Math 251

Final Exam - Fall 2015 - Problem List

Note: The wording of the problems here is not identical to that on the exam as administered. The total number of points was 100, so the point values are about half of what similar questions would be on a midterm.

1. (8 points; point values of parts as shown) A small spacecraft takes off from the surface of a planet, reaches a maximum height, and then crashes. Its position at time t is given by $y(t) = 9t^2 - 4t^3$, where $y(t)$ is measured in kilometers (km) above the surface and t is measured in minutes (min). Answer the following questions, being careful to give correct units when called for.

(a) (2 points) Find the speed of the spacecraft at time $t = 1$.

(b) (2 points) How long will it take for the spacecraft to reach its maximum height?

(c) (4 points) At time $t = 2$, what is the acceleration of the spacecraft? Would a person in the craft feel himself speeding up or slowing down at this moment? Explain.

2. (6 points) The derivative of the function $f(x) = (x^3 + 3x^2 + 3x + 5)e^{-x}$ is given by $f'(x) = -(x+2)(x-1)^2 e^{-x}$. Find the critical numbers of f and for each one determine if it is a local minimum, local maximum, or neither.

3. (8 points) A pendulum swings back and forth on the surface of the earth. The relation between the period T and the length l of the pendulum can be modelled by the equation $T = \sqrt{kl}$, where T is measured in seconds, l is measured in meters, and $k = 4\,\mathrm{s}^2/\mathrm{m}$. Answer the following questions, being careful to include units when appropriate.

(a) (1 point) When l is 25 meters, compute the period of the pendulum.

(b) (4 points) Using part (a) as your base value, give a linear approximation for the period of the pendulum when the length l is changed to 27 meters.

(c) (3 points) Suppose you do a linear approximation for $l = 205$ meters, based on the starting point $l = 200$. Is the linear approximation greater or less than the actual period? Explain (perhaps using a picture).

4. (4 points) Let f and g be functions such that:

$$g(1) = 2, \quad g'(1) = 1, \quad f(1) = 2, \quad \text{and} \quad f'(1) = 6$$

and

$$g(2) = 1, \quad g'(2) = 5, \quad f(2) = 3, \quad \text{and} \quad f'(2) = -1.$$

Let $h(x) = f(g(x))$. Find $h'(2)$. (You will not need to use all the information provided.)

5. (11 points; point values of parts as shown) Air Krill runs daily flights from the Eugene Airport to the Falkland Islands. When the company charges \$400 per ticket, 24 people fly. Market research has shown each \$10 that the fare is increased, 1 fewer person flies. Conversely, for each \$10 the fare is lowered, 1 more person flies.

(a) (4 points) Let c denote the price charged for each ticket. ("c" stands for "cost".) Let $T(c)$ be the number of tickets sold at that price, and let $R(c)$ be the revenue at that price. Write down formulas for $T(c)$ and $R(c)$.

(b) (3 points) How much should Air Krill charge per ticket to maximize total revenue?

(c) (2 points) Suppose that the Air Krill airplanes can only seat 30 people. Write down an inequality involving $T(c)$ that represents this, and deduce a corresponding inequality for c.

(d) (2 points) Does the condition in (c) change the answer for (b), and if so what is the new answer? Explain.

6. (3 points/part) Find the exact values of the following limits (possibly including ∞ or $-\infty$), or explain why they do not exist. Show supporting work.

(a) $\displaystyle\lim_{x\to 0} \frac{x^4}{e^{7x^4}-1}$.

(b) $\displaystyle\lim_{x\to 1} \frac{x-2}{x^2-x-6}$.

(c) $\displaystyle\lim_{x\to 0} \frac{x^2}{\cos(2x)-1}$.

(d) $\displaystyle\lim_{x\to\infty} \frac{3x+1}{5x^2-9}$.

7. (7 points) At noon Horton is 10 miles due west of Gertrude. Horton walks north and Gertrude walks west; neither walks at a steady rate. Both have devices that can continuously measure the distance between them. After one hour, Horton has walked 3 miles and Gertrude has walked 6 miles. Horton's device tells him that he is at that moment walking at 5 miles per hour, and the distance between him and Gertrude is increasing at a rate of 2 miles per hour. How fast is Gertrude walking at this moment? Include units in your answer.

8. (6 points) Find the equation of the tangent line to the curve $x^2 + xy + y^2 = 3$ at the point $(1, 1)$.

9. (4 points/part) In each part below, find the derivative of the given function.

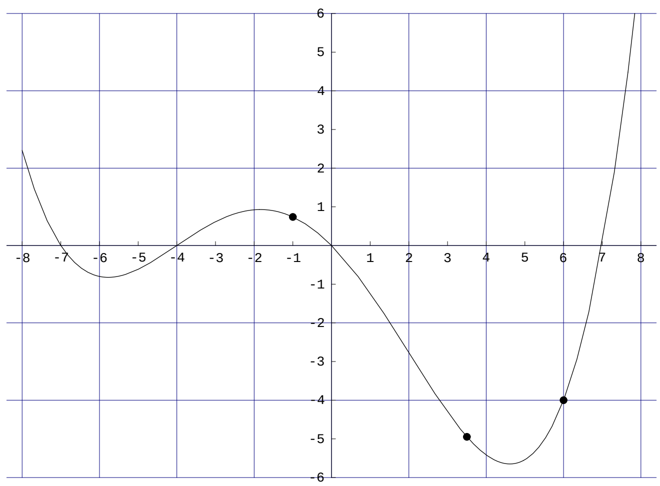
(a) $f(x) = \sin(x^2) + (\sin(x))^2$.

(b) $g(x) = \tan(5x) + \ln(7)x^3 + \pi^2$.

(c) $q(t) = e^{3t}\sqrt{t}$.

(d) $h(s) = \dfrac{7s+3}{3s-7}$.

10. (2 points/part) The picture below is the graph of the *DERIVATIVE* $y = f'(x)$ for a certain function f. **CAUTION:** You are given the graph of the *derivative* $f'(x)$, *not* the graph of $f(x)$, but you are asked questions about $f(x)$. The points referred to in parts (a) and (b) are marked on the graph with dots.



(a) Is f increasing, decreasing, or nearly flat at $x = -1$, or is there not enough information provided to determine this? Why?

(b) Is f concave up or concave down at $x = 3.5$, or does f (nearly) have an inflection point at $x = 3.5$, or is there not enough information provided to determine this? Why?

(c) At which values of x in $[-8, 8]$ (the interval shown) does f have a local minimum? Explain.

(d) Is $f(0)$ positive, negative, or zero, or is there not enough information to determine this? Explain.

11. (8 points) You want to build a box with a square base. Let b be the length of one side of the base, and let h be the height, both measured in meters. Given the constraints that $10 \le b \le 25$ and $2b + h = 60$, what are the biggest and smallest volumes for such a box?

12. (3 points/part) The function $P(t) = (6t+1)e^{k(t-1)}$ models the population of a colony of bacteria at time t, where P is measured in hundreds of bacteria and t is measured in hours. Observations indicate that after one hour there are 700 bacteria, and at that time the colony is growing at a rate of 200 bacteria per hour.

(a) Find k.

(b) What happens to the population of bacteria in the long run, as $t \to \infty$? Explain.