**The Costs of Being Wrong about Lags in Monetary Policy**

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**Abstract:** This paper examines the losses associated with conducting monetary policy based upon an incorrect estimate of the delay between a change in policy and subsequent changes in macroeconomic variables. The paper finds that the costs of overestimating or underestimating the delay depend upon the particular loss function over outcomes for inflation and output used to evaluate policy. When inflation variability is weighted heavily in the loss function, it is less costly to assume the policy delay is too long than it is to assume the delay is too short. However, when output variation is sufficiently important in the loss function the results reverse and if an error is made, it is better to assume the delay is too short rather than too long. Finally, when the weight on output variation is large enough, it can be optimal to conduct policy as though the delay is short even if it is known that the true delay is longer.

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## 1. Introduction

Getting the timing of monetary policy correct is a difficult task and Federal Reserve policymakers seem to be criticized no matter what they do. If they wait until they have as much information as possible before setting monetary policy, they are subject to criticism for overshooting, i.e. sticking with current policy too long. For example, if there is a six-month delay between the time monetary policy changes and its subsequent impact on the economy, and if the Fed waits until one month before the onset of a recession to begin lowering interest rates, there will be a period of time when the Fed's policy is working against its stabilization goals.

If the Fed tries to avoid this problem by changing policy well in advance of any anticipated change in economic conditions, then the Fed is subject to criticism for tightening or loosening policy too soon. For example, if the Fed begins easing policy nine months in advance of a turning point when there is a six-month policy lag, then this will also work against its stabilization goals.

And even if the Fed happens to get the timing exactly right, there are uncertainties in the empirical literature about the exact nature of policy lags, so the Fed would still likely face criticism due to differences in estimates of the optimal policy response.

Does the Fed make systematic errors on one side or the other? Because there have been so few business cycles since modern central bank operating procedures have been in place, policy errors can lead to perceptions that the monetary authority tends to consistently miss on one side or the other of the optimal turning point. That is, a small number of business cycles combined with the difficulty the monetary authority faces in getting the monetary policy timing correct makes it easy to find a pattern in the policy responses even if none is really there. So this is a difficult question to answer.[1]

Determining when to alter the stance of policy is a difficult problem for the Fed because of data lags, uncertainties about which measure of variables such as prices or aggregate activity to use, the time it takes to interpret and revise data, uncertainties about the underlying theoretical structure, uncertainties about policy lags, uncertainties about the public's perceptions, and uncertainties associated with forecasting the future.

This paper focuses on one of these difficulties, the consequences of the monetary authority having an incorrect estimate of the lag between the time monetary policy is changed and the time the policy begins to impact the economy and show up in macroeconomic aggregates. If, for example, the monetary authority believes there is a longer lag between changes in policy and changes in variables such as inflation, employment, and output than actually exists, it will change policy too soon even if it estimates future turning points exactly. If it believes the lag is shorter than the true lag then the opposite will happen, it will fail to change policy soon enough.[2]

This is a problem that concerns the Fed,[3] and recent empirical work supports the Fed's concern about this issue. For example, the response of inflation after a monetary policy shock is typically estimated to occur with a one to two quarter delay, peaking after

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[1] William Poole (2002) suggests there is no systematic relationship.

[2] Let the true lag be two periods, and let the Fed believe the lag is four periods so that the lag is overestimated. Then, if the economy is correctly forecast to turn at $t=t_0$, the Fed will change policy four periods in advance, which is too soon, rather than two periods in advance as is optimal. When the lag is underestimated the opposite occurs, policy is changed too late relative to the optimal response. Uncertainties about the timing of turning points, a common problem, add to the Fed's difficulties.

[3] For example, president of the Federal Reserve Bank of Kansas City, Thomas Hoenig, in an interview with the *Wall Street Journal* noted that, "A Federal Reserve policy maker ... has to be mindful of the lagged impact of previous increases in interest rates and the risks of "overshooting" with future rate increases..." -- [Against 'Overshooting' on Rates, by Greg Ip, Wall Street Journal, May 19, 2006]

around a year and a half, and returning to its original level a year to a year and a half after the peak effect (see, e.g, Christiano, Eichembaum, and Evans (2005) for more details). Thus, this work implies a fairly sluggish response of prices and other variables in response to monetary policy shocks.

However, recent work by Boivin, Giannoni, and Mihov (2007) shows that separating shocks into economy-wide and sector specific components is important for understanding price rigidities, and that this separation suggests that price adjustment lags may not be as long as commonly assumed in the past. Further reflecting the uncertainty on this issue, Kehoe and Midrigan (2207) note that if temporary changes in prices due to sales are removed from the data, the lag increases substantially. Taken together, these papers imply considerable uncertainty about the timing of the response of macroeconomic aggregates to changes in monetary policy.

This paper assesses the consequences of the monetary authority over or under estimating the delay parameter within the context of a New Keynesian model that allows for delays between policy changes and the subsequent response of macro variables. The results show that the costs of over or under estimating the delay depend upon the particular loss function over outcomes for inflation and output used to evaluate policy. When inflation variability is weighted heavily in the loss function, it is less costly to assume the policy delay is too long than it is to assume the delay is too short. However, when output variation is sufficiently important in the loss function the results reverse and if an error is made, it is better to assume the delay is too short rather than too long. Finally, when the weight on output variation is large enough, it can be optimal to conduct policy as though the delay is short even if it is known that the true delay is longer.

## 2. The Model

As just noted, the impact of a monetary policy shock on the economy is not immediate, there is a delay before the change is reflected in macroeconomic variables such as inflation, employment, output, and interest rates. Standard off-the-shelf New Keynesian models do have such a delay built into them, so the most elementary forms of these models are inconsistent with the empirical evidence. This, of course, has prompted a search for ways to extend the basic model so that it is able to explain this feature of the data.

Following Woodford (2003), one way to build this type of policy lag into theoretical models is to assume that many expenditure decisions made by firms and households must be made in advance. This is similar in spirit to the assumption that prices must be set in advance when modeling firms' price-setting behavior.[4] The particular assumption used here is that expenditures at time t are a function of time t-d information about interest rates. Thus, only monetary shocks dated t-d and earlier can affect current expenditure decisions.

Woodford argues that this is a plausible model because many interest sensitive real-world expenditures are subject to such delays. For example, he cites the "time to build" literature where investment projects have planning lags and require spending to be distributed over time. Woodford also notes that this is in the spirit of models of household consumption such as Gabaix and Laibson (2002) where it is optimal for households to change their consumption plans only intermittently.

[4] An alternative, interchangeable assumption is that the decisions are based upon old information.

The result of this assumption about price delays is an "IS curve" of the form[5]:

$$\hat{y}_t = b + (1-\eta) E_{t-d} \hat{y}_{t+1} + \eta \hat{y}_{t-1} - \phi(i_t - E_{t-d}\pi_{t+1}) + g_t \tag{1}$$

where $\pi_t$ is the inflation rate, *d* is the delay as described above, $\hat{y}_t$ is the output gap, and $i_t$ is the nominal interest rate. The term $g_t$ captures demand shocks and other sources of shifts in the IS curve.

The model is discussed in detail in Clarida, Gali, and Gertler (1999). Equation (1) is an "IS curve" where the output gap at time $t$, $\hat{y}_t$, depends negatively upon the real interest rate and positively upon expected future output (since it is assumed that $\eta \leq 1$).

The presence of lagged output on the right hand side of equation (1) allows for endogenous persistence. As noted by Clarida, Gali, and Gertler, the primary justification for including these lagged terms is empirical though they can be motivated by the presence of some types of adjustment costs. The dependence of current output on expected future output arises from consumption smoothing, and the presence of the real interest rate is included to capture intertemporal consumption decisions.

The delay parameter, *d*, in equation (1) arises from consumers and firms making decisions in advance. Another basis for incorporating a delay into these models comes on the supply-side and is also from Woodford (2003). This class of models incorporates staggered price setting with Calvo pricing that assumes that there is a delay between the time a price or wage is set and the time that it takes effect. Under this formulation, there will be a delay between the time that a monetary shock hits and the subsequent change in inflation, but output can respond before inflation responds, a result consistent with the

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[5] This and the other equations of the model used here are essentially the same as the model used by Clarida, Gali, and Gertler (1999) and by Honkapohja and Mitra (2003) with the delay added based upon Woodford (2003).

empirical evidence.[6]

Consistent with this assumption, the aggregate supply curve used in this paper is:

$$\pi_t = c + \beta\,[(1-\xi)\,E_{t-d}\pi_{t+1} + \xi\,\pi_{t-1}] + \lambda\,E_{t-d}(\hat{y}_t - \hat{y}_t^n) + E_{t-d}\,u_t \qquad (2)$$

where $\hat{y}_t^n$ is the natural rate of output and all other variables are as previously defined. The term $u_t$ captures marginal cost or markup shocks and brings about shifts in the relationship between inflation and the output gap.

The aggregate supply curve (2) is derived from a model of staggered nominal price setting where inflation depends positively upon the output gap and positively upon expected future inflation. This differs from the standard Phillips curve formulation in that expected future rather than expected current inflation affects current inflation, a difference that has important implications. As with the IS curve formulation in equation (1), the presence of lagged inflation on the right-hand side of equation (2) is motivated mainly by empirical considerations.[7] By assumption, $\xi \le 1$.

The monetary policy rule is assumed to be a Taylor type rule with interest rate smoothing:

$$i_t = \alpha_i\,i_{t-1} + (1-\alpha)[\alpha_0 + \alpha_\pi\,E_{t-d}\,\pi_t + \alpha_y\,E_{t-d}\,\hat{y}_t] + w_t \qquad (3)$$

Equation (3) is a standard Taylor rule augmented by lagged interest rate terms to capture interest rate smoothing, and by the delay as described above.[8] It is assumed that $\alpha_i \le 1$ and that $\alpha_\pi \ge 1$. In this specification, the federal funds rate is increased when inflation or

[6] The same model can be obtained by assuming that only information dated t-d and earlier is available when prices are set. Under this assumption, the same optimality conditions and the same aggregate supply curve would result.

[7] Further discussion of the reasons for including lagged or inertial terms in the AS and IS equations can be found in Woodford (2003) and Evans and McGough (2004). Evans and McGough also reference additional papers where inertia is explicitly modeled. A paper by Smets (2003) fits a theoretical model in which lags appear to fit European data.

[8] Orphanides (2003) discusses interest rate smoothing in detail.

output rises above target with the strength of the response depending upon the values of the parameters of the policy rule, $\alpha_\pi$ and $\alpha_y$.

The final three equations specify the processes followed by shocks to the IS, AS, and policy rule equations. Shocks to the IS and AS curves are assumed, as is standard, to follow a first-order autoregressive process and the shock to the policy rule is assumed to be white noise:

$$g_t = \theta_g g_{t-1} + \varepsilon_{gt}, \qquad \varepsilon_{gt} \sim N(0,1) \tag{4}$$

$$u_t = \theta_u u_{t-1} + \varepsilon_{ut}, \qquad \varepsilon_{ut} \sim N(0,1) \tag{5}$$

$$w_t = \varepsilon_{wt}, \qquad \varepsilon_{wt} \sim N(0,1) \tag{6}$$

## 2.1 Solving the Model

The RE solution is obtained by first writing the model in the form

$$AE_t x_{t+1} = Bx_t + Cz_t$$

where $x_t = \left[y_t^T, k_t^T\right]^T$ is a vector of endogenous variables with $y_t$ a vector of non-predetermined variables and $k_t$ a vector of predetermined variables. Letting $n_x = n_y + n_k$ be the length of the vectors $x_t$, $y_t$, and $k_t$, the dimension of the matrix $A$ is ($n_x x n_x$) as is the dimension of the matrix $B$. The dimension of the matrix $C$ is ($n_x x n_y$). The vector $z_t$ contains exogenous variables and is assumed to follow a vector AR(1) process

$$z_t = \Phi z_{t-1} + w_t \tag{7}$$

where the matrix $\Phi$ has dimension ($n_y x n_y$). The solution to this is the Markov decision rule

$$y_t = Mk_t + Nz_t \tag{8}$$

$$k_{t+1} = Pk_t + Qz_t \tag{9}$$

where the matrices $M$, $N$, $P$, and $Q$ have dimensions ($n_y x n_k$), ($n_y x n_y$), ($n_k x n_k$), and ($n_k x n_y$). When a solution exists, it is derived using the techniques described in McCallum (1998, 1999) and in Klein (2000).[9] The Markov decision rule is used to simulate data for the RE model.

In the simulations below, three versions of the model are examined, one where the Fed underestimates the value of the delay, *d*, one where it overestimates the value, and one where it gets the value of the delay correct. As noted in the introduction, the goal is to evaluate whether the Fed faces a symmetric tradeoff and, if not, whether there is any reason to prefer to be wrong on one side or the other due to asymmetric losses around the true value of *d*.

## 3. Simulation

This section explains the simulation procedure, then discusses how the results of the simulation change when different assumptions are imposed about the Fed's error in calculating the delay parameter. The section also discusses how the results change as the parameters of loss function and the Taylor rule change.

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[9] See "Software for RE Analysis" by Bennett T. McCallum August 23, 2001 (Revised 02-17-04) at http://wpweb2k.gsia.cmu.edu/faculty/mccallum/Software%20for%20RE%20Analysis.pdf. The description of the solution in this section is based upon this material.

**3.1 Simulation Procedure**

The simulation is conducted over a range of values for three parameters in the model. The first parameter is the value of *d*, the delay parameter. The true value is assumed to be *d*=1, and three different values of *d* are assumed in the simulations, *d*=0, *d*=1, and *d*=2. When *d*=0, policy “overshoots,” and when *d*=2 policy “undershoots” by the definitions given above.

The second and third parameters allowed to vary across the simulations are the coefficients attached to inflation and output in the monetary policy rule, $\alpha_y$ and $\alpha_\pi$. The simulations allow $\alpha_y$ to vary between 0.0 and 5.0 and $\alpha_\pi$ to vary between 1.0 and 6.0, both in increments of .25.[10]

For each value of *d*, $\alpha_y$, and $\alpha_\pi$, the model is simulated for T=2,500 time periods.[11] The variance of inflation, output, and the interest rate are calculated, and these are used to evaluate the loss functions given below in equations (10) and (11). Thus, for each set of parameter values *d*, $\alpha_y$, and $\alpha_\pi$, the simulations give an estimate of the variances for output, inflation, and the interest rate based upon 2,500 observations.[12]

The next step is to evaluate the outcome for a given set of parameter values. This is done through a quadratic loss function of the form (see Woodford 2003):

$$L = (\tau - \tau^*)' W (\tau - \tau^*) + \lambda (i - i^*)^2 \tag{10}$$

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[10] Thus, there are 11 by 11 or 121 points in the parameter space for $\alpha_y$ and $\alpha_\pi$. The value of $\alpha_\pi$ is assumed to be greater than one to avoid indeterminacy. For this reason, the smallest value used for $\alpha_\pi$ is 1.01, followed by 1.25, 1.50, ..., 6.0.

[11] This is equivalent to, say, simulating a model with 100 observations 25 times and then averaging the outcomes across models.

[12] Overall, then, there are 3x11x11x2500 = 907,500 simulations.

The specific form of this general loss function examined here is:

$$L = \mu\sigma_{\hat{y}}^2 + (1-\mu)\sigma_\pi^2 \tag{11}$$

As noted above, the variances are obtained from the simulations. The parameter $\mu \le 1$ captures the relative weight on output and inflation stability. Various values for $\mu$ are examined to see how the results change with changes in the loss function for the economy.[13, 14]

While the values of $d$, $\alpha_y$, and $\alpha_\pi$, are allowed to vary across the simulations, the remaining parameters in the model are fixed. The values used are shown in the following table:

**Table 1**

**Parameter Values in the Simulations**

| IS Curve | Aggregate Supply | Policy Rule |
|---|---|---|
| $b = 2.0$ | $c_0 = .02$ | $\alpha_0 = 1.5$ |
| $\eta = .35$ | $\beta = .99$ | $\alpha_\pi = 1.0$ to $6.0$ |
| $\phi = 1.0$ | $\xi = .35$ | $\alpha_y = 0.0$ to $5.0$ |
| $\theta_g = .3$ | $\lambda = .30$ | $\alpha_i = .5$ |
| | $\theta_u = .30$ | |

[13] It is assumed that, for a given weight $\mu$, the monetary authority attempts to choose the parameters of the money rule, $\alpha_y$ and $\alpha_\pi$ to minimize the loss function. More on this below.

[14] A second loss function is also examined but is not presented here, one that assumes the monetary authority also cares about interest rate stability and thus seeks to minimize:

$$L = \mu_1\sigma_{\hat{y}}^2 + \mu_2\sigma_\pi^2 + (1-\mu_1-\mu_2)\sigma_i^2$$

The results are very similar to those presented in the paper due to the similarity in the behavior of the variances of output and the interest rate as described below.

The parameter values are from various sources. Clarida, Gali, and Gertler (1999) use $\phi = 1.0$, $\beta = .99$, and $\lambda = .3$. The values of $\theta_g = .3$ and $\theta_u = .3$ are close to the values of .4 used in Evans and Honkapohja (2003), but there is no firm guidance in that paper as to the correct values to select. The classic Taylor rule is $\alpha_\pi = 1.5$ and $\alpha_y = .5$ which is within the simulated grid for these two parameters. Though the values of the smoothing parameter assumed here, $\alpha_i = .5$, is lower than the one lag estimates of .75-.91 in Orphanides (2003), this does not appear to be critical for the results and produces second moments that accord more closely with actual data. Little guidance is available for choosing values for the $\eta$ and $\xi$ and .35 is chosen in both cases. Finally, the parameters for the loss functions will vary and are discussed further below.

## 4. Results

Because one of the goals is to understand how the conclusions change as the loss function changes, and since the loss functions used here involve the variances of inflation, output, and the interest rate, the first step is to characterize how the variances of these variables change as the parameters of the model change.

*The Variance of Inflation*

These three figures show the variance of the inflation rate for various values of the coefficients of the monetary policy rule across the three models *d*=0, *d*= 1, and *d*=2:[15]

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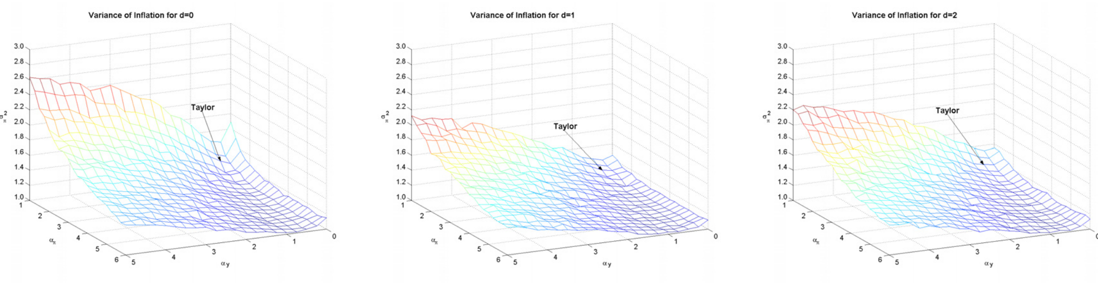
[15] The figures are smoothed at the minimum values for $\alpha_y$ and $\alpha_\pi$, i.e. when $\alpha_y$=0.0 and $\alpha_\pi$=1.01 due to instability and high variances at these values. In all diagrams, unless otherwise noted, moving from right to left from the viewer's perspective is an increase in $\alpha_y$, and moving from top to bottom is an increase in

**Figure 1**




(a) *d*=0 (b) *d*=1 (c) *d*=2

The figures show that, in general, as $\alpha_\pi$ increases and the monetary authority places more relative weight on deviations of inflation from target, the variance of inflation falls as expected. Similarly, as $\alpha_y$ increases and the relative weight shifts to deviations of output from target, the variance of inflation increases. Again, this is as expected.

The figures also show that the variance is generally lowest when *d*=1, i.e. when the monetary authority gets the delay right, and highest when *d*=0, i.e. when the monetary authority assumes the lag is shorter than it actually is and waits too long to set policy. The case in the middle occurs when *d*=2 and the monetary authority tends to change policy too soon rather than too late.[16]

*The Variance of Output*

These next three graphs show the variance of output for the three models *d*=0, *d*=1, and *d*=2:

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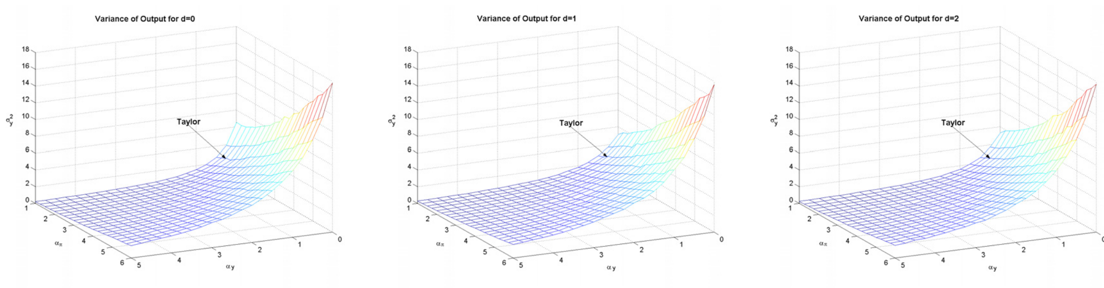
$\alpha_\pi$. The largest values for both parameters are in the corner closest to the viewer and furthest from the walls in the diagram.

[16] Plots of the differences between the diagrams, which are not included, can be used to demonstrate this result.

**Figure 2**




(a) *d*=0 (b) *d*=1 (c) *d*=2

The results are, once again, as expected with the variance of output declining as $\alpha_y$ increases, and increasing as $\alpha_\pi$ becomes larger.

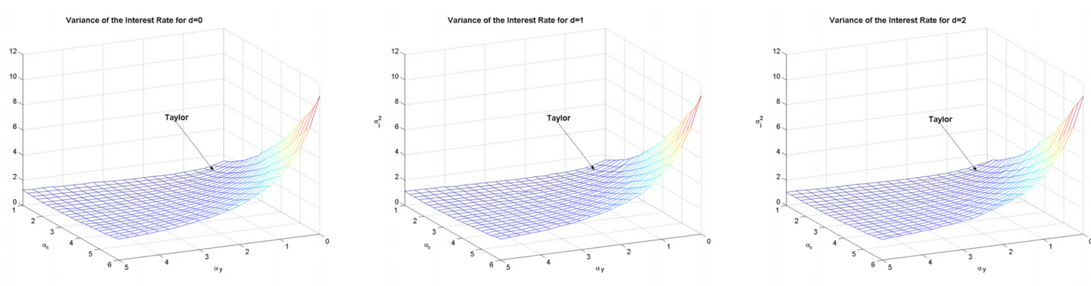
Though the differences are not large so it's difficult to see in the graphs, the variance of output is generally highest when *d*=1, smallest when *d*=0, and in between when *d*=2.

These results for the variance of output are the opposite of the results for inflation and together they represent the tradeoff between inflation and output stability present in the model. When moving away from the case where *d*=1, i.e. the case where the delay is correct, there is a gain in output stability but a loss in inflation stability when moving to either the *d=0* case or *d=2* case. An implication is that a loss function that places very little weight on inflation stability relative to output stability would imply that the monetary authority should wait until the last moment before changing policy since the variance of output is generally smallest (and the variance of inflation generally largest) when *d*=0. Loss functions that weight inflation heavily would result in *d=1* where the variance of inflation is smallest.

*The Variance of the Interest Rate*

Finally, here are the graphs showing changes in the variance of the interest rate for the same three models:

**Figure 3**




(a) *d*=0 (b) *d*=1 (c) *d*=2

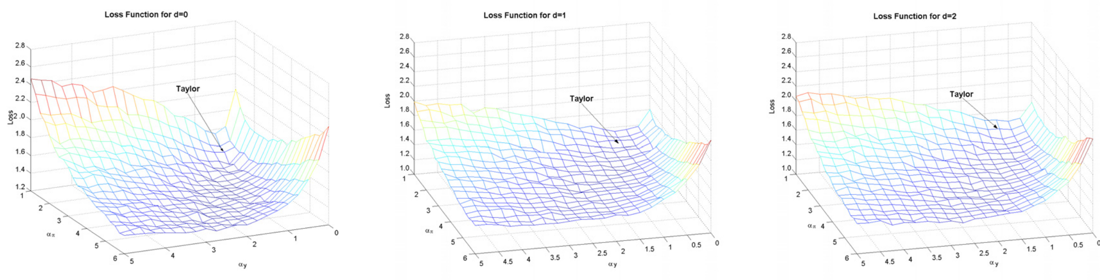
In this case, the variance of the interest rate increases when $\alpha_\pi$ increases, and decreases when $\alpha_y$ increases. Thus, the behavior of the variance of the interest rate as the policy parameters change is very similar to the behavior of the variance of output.[17] Across the three cases *d*=0, *d*=1, and *d*=2 the difference in the variances are small, which may be due to the lack of variation in the parameter $\alpha_i$ across the simulations.

*Losses Under Different Delay Assumptions*

This section presents an example for a particular loss function to help to illustrate the results. Figure 4 shows the results for the three different values of *d* for a loss function that is weighted heavily toward inflation stability, i.e. a loss of $L = .07\sigma_{\hat{y}}^2 + .93\sigma_\pi^2$. As above, the figures show the value of the loss function for each of the simulated pairs of monetary policy parameters $\alpha_y$ and $\alpha_\pi$. The figures also display the classic Taylor rule values as reference points:

[17] This is why adding the variance of the interest rate to the loss function does not have much of an effect on the results sown below, it is essentially equivalent to increasing the weight on the output term.

**Figure 4**




(a) *d*=0 (b) *d*=1 (c) *d*=2

Across all three figures:

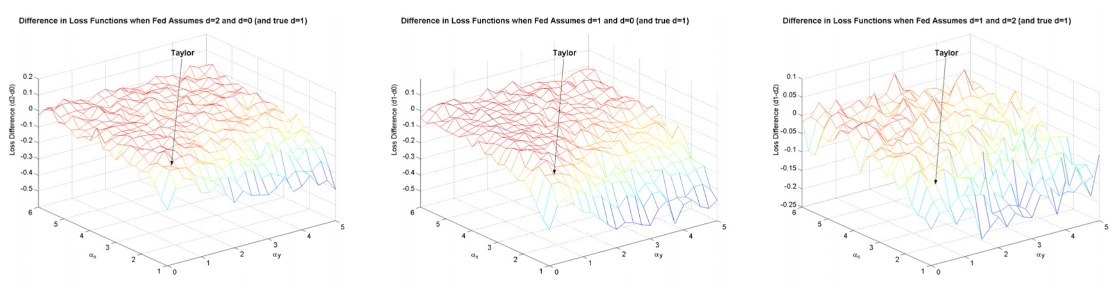
(1) As $\alpha_y$ increases from 0.0 to 5.0 (i.e. moving right to left from the viewer's perspective, the axis showing $\alpha_y$ values is on the bottom and the orientation is such that 0.0 is on the right), the loss initially falls, then rises again. For smaller values of $\alpha_\pi$, i.e. values near 1.0, the increase in the loss as $\alpha_y$ increases is bigger than for larger values of $\alpha_\pi$. That is, unlike the relatively steep increase in the loss when $\alpha_\pi$ is small, when $\alpha_\pi$ is near 6.0 the function is relatively flat as $\alpha_y$ increases.

(2) As $\alpha_\pi$ increases, the change in the loss function depends upon the value of $\alpha_y$. When $\alpha_y$ is large, the loss falls at a fairly uniform rate, and this happens over most of the range for $\alpha_y$. However, when $\alpha_y$ is near zero, the value of the loss increases after an initial drop.

(3) The Taylor rule parameters, while not quite optimal, give a loss that is very near the minimum. The Taylor values are among the smallest values of both monetary policy parameters for which the loss is near the minimum.

Turning next to the differences in the loss surfaces for the three models, Figure 5 shows the differences between the surfaces in Figure 4 for (a) *d*=2 and *d*=0, (b) *d*=1 and *d*=0, and (a) *d*=1 and *d*=2. Looking at the differences (a) determines which type of error is more costly, overestimating the delay parameter or underestimating the delay parameter, (b) illustrates how the loss varies between getting the delay correct and undershooting, and (c) shows how the loss varies between getting the delay correct and overshooting.

**Figure 5**




(a) *d*=2 minus *d*=0 (b) *d*=1 minus *d*=2 (c) *d*=1 minus *d*=0

The figures show that[18]:

(1) The difference in the loss functions is uniformly negative across all three figures.[19] The negative valued difference in the loss functions in the first case means that the loss when the monetary authority believes that *d*=2 is smaller than the loss when it is believed that *d*=0.[20]

(2) The negative valued difference in the loss functions in the middle figure, the case that compares the loss functions for *d*=1 and *d*=0, means that the loss is smaller when the monetary authority believes (correctly) that *d*=1 than when it believes that *d*=0.

(3) The negative valued difference in the loss functions in the right-hand figure, the case that compares the loss functions for *d*=1 and *d*=2, means that the loss is smaller

[18] In these figures, the axes are reversed so that the smallest values of the parameters are in the bottom corner of the diagrams rather than the largest values as before, however the parameters are an the same individual axes as before.
[19] Errors in estimating each surface due to only being able to examine a finite number of simulations allow an occasional node in the figure cross into positive territory.
[20] When evaluated at the same values of the coefficients on output and inflation in the money rule

when the monetary authority believes (correctly) that *d*=1 than when it believes (incorrectly) that *d*=2.

(4) The shape of the loss function is similar across the three figures. In general, as $\alpha_y$ changes there is very little change in the difference in the loss functions, though there is some hint that the loss difference increases as $\alpha_y$ increases when $\alpha_\pi$ is small. The difference does respond to $\alpha_\pi$. As $\alpha_\pi$ decreases (top to bottom in the figures), the difference in the loss functions also increases. Thus, the less weight that is placed on inflation in the money rule, the more there is to be gained from getting the delay parameter correct.

Putting (2) and (3) together, the loss is smallest when the delay parameter is estimated correctly, i.e. when *d*=1, and, from (1) we know that *d*=2 dominates *d*=0. It is best to be right, but if an error must be made, then *d*=2, i.e. changing policy before it is optimal, is better than *d*=0, i.e. waiting too long to respond. Finally, (4) says that the differences are larger when inflation receives less weight in the money rule.

**4.1 Robustness to Different Loss Functions**

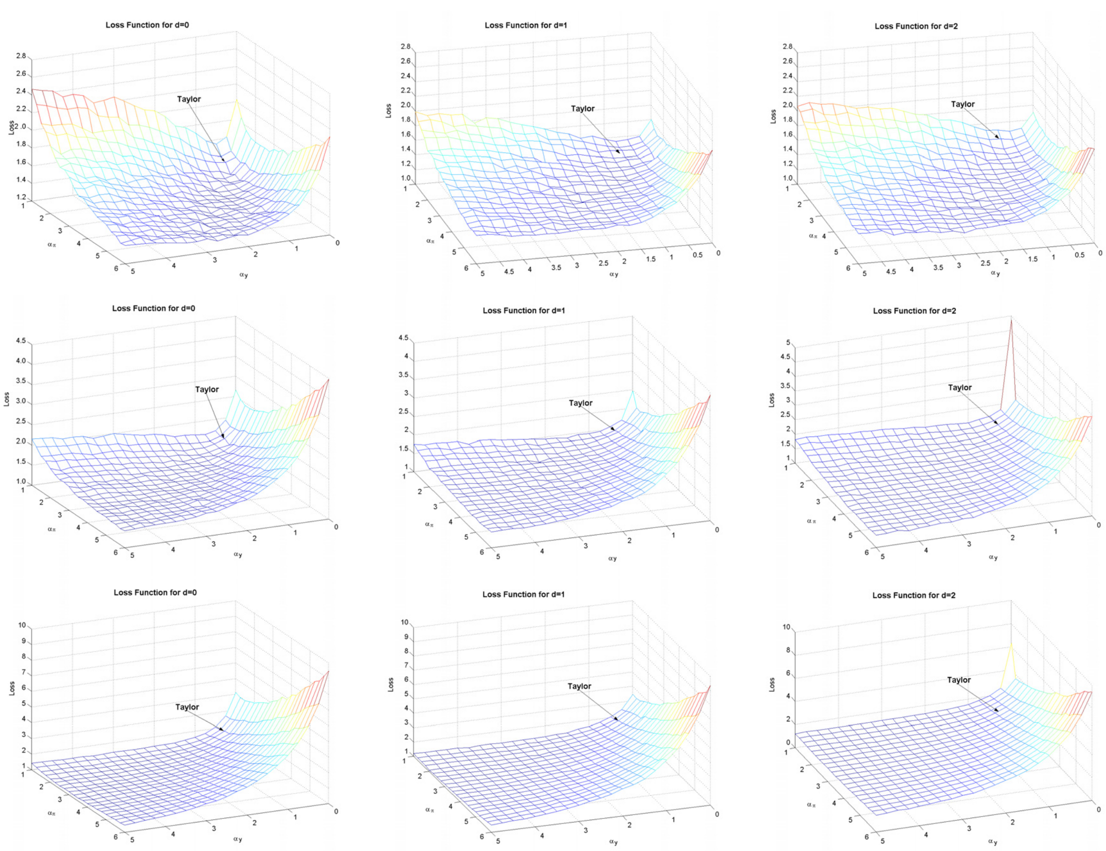
The loss function used so far, $L = .07\sigma_{\hat{y}}^2 + .93\sigma_\pi^2$, places considerable weight on the variance of inflation relative to the variance out output. How robust are the results to variations in the loss function? To answer this, two additional loss functions are examined. The set, including the loss function used above, is:

$$\begin{aligned} L1 &= .07\sigma_{\hat{y}}^2 + .93\sigma_\pi^2 \\ L2 &= .20\sigma_{\hat{y}}^2 + .80\sigma_\pi^2 \\ L3 &= .50\sigma_{\hat{y}}^2 + .50\sigma_\pi^2 \end{aligned} \tag{12}$$

Moving beyond *L*3, i.e. increasing the relative weight on output even further, does not have much effect on the outcome, i.e. the shape of the loss function changes very little from the *L*3 outcome.

Here are the figures for each case, *d*=0, *d*=1, and *d*=2 for each of the three loss functions. The first row shows the results for *L*1, the second row the results for *L*2, and the third row the results for *L*3. The columns show the results for *d*=0, *d*=1, and *d*=2:

**Figure 6**




(a) *d*=0 (b) *d*=1 (c) *d*=2

The figures show the shape of the loss function depends upon the parameters of loss function.[21]

[21] A much wider array of loss functions shapes can be obtained from the general loss function in equation (10) which includes cross-product terms.

Because there are many possible loss functions, and because each one has a different set of optimal policy coefficients $\alpha_y$, $\alpha_\pi$, and $\alpha_i$ that minimize the loss function, and because little is known about what values of $W$ and $\lambda$ in equation (10) to assume when specifying the loss function, no attempt is made to take a position on a particular loss function. Instead, a general characterization of how the results change with changes in the weights is the focus.[22]

**4.2 Interpretation of Results**

The loss function $L1$ used above weights inflation variation heavier than it weights variation in output, thus the loss function is lower in the $d$=2 case as compared to the $d$=0 case, and even lower when $d$=1. Because the weight on inflation variation is large, this simply echoes the results shown above for the variance of inflation. However, the result that $d$=1 is optimal is a consequence of the particular loss function used in the simulations. If the loss function is altered to place more weight on output relative to inflation, then the results change. In particular, it is possible for the loss function to be smallest for $d$=0 even when the true delay is $d$=1 since the variance of output falls as the delay gets shorter for a given money rule.[23]

This means that the optimal strategy for the monetary authority to pursue depends upon which type of variation is most important when evaluating losses in the economy, and which type of variation is most important depends upon underlying preferences. If

[22] There is also no guarantee that the policy parameters actually chosen by the monetary authority will be the optimal values so the results look at the loss across a variety of monetary policy parameter combinations rather than restricting the analysis to just the optimal pair.
[23] Because the variance of the interest rate behaves like the variance of output, it will not be discussed separately, i.e. when relatively more weight is placed on output, the same qualitative result would obtain if the weight were placed on interest rate variation instead of output variation.

the economic consequences of variation in inflation are relatively important to agents, then if a mistake is made *d*=2, i.e. changing the interest rate too soon relative to the optimal response, is preferred to a policy that assumes *d*=0 and changes policy too late (*d*=1, i.e. getting it correct, is optimal). But when the consequences of output variation are relatively important, then making the error of assuming that *d*=0 is preferred to the error of assuming *d*=2, and *d*=0 can also be preferred to *d*=1 when the parameter on output variation in the money rule is large enough.

**5. Conclusion**

In answer to the main question posed in the introduction, is it better for the monetary authority to overshoot or undershoot, it depends upon the preferences of agents and the responsiveness of the variance of the inflation rate, output, and the interest rate to changes in the parameters of the money rule.

More particularly the paper showed that, consistent with expectations, as $\alpha_\pi$ increases the variance of inflation falls and the variance of output and the interest increase. Conversely, as $\alpha_y$ increases, the variance output and the interest rate go down while the variance of inflation goes up.

The results also show that across that values of *d*, the variance of inflation is smallest when *d*=1, largest when *d*=0, and in between when *d*=2. For output, the results differ and the variance is inflation is smallest when *d*=0, largest when *d*=1, and in between when *d*=2. Because of this difference in the movements of the variances of inflation and output as *d* changes, the optimal value of *d*, i.e. the value where the loss is smallest, depends upon the weight attached to the variances in the loss function. In particular, when inflation is weighted heavily, *d*=1 is optimal, and d=2 is better than d=0, but when most of the weight is on output variation, *d*=0 can be optimal since this minimizes the output variance.

Thus, the parameters on inflation and output variation in the loss function determine which of the *d*= 1, *d*= 2, and *d*= 3 cases has the smallest loss, and the loss function itself will depend upon the preferences of agents. When those preferences result in a large weight on output variation, the monetary authority's job is relatively easy since it does not have to worry about the delay. Since *d*=0 is optimal policy in this case even though the actual delay is *d*=1, the monetary authority should collect as much information as possible and wait as long as possible before changing policy.

But when preferences result in a large weight on inflation variation in the loss function, the monetary authority's job is more difficult since the delay parameter must be estimated correctly for policy to be optimal. In the face of uncertainty over the delay parameter and the potential for incorrect estimates of *d*, when inflation variation is the predominant concern and the delay is estimated incorrectly, it is better for the monetary authority to change policy too soon rather than too late.

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