Math 251 Midterm 1 NAME: PROBLEM LIST
F 20 Oct. 2017 Student ID:________________

1. (1 point) Are you awake?

2. (a) (6 points) State carefully the definition of the derivative of a function.

(b) (14 points) If $f(x) = \dfrac{1}{8-x}$, compute the derivative $f'(2)$ *directly from the definition*. (You can check your answer using a differentiation formula, but no credit will be given for just using the formula.)

3. (9 points/part) Differentiate the following functions:

(a) $g(t) = ae^t - \dfrac{7}{t^2} + \sqrt{t} + \pi^2$. ($a$ is a constant.)
(b) $h(x) = \sin(6x^2 - 11x)$.

4. (9 points) Find the equation of tangent line to the graph of $f(x) = x^2 - 2x$ at $x = -3$. You need not calculate the derivative directly from the definition.

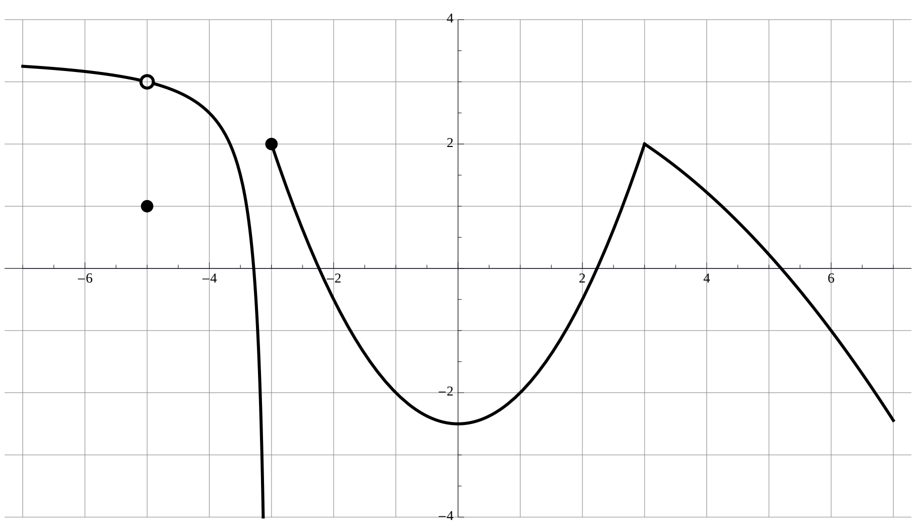
5. (9 points/part) Find the exact values of the following limits (possibly including ∞ or $-\infty$), or explain why they do not exist or there is not enough information to evaluate them. Give reasons in all cases.

(a) $\displaystyle\lim_{x\to 1} \frac{x-1}{x^2-x-6}$.

(b) $\displaystyle\lim_{x\to 10} \frac{x-10}{3(\sqrt{x}-\sqrt{10})}$.

(b) $\displaystyle\lim_{x\to \infty} \frac{x+109}{7x+1}$. (Be sure to show your work!)

6. For the function $y = k(x)$ graphed below, answer the following questions:



(a) (4 points.) Find $\lim_{x\to -5} k(x)$.

(b) (4 points.) Which of the following best describes $k'(4)$?
(1) $k'(4)$ does not exist.

(2) $k'(4)$ is close to 0.
(3) $k'(4)$ is positive and not close to 0.
(4) $k'(4)$ is negative and not close to 0.

8. (4 points/part) A traffic reporter's helicopter is hovering over a freeway interchange. Its height above the ground varies. During the period from 8:00 am to 8:22 am, its height $y(t)$ above the ground, measured in meters, at time t, measured in minutes (min) after 8:00 am, is given by $y(t) = t^3 - 5t^2 + 110$.

(a) Is the helicopter falling or rising 2 minutes after 8:00 am? How fast?

(b) What is the average upwards velocity of the helicopter between 8:00 am and 8:02 am?

8. (9 points) If $xy = \cos(x+y) + \sin(6)$, find $\dfrac{dy}{dx}$ by implicit differentiation. (You must solve for $\dfrac{dy}{dx}$.)

EC1. (5 extra credit points) Let $f(x) = \cos(3x)$. Find the 1033th derivative $f^{(1033)}(x)$.

EC2. (10 extra credit points) We will see later this quarter that if g is a differentiable function on an open interval (a,b), and if $g'(x) = 0$ for all x in (a,b), then g is constant. By considering the function $g(x) = \dfrac{f(x)}{e^x}$, prove that if f is a function on (a,b) such that $f'(x) = f(x)$ for all x, then there is a constant c such that $f(x) = ce^x$ for all x.

EC3. (5 extra credit points/part) For each of the following parts, find a function f whose derivative is as given. Check your function to be sure its derivative really is what you think it is.

(a) $f'(x) = xe^{-x^2}$.

(b) $f'(x) = x\sin(x)$.