Math 251 Midterm 1 NAME:_____
F 20 Oct. 2017 Student ID:_____
GENERAL INSTRUCTIONS

1. DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.
2. The exam pages are **two sided**.
3. Closed book, except for a 3×5 file card.
4. The following are all prohibited: Calculators (of any kind), cell phones, laptops, iPods, and any other electronic devices or communication devices. All electronic or communication devices you have with you must be turned completely off and put inside something (pack, purse, etc.) and out of sight.
5. The point values are as indicated in each problem; total 100 points.
6. Write all answers on the test paper. Use the back of the last page for long answers or scratch work. (If you do write an answer there, indicate on the page containing the problem where your answer is.)
7. Show enough of your work that your method is obvious. Be sure that every statement you write is correct. Cross out any material you do not wish to have considered. Correct answers with insufficient justification or accompanied by additional incorrect statements will not receive full credit. Correct guesses to problems requiring significant work, and correct answers obtained after a sequence of mostly incorrect steps, will receive no credit.
8. Be sure you say what you mean, and use correct notation. Credit will be based on what you say, not what you mean.
9. When exact values are specified, give answers such as $\frac{1}{7}$, $\sqrt{2}$, $\ln(2)$, or $\frac{2\pi}{9}$. Decimal approximations will not be accepted.
10. Final answers must always be simplified unless otherwise specified.
11. Grading complaints must be submitted in writing at the beginning of the class period after the one in which the exam is returned (usually by the Tuesday after the exam).
12. Time: 50 minutes.

1	2	3	4	5	6	7	8	TOTAL	EC
1	20	18	9	27	8	8	9	100	

1. (1 point) Are you awake?

2. (a) (6 points) State carefully the definition of the derivative of a function.

(b) (14 points) If $f(x) = \dfrac{1}{8-x}$, compute the derivative $f'(2)$ *directly from the definition*. (You can check your answer using a differentiation formula, but no credit will be given for just using the formula.)

3. (9 points/part) Differentiate the following functions. (You need not compute the derivatives directly from the definition.)

(a) $g(t) = ae^t - \dfrac{7}{t^2} + \sqrt{t} + \pi^2$. ($a$ is a constant.)

(b) $h(x) = \sin(6x^2 - 11x)$.

4. (9 points) Find the equation of tangent line to the graph of $f(x) = x^2 - 2x$ at $x = -3$. You need not calculate the derivative directly from the definition.

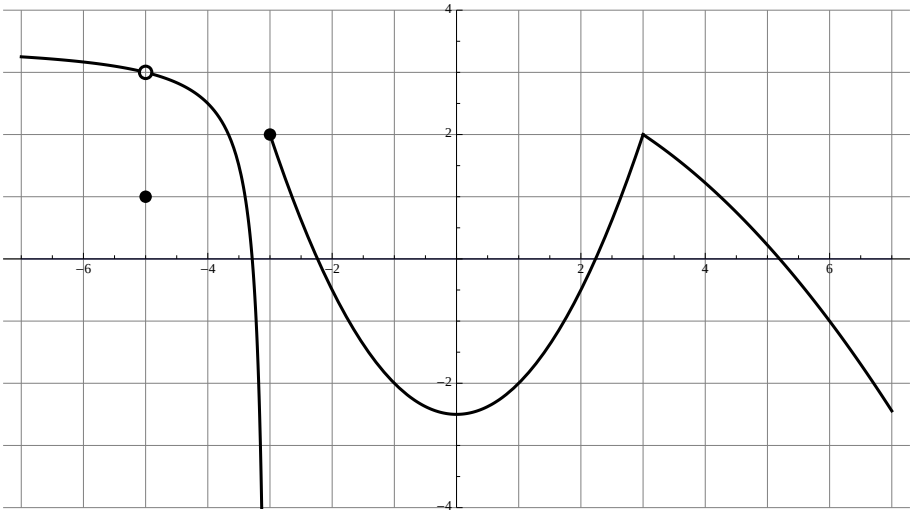
5. (9 points/part) Find the exact values of the following limits (possibly including ∞ or $-\infty$), or explain why they do not exist or there is not enough information to evaluate them. Give reasons in all cases.

(a) $\lim_{x \to 1} \dfrac{x - 1}{x^2 - x - 6}$.

(b) $\lim_{x \to 10} \dfrac{x - 10}{3(\sqrt{x} - \sqrt{10})}$.

(c) $\lim_{x \to \infty} \dfrac{x + 109}{7x + 1}$. (Be sure to show your work!)

6. For the function $y = k(x)$ graphed below, answer the following questions:



(a) (4 points.) Find $\lim_{x \to -5} k(x)$.

(b) (4 points.) Which of the following best describes $k'(4)$?
(1) $k'(4)$ does not exist.
(2) $k'(4)$ is close to 0.
(3) $k'(4)$ is positive and not close to 0.
(4) $k'(4)$ is negative and not close to 0.

7. (4 points/part) A traffic reporter's helicopter is hovering over a freeway interchange. Its height above the ground varies. During the period from 8:00 am to 8:22 am, its height $y(t)$ above the ground, measured in meters, at time t, measured in minutes (min) after 8:00 am, is given by $y(t) = t^3 - 5t^2 + 110$.

(a) Is the helicopter falling or rising 2 minutes after 8:00 am? How fast?

(b) What is the average upwards velocity of the helicopter between 8:00 am and 8:02 am?

8. (9 points) If $xy = \cos(x + y) + \sin(6)$, find $\dfrac{dy}{dx}$ by implicit differentiation. (You must solve for $\dfrac{dy}{dx}$.)

Extra credit. (Do not attempt these problems until you have done and checked your answer to all the ordinary problems on this exam. They will only be counted if you get a grade of B or better on the main part of this exam.)

Do these problems below or on the the back of this page.

EC1. (5 extra credit points) Let $f(x) = \cos(3x)$. Find the 1033th derivative $f^{(1033)}(x)$.

EC2. (10 extra credit points) We will see later this quarter that if g is a differentiable function on an open interval (a, b), and if $g'(x) = 0$ for all x in (a, b), then g is constant. By considering the function $g(x) = \dfrac{f(x)}{e^x}$, prove that if f is a function on (a, b) such that $f'(x) = f(x)$ for all x, then there is a constant c such that $f(x) = ce^x$ for all x.

EC3. (5 extra credit points/part) For each of the following parts, find a function f whose derivative is as given. Check your function to be sure its derivative really is what you think it is. (Caution: These are tricky.)

(a) $f'(x) = xe^{-x^2}$.

(b) $f'(x) = x\sin(x)$.