Math 251, Practice Questions for Final
Spring 2016

General information: The final exam will be closed book, and you will not be allowed to use a calculator. You may have one 3×5 notecard (front and back). You will have two hours to complete the exam.

This review sheet gives questions that are similar in content and difficulty to what will be on the exam. The exam will be much shorter than this review sheet.

1. Let
$$f(x) = \frac{(x+1)^2}{(x+2)^2}$$
Find all horizontal/vertical asymptotes, critical points, local minima/maxima, intervals on which $f(x)$ is increasing and intervals on which $f(x)$ is decreasing, inflection points, and intervals of concavity. Use this information to sketch a graph of $f(x)$.

2. For each of the functions $f(x)$ given below, compute the derivative $f'(x)$.
   (a) $f(x) = 3x^5 + 2x\sin(3x) + 7e^2$
   (b) $f(x) = 3e^{4x^2-1}$
   (c) $f(x) = \ln\left(\sqrt{3x^2-1}\right)$
   (d) $f(x) = (x-2)^2 \sin(x^3)$
   (e) $f(x) = \frac{\sin x}{x^2+1}$

3. At $a = 3$, the equation for the tangent line to the curve $y = f(x)$ is given by $y = 5x + 8$. Find $f(3)$ and $f'(3)$.

4. Find the linear approximation of the function $g(x) = e^x$ at $a = 0$ and use it to approximate the number $e^{-0.001}$. Is your approximation greater than or less than the true value?

5. Evaluate each of the following limits:
   (a) $\displaystyle\lim_{x\to 0+} \frac{x^3 + x^2}{\sin(x) - x}$
   (b) $\displaystyle\lim_{x\to 3} \frac{x^3 - 2x + 1}{x^2 - 5}$

(c) $\lim_{x\to\infty} \dfrac{\sqrt[3]{x^6+2x-1}}{5x^2+8x+2}$

(d) $\lim_{x\to 2+} \dfrac{\sqrt{x^2-4}}{10-x-x^3}$

(e) $\lim_{t\to-\infty} \dfrac{t^2+3t-1}{5t^2-6t+7}$

6. Suppose $2x^3 + x^2y - xy^3 = 2$. Find the equation of the tangent line at $(1, 0)$.

7. Consider the curve defined by $5x^2 + 6xy + 5y^2 = 1$.

   (a) Find all points on the curve where the tangent line is horizontal.

   (b) Find all points on the curve where the tangent line is vertical.

8. Find the absolute maximum and absolute minimum values of the function $f(x) = x - \ln(x)$ on the interval $[e^{-1}, e^{10}]$.

9. A man walks along a straight path. A searchlight is located on the ground 15 feet from the path, and follows the man as he walks. When the man is 20 feet from the point on the path closest to the searchlight, he is walking at a speed of 5 feet per second. At this moment, at what rate is the searchlight rotating?

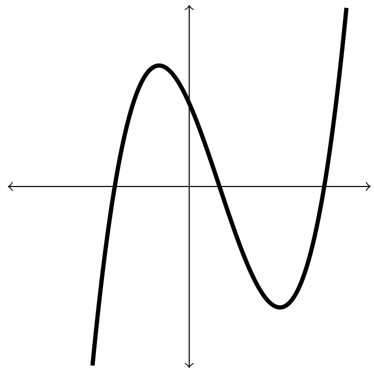
10. Suppose $f(x)$ is an unknown function and $g(x) = \cos(xf(x))$. Determine a formula for $g'(x)$ in terms of $f(x)$ and $f'(x)$.

11. An object is moving along a straight line with position $y = 5t + 10t^3 - 3t^5$ after t seconds (for small values of $t \geq 0$).

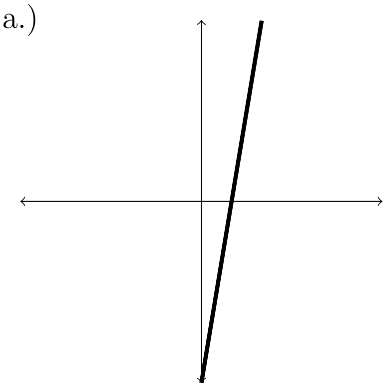
    1. What is the object's acceleration after t seconds?
    2. When does the object reach its maximum velocity?

12. A cylindrical can is to be made to hold 1 L (1000 cm^3) of oil. Find the dimensions (radius and height) that will minimize the amount of metal needed to manufacture the can. Note that the volume of a cylinder is $\pi r^2 h$ and the surface area is $2\pi r^2 + 2\pi rh$.

13. Given the graph of the function below, identify which of the following represent its derivative.



a.)



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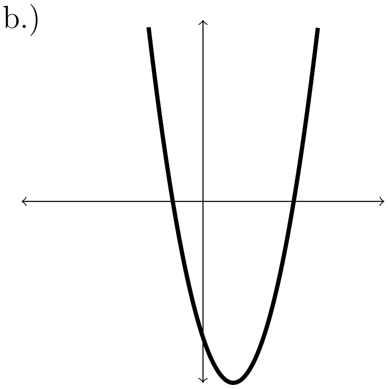

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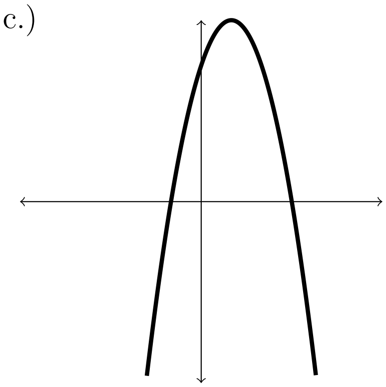
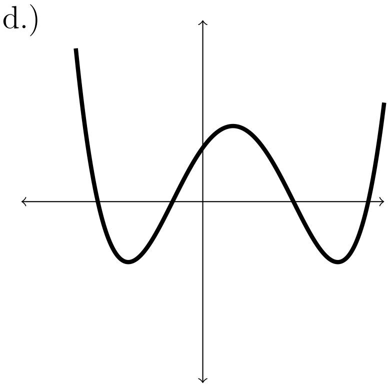


d.)



14. If $y = 5\sin(x^2+1)$ find $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$.

15. For each of the functions $f(x)$ given below, compute the derivative $f'(x)$.
   (a) $f(x) = x^2 e^x$
   (b) $f(x) = \dfrac{x}{\sin(x)} + x$
   (c) $f(x) = 3^x$
   (d) $f(x) = \sqrt[6]{x^3 + 3x - 1}$
   (e) $f(x) = \cos(\ln(x + x^2))$

16. Find the equation for the tangent line to the curve $y^2 = x^3 + 3x^2$ at the point $(1, -2)$.

17. If $\sin(x + y) = ye^x + 2y$, find dy/dx.

18. The table below gives a few values of the function $f(x)$ and its derivative $f'(x)$

x	1	2	3	4	5
$f(x)$	3	2	1	5	2
$f'(x)$	4	2	3	1	4

   Let $g(x) = f(f(x))$. Find $g'(4)$

19. Joe drives north from Eugene, leaving at noon. If t is the number of hours since noon and $d(t)$ is his position measuring north from Eugene, his velocity is given by

$$d'(t) = t^3 - 11t^2 + 24t \quad \text{mi}$$

   for $0 \le t \le 10$.
   (a) For how long is he driving *southbound*?
   (b) At time $t = 1$, is Joe pressing down on the car's accelerator pedal or easing up on it?
   (c) What is the greatest speed Joe reaches over the course of his journey (from $t = 0$ to $t = 10$), and is this speed northbound or southbound?
   (d) From the perspective of an observer on the ground, is Joe speeding up or slowing down at time $t = 2$?

20. Evaluate each of the following limits:
   (a) $\lim_{x \to \infty} x^{\frac{1}{x}}$

(b) $\lim_{x\to\infty} \frac{\sin(x^2)}{5x}$

(c) $\lim_{x\to 100+} \frac{e^{-x}}{\sqrt{x}-10}$

21. Let $f(x) = 3x^2 - 5x + 4$. For which value(s) of x does the tangent line to the graph of $f(x)$ at x have a slope of 17?

22. Give the definition of $f'(a)$ in terms of limits.

23. If a and b are the sides of a right triangle, then the hypotenuse is $c = \sqrt{a^2 + b^2}$. For $a = 3$ and $b = 4$, one has $c = 5$. If the a-side is kept constant at 3 and the b-side is increased by 0.1, give a linear approximation for the length of the new hypotenuse.

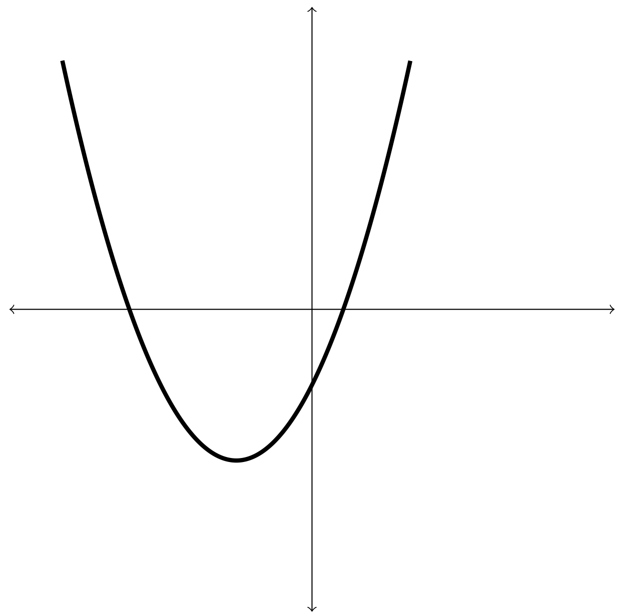
24. Find the local minima and maxima of $x^3 - 5x^2 - 8x + 10$. State which are minima and maxima. Find the global minimum and maximum values of this polynomial on $[0, 10]$.

25. You are told that $f(x)$ is a function with domain all real numbers except -2, and whose derivative is $f'(x) = \frac{x^2 + 5x + 4}{(x+2)^2}$. Find the critical numbers of $f(x)$ and identify each of them as a local minimum, local maximum, or neither.

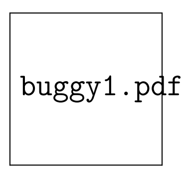
26. Find the horizontal asymptotes (if any) of the function $f(x) = \frac{\sqrt{16x^2+1}}{3x-5}$.

27. A car starts 3 miles directly south of a library. It then drives due east at a speed of 10 mi/h. At what rate is the distance between the car and the library increasing when the car has driven for 5 miles?

28. Below is the graph of $f'(x)$. On the same pair of axes given below, draw $f(x)$ such that $f(0) = 0$.



29. A paved road runs east through the desert into a town. A dunebuggy is located 5 miles west and 2 miles south of the town, in the desert. (A dunebuggy is sort of a car made to go on dirt.) The buggy can travel 30 mph in the desert, and 50 mph on the road. Find the route that gets it into town the fastest. In particular, in the picture below find the value of x that results in the fastest route for the dunebuggy.



30. Consider a flat sheet of cardboard that is 17 inches wide and 12 inches tall. Suppose that you cut out a square of equal size out of each corner of the sheet. Then you can fold the remaining sides (flaps) of the cardboard up to make a rectangular box (with no top). If you want to create box that can hold the most stuff, what is the side-length of each square that is cut out?

31. A Florida citrus grower estimates that if 60 orange trees are planted, the average yield per tree will be 400 oranges. The average yield will decrease by 4 oranges per tree for each additional tree planted on the same acreage. How many trees should the grower plant to maximize the total yield?

32. Let $f(x) = x^2 + \dfrac{8}{x}$. Determine all of the following: horizontal and vertical asymptotes, the critical points, intervals on which f is increasing or decreasing, the inflection points, and the intervals on which f is concave up or concave down. Finally, sketch the graph of $f(x)$.

33. (a) Find $\dfrac{d^2u}{dr^2}$ for $u = \ln(\cos r)$.

(b) Find $\dfrac{d^2u}{dt^2}$ for $y = \cos^2(2t)$.