes in the parameters of the Taylor rule.

3. Results

Fig. 1a–d plot the coefficients of the Taylor rule along with two standard deviation confidence bounds.

These parameters exhibit notable variation through time, but there is a theoretical reason for using a transformation of the estimated parameters. In theoretical models such as Gertler, and Clarida et al. (1999) and in Honkapohja and Mitra (2003), the policy reaction function for the quarterly case is

$$i_t = (1 - b_1)(b_0 + b_\pi \pi_t + b_{\tilde{y}} \tilde{y}_t) + b_1 i_{t-1} + w_t. \tag{2}$$

This is a standard Taylor rule augmented by a lagged interest rate term to capture interest rate smoothing. It is assumed that $b_1 \leqslant 1$ and that $b_\pi \geqslant 1$. In this specification, the federal funds rate is increased when inflation or output rises above target with the strength of the response depending upon the values of the parameters of the policy rule.

The parameters estimated using Eq. (1) must be transformed to match those from Eq. (2).[7] The transformations are

$$\hat{b}_0^* = \hat{b}_0/(1 - \hat{b}_1) \tag{3}$$

$$\hat{b}_\pi^* = \hat{b}_\pi/(1 - \hat{b}_1) \tag{4}$$

$$\hat{b}_y^* = \hat{b}_y/(1 - \hat{b}_1). \tag{5}$$

These parameters, along with the estimated lag lengths for comparison, are shown in Fig. 2a–c.

The last graph (Fig. 2c) showing the relationship between the coefficient on the output gap in the Taylor rule and the estimated lag lengths from a three variable output, inflation,

[5] Other versions of the policy equations that use the lagged rather than current values of the right-hand side variables were also investigated.

[6] This is a standard baseline model in the empirical monetary policy and income causality literature. For example, see the discussion on page 20 of the recent paper by Bernanke et al. (2004).

[7] This simply corrects expected values for the AR(1) term on the right-hand side.

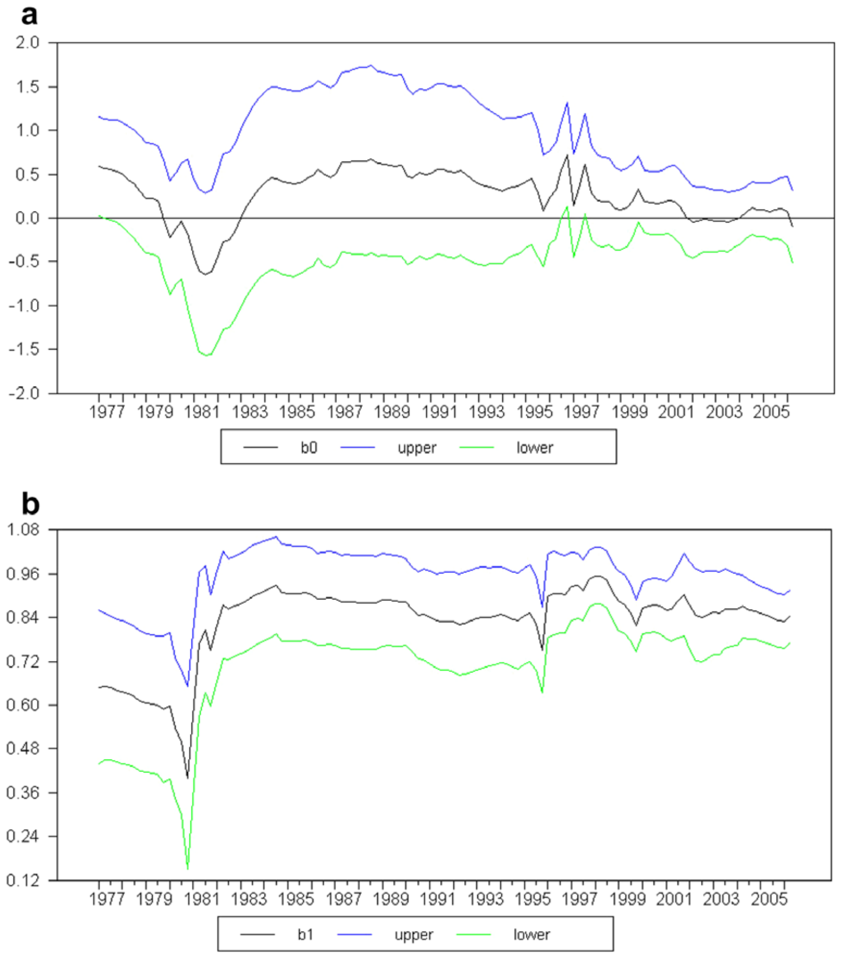

Fig. 1. (a) Taylor rule constant over time, (b) Taylor rule AR1 coefficient over time, (c) Taylor rule inflation coefficient over time and (d) Taylor rule output coefficient over time.

and federal funds rate VAR shows the clearest association.[8] If these two series are smoothed with a rectangular window seven quarters wide to remove high frequency variation, the association is more evident as shown in Fig. 3.

There are several results to note from the figures. First, with respect to the coefficient on the inflation term, theory implies that indeterminacy will arise if the coefficient is too small, and in many models, a coefficient value less than one defines too small. These results show that the coefficient was below 1.0 in the late 1970, consistent with Clarida et al. (2000). After 1980, the value of the coefficient generally stays above 1.0, but not uniformly so. In particular, the coefficient dips into the indeterminacy range in the mid and late

[8] If the level of output in the VAR model is replaced by the output gap so that the definition of output is identical in the monetary policy rule and the VAR model, the results are very similar to those shown in the graph, i.e. the estimated lag lengths in the VAR model are very similar.

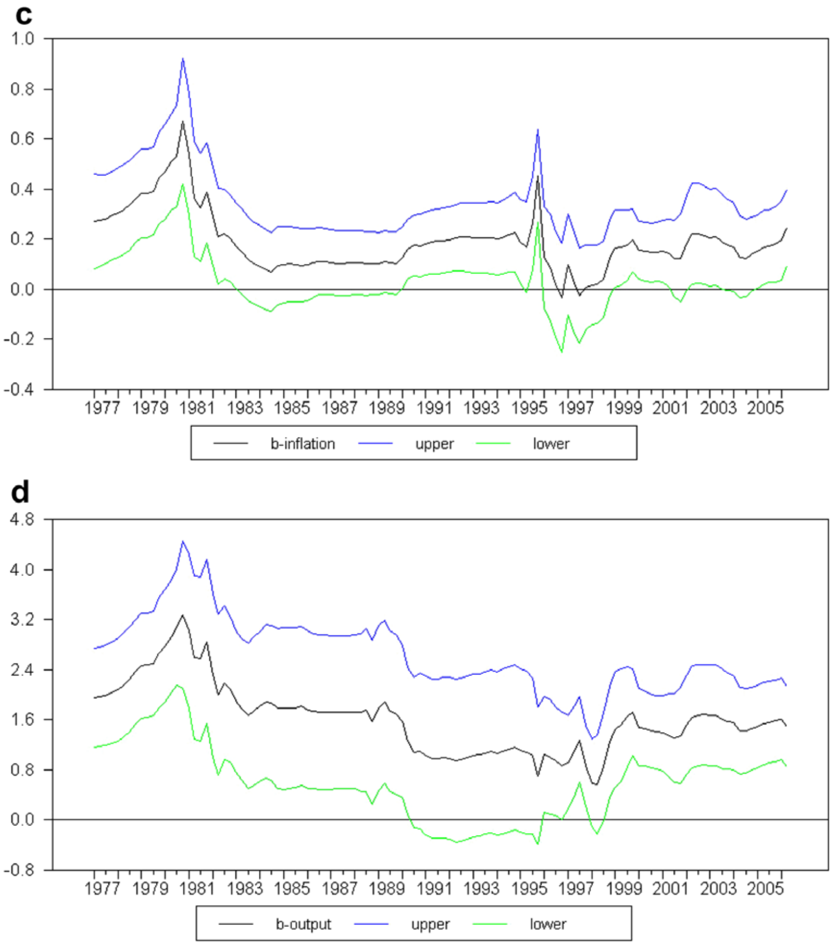

Fig. 1 (*continued*)

1980s, for a period in the mid 1990s, and for a brief period around 2004. However, recently the coefficient has been increasing reflecting, perhaps, increasing inflation vigilance.

Second, the constant term also exhibits variation through time, and the movements occur at points in time that are similar to those where the inflation coefficient moves. The third result is the close association between the estimated lag lengths and the coefficient on the output gap. The estimated lag length is initially four, falls quickly to three, rises to five during the 1980s, and then falls back to three thereafter (though it does return to four for brief periods in the early 1990s). Thus, the 1980s are a time period when the estimated lag length for the VAR is longer than for other periods, and it corresponds to a time period when the coefficient on the output gap in the Taylor rule is uncharacteristically large.

4. Discussion of the empirical results

Are changes in policy responsible for changes in the estimated dynamic structure of the VAR model? As noted above, reasons to expect that a change in policy can affect esti-

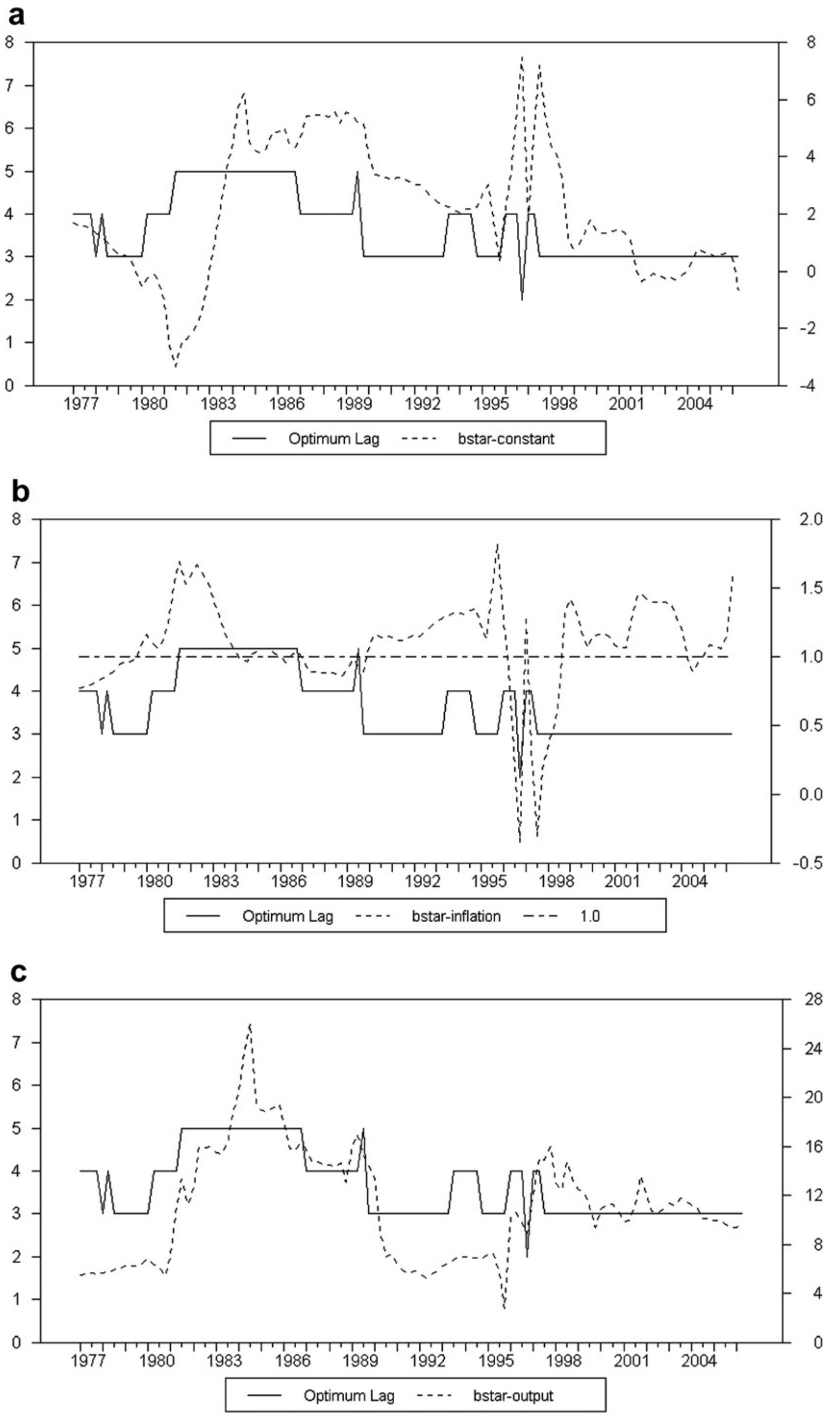

Fig. 2. (a) Optimum lag lenght and the constant term, (b) optimum lag length and the inflation coefficient and (c) optimum lag lenght and the gap coefficient.

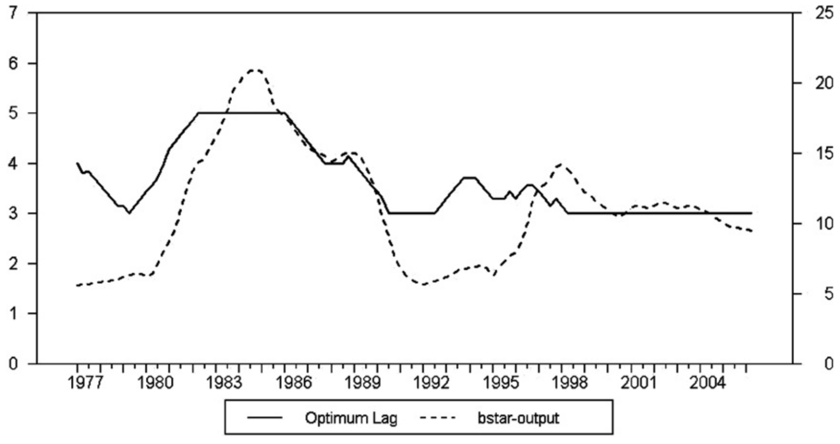

Fig. 3. Smoothed optimum lag length and smoothed gap coeffiecient.

mated lag lengths in empirical VAR models include: (i) policy parameters consistent with indeterminacy for some time periods, (ii) changes in policy that change the number of lags in the policy equation, (iii) agent's use of learning equations can bring variable numbers of lagged values into the solution causing estimated lag lengths to change, (iv) changes in the policy parameters causing structural breaks that, if unaccounted for in the econometric model, can cause changes in the estimated lags, and (v) changes in the parameters of the money rule causing changes in the degree of real rigidity and changes in the amount of persistence after shocks hit the economy. Finally, there are also reasons to think causality runs in the other direction, that is, that (vi) changes in the dynamic structure of the economy can bring about an altered policy response.

Taking these in turn, the first reason lag lengths might vary with changes in policy is that policy might be consistent with indeterminacy during some time periods. It can be shown that the solution under indeterminacy can contain more lags than the solution for parameter ranges with a determinate solution so that variation between these parameter ranges can cause variation in both estimated and theoretical lag lengths. In addition, there may be multiple solutions with different persistence characteristics leading to different estimated lag lengths over time.[9] However, since there is no clear association between changes in the estimated lag lengths and the time periods when the inflation coefficient falls below one in the figure above, it does not appear that indeterminacy is the source of the estimated variation in the estimated lag length.

The second reason is changes in the number of lags in the policy equation causing changes in the persistence of output, inflation, and the interest rate in the estimated VAR model. However, there is little evidence that the number of lags involved in the policy equation has changed substantially. For example, one lag of the interest rate is sufficient to remove serial correlation in the policy equation throughout the sample.

The third reason that estimated lag lengths might vary with changes in policy parameters arises in learning models. In learning models, parameter values are estimated by

[9] The paper by Sims and Zha (2004) discusses this issue and argues that allowing for stochastic volatility and relaxing very strong identifying restrictions overturns the Clarida et al. (2000) indeterminacy result.

economic agents using linear models and a particular window of data, i.e. a particular gain.[10] These regressions bring lagged values of variables into the solutions. In models such as Evans and Ramey (2003) the gain varies through time according to the frequency of policy changes. For example, in the learning regressions, frequent changes in policy parameters can cause agents to place more weight on more recent data, and less weight on data that is more distant thus changing the gain. As the gain changes, the number of lagged variables in the learning regressions changes which can bring about a change in estimated lag lengths. Again, however, this seems to be an unlikely explanation for the results since the lag length changes appear to be associated with the magnitude of the parameter, not the frequency with which it changes.

Fourth, changes in the policy parameters themselves may be responsible for changes in the estimated number of lags. A simulation is used to examine this possibility. In the simulation, artificial data are generated from an AR(3) with lag coefficients of .5, .25, and .125. The constant term captures the structural change imposed on the simulation and is equal to 1.0 for the first 500 observations, and 2.0 for the second 500 observations.[11] The error term is drawn from a unit normal distribution.

The graph in Fig. 4 shows the estimated lag length at each date determined by mimicking the procedure above, i.e. by rolling a window of 60 observations through the data and estimating the lag length for each sample window. As is evident, in periods when both parts of the sample are in the data set, the lag length estimates are affected by the break in the constant value.[12]

The fifth explanation, like the fourth, involves changes in the coefficients of the money rule driving changes in the lag structure. This explanation relies on theoretical work by Bratsiotis and Martin (2005) discussed below showing that changes in the parameters of the money rule can cause changes in the degree of real rigidity, and this in turn can cause changes in the degree of persistence and changes in the estimated lag lengths.

The theoretical model also provides an explanation for a challenging aspect of the results. Why do the estimated lag lengths vary with changes in the output coefficient, but not with changes in the inflation coefficient? One potential answer is that it is the ratio of the two coefficients that matters, i.e. it is the relative strength of the monetary authority's response to inflation and output changes rather than either coefficient individually that is important, and looking at the individual coefficients masks this relationship.

Fig. 5 plots the ratio of the output gap parameter to the inflation parameter along with the estimated lag lengths. The figure shows a strong association between movements in the ratio and changes in the estimated lag length. But the fact that it looks a lot like Fig. 3c in the paper says that most of the movement in the ratio is driven by variation in the output

[10] See Orphanides and Williams (2003) for a discussion of the equivalence between a fixed gain under a Kalman filtering approach and a fixed window length under a rolling regression approach. The gain, which measures the sensitivity of the estimates to recent data points, is inversely related to the window length.

[11] If this is interpreted as a reduced form policy equation, one source of structural change of this type would be a change in the federal funds rate, say to tighten policy, at all values of income and the interest rate. Thus, this would represent a general tightening of policy. In addition, an AR(3) is not crucial. Models of different order behave similarly when there is a break in the constant.

[12] An AR(3) is chosen because the reduced form policy equation, i.e. the equation obtained from substituting for the current values of inflation and output in this equation, will contain more than one lag if the reduced form equations for inflation and output have more than one lag, and the results of the LR tests above indicate that they do.

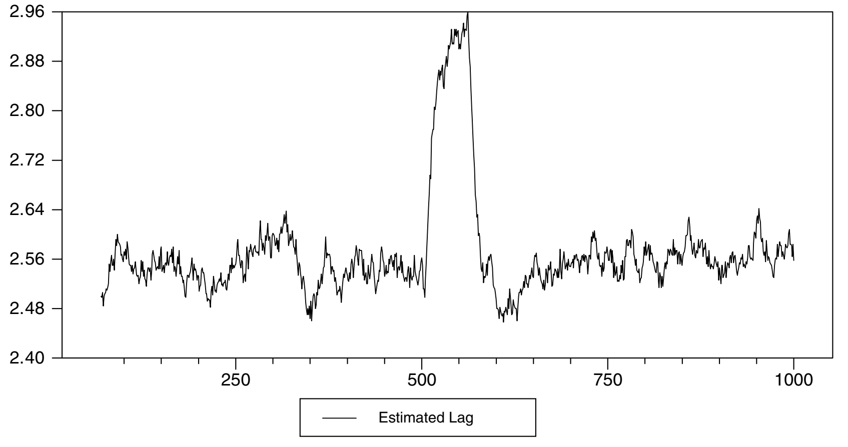

Fig. 4. Estimated lag length with structural change.

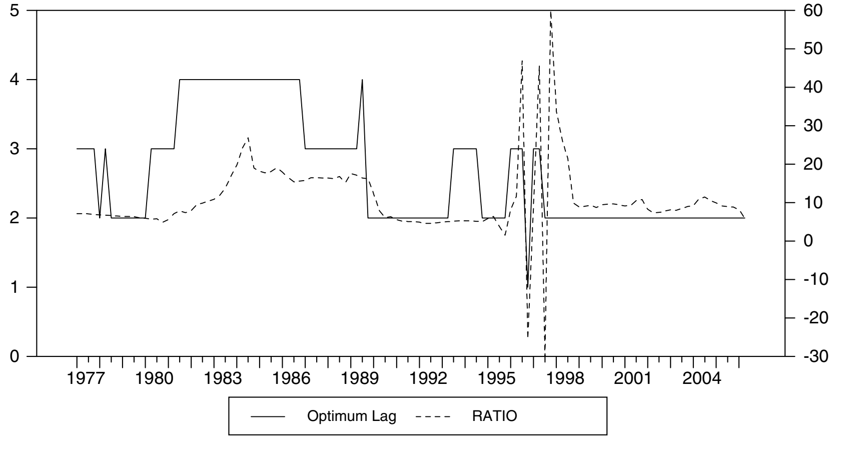

Fig. 5. Optimum lag length and coefficient radio (Gap/Inflation).

term, and further investigation does not support that the coefficient ratio explains the variation in the lag length estimates any better than the output coefficient alone.[13]

Since the ratio does not fit better, the task is to explain why the coefficient on output variation matters for lag lengths while the inflation parameter does not. An answer comes from a paper by Bratsiotis and Martin (2005), a paper that can explain either a statistical relationship between the ratio and the lag length as in Fig. 5, or a relationship between the lag length and one of the two parameters alone as in Fig. 3c.

[13] For example, adding the ratio to a regression model that already includes the coefficient on the output gap provides only a very slight improvement in fit. Alternative assessment methods lead to the same conclusion.

The paper shows that, within a standard New Keynesian framework, an increase in the parameter on the output gap or a decrease in the coefficient on inflation, both of which increase the ratio of the two coefficients, increases the degree of real rigidity and increases macroeconomic persistence. Thus, the ratio of the gap parameter to the inflation parameter should be positively related to changes in lag length.

To explain how the output parameter can matter while the inflation parameter does not, note that the effect of changes in each parameter on macroeconomic persistence depends upon other parameters in the model, so it is possible for one parameter of the money rule to have a much larger impact on persistence, and hence estimated lag lengths, than the other.

In particular, the effect that variations in the coefficient on inflation have on persistence depends upon the probability that prices will be reset in the Calvo pricing mechanism used in the model, but this probability does not impact the relationship between the gap parameter and lag lengths. Thus, changes in the probability that prices will be reset can impact the relationship between the inflation parameter and persistence without affecting the relationship between the gap parameter and persistence. Similarly, the effect of the output gap coefficient on persistence depends upon the responsiveness of real wages to changes in output, i.e. the degree of real rigidity on the supply-side of the model, but this parameter is independent of the relationship between the inflation parameter and persistence.

More particularly, changes in the inflation coefficient will have a larger impact on persistence when the probability that prices will remain constant increases, and changes in the gap parameter will have a larger impact on persistence when the responsiveness of real wages to output falls. This means that, to be consistent with the results above, the labor supply rigidly must be relatively large so that persistence responds to changes in the output gap parameter, and that the degree of price rigidity must be relatively small so that persistence does not change much when the inflation parameter changes.[14] Thus, under the appropriate parameterization this model can explain the relationship between lag lengths and the coefficients of the monetary policy rule shown in the diagrams above, in particular the responsiveness of the estimated lag length to the output gap parameter, and the lack of response to the coefficient on inflation.

Finally, there is the possibility that causality runs in the other direction, from changes in the dynamic structure of the economy to changes in the policy parameters. To examine this possibility, Granger causality tests are used to look at the causal structure between changes in lag length and changes in the parameter on the output gap in the policy equation. The results are shown in Table 1.

The results show that causality is strongest from changes in the number of lags to changes in parameter values indicating that it may be changes in the dynamic structure

[14] Without actually estimating the model, it is not possible to state the precise values of these parameters that would be required to be consistent with the empirical results. On the degree of price rigidity, recent work by Boivin et al. (2007), using evidence from a factor augmented VAR model, finds that previous work indicating substantial price rigidities confounds the effects of sector-specific and economy-wide shocks. When shocks are identified and considered separately, prices exhibit a high degree of flexibility in response to sector specific shocks, but show more persistence when aggregate shocks hit the economy. But since aggregate shocks explain only about 15% of the variation in sector specific prices, this result implies that prices display considerable flexibility. However, Nakamura and Steinsson (2006) in turn show that if you exclude price changes due to sales, sectoral prices show more rigidity. Thus, the degree of price rigidity is an open question and an active area of research.

Table 1
Causality tests

Causality direction	Test statistic
Lags → parameter	3.22 (.03)
Parameter → lags	1.15 (.32)

Note: *F*-statistics are shown with *p*-values in parentheses.

of the economy that is driving changes in the policy responses rather than changes in policy changing the dynamics of the system.

5. Conclusion

This paper investigates the relationship between changes in the policy rule for the federal funds rate and the estimated lag length in a typical VAR model involving output, inflation, and the federal funds rate. The paper finds an association between changes in the estimated lag length of the VAR model and changes in the coefficient on the gap term in a Taylor rule.

After discussing several reasons why lag length changes might be associated with changes in the policy rule, the paper finds that the two most likely explanations for the association between lag lengths and parameter values are the last two given above, one where structural change in the economy changes the dynamic structure, and this brings about a change in the policy response to deviations of output from its target value, and the other where changes in the coefficient on the output gap in the monetary policy rule cause changes in the degree of real rigidity and the degree of persistence.

But whatever the reason for the change in the estimate lag lengths and the direction of causality, the main message is that the typical means of estimating structural change in empirical models where parameter values are allowed to change while holding the dynamic structure constant may not fully capture the structural change. The results of this paper indicate that when allowing for changes in the structure of the economy in empirical investigations, particularly changes in the parameters of the monetary policy rule, it may be beneficial to allow for changes in the lag length of the empirical model as well.

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