Page 467

Page 468

Page 469

Page 470

Page 471

Page 472
line 1: H should be K.
line -6: It's more correct to replace P_j by P_j^* in the formula.

Page 473
line 1: D and X(m) are defined on page 486.
line 10: The little square is a picture of the Young tableau with only one box.
line 10: The letters f and g now denote Young tableaux, where as 5
lines before they were used for functions on S^1.
line 11: The number Delta is defined on page 518.
line 13: The indices of a^* are g_1 and f.
line 13: The first sum is indexed over all Young tableaux f_1 subject
to h < f_1 < g.
The second sum is indexed over all Young tableaux h that are
simultaneously obtainable by adding one box to g, or one box to g_1.
line -1: The sum is indexed over all g that are obtainable by adding a box to f.

Page 474
line -11: "less refined" refers to the fact that the braiding map
introduced here would still need to be corrected by a phase.

Page 475
line 12: The notation L^AG is introduced on page 504.

Page 476
line 21: The third word of the line has an extra letter.
line -15. SU(N)_\ell x SU(\ell)_N --> SU(N\ell)_1 is more precise.

Page 477

Page 478
line 10: Note that \lambda^N V is trivial as SU(N) representation. It
would only make sense to keep that term if we were dealing with U(N)
representations.
line 20: >_h should be >_k.
line -3: n should be N.

Page 479
line 4: >_h should be >_k.
line 11: Remember that g > f means that g is bigger than f by exactly one box.
line 12: X_k means the same as X_{[k]}.

Page 480
line -12: Note that the element e_{i_1} \wedge e_{i_2} \wedge ....
contains "almost every" basis vector of the orthogonal of P, and only
finitely many from P.

Page 481
line 11: the first F_Q should be F_P.
line 11: The \mu_i were called \lambda_i in the previous proof.

Page 482
lines 14 and 15: See the equation on line -16 of page 501.
line -18: SU_\pm(1,1) is a double cover of the group of Mobius
transformations of S^1.
line -13: Note the (\alpha-\bar\beta z)^{-1} is a square root of the
derivative of g^{-1}. Therefore, f is secretely a section of the

spinor bundle (\Omega^1_\CC)^{otimes 1/2}.
line -2: "multiplication by z" refers to the scalar operator z times identity.
line -1: The grading operator is U_{-1}.

Page 483
line 1: The operator U_z acts by z^n on the "charge n" subspace of the
Fock space. So the lemma says that LSU(N) preserves the charge.
line -16: This is the definition of \mathcal{L}G.
line -13: Note that Rot(S^1) is not a subgroup of SU_\pm(1,1): compare
the formulas on line 18 and -13 of page 482. The "correct" circle
action is the one coming from the maximal torus in SU_\pm(1,1). But
since LSU(N) preserves the charge, the discrepancy is actually
irrelevant.

Page 484
line 20: The T_n in the integral should be T.
line 21: T should be T_n.
line 21: T_0 is scalar because it commutes with <U_t>, and H is
irreducible as \Gamma\rtimes T- module.
line 22: Remove the sentence part "Tv cannot be a multiple of v and therefore".

Page 485
line 2: Note the weird definition: level l representations are defined
to be subreps of F_P^{\otimes l}. It turns out that all positive
energy representations of LSU(N) occur as subreps of F_P^{\otimes l},
but this is not proved in this paper.
line -10: the dot denotes a right action of \gamma^1 on g.
line -3: "\otimes I" should be added at the end of the right hand side.

Page 486
On this page, there is a big confusion between the two possible ways
z=e^{i\theta} of parametrizing S^1. The operator d is then given by
either -id/d\theta, or by zd/dz.
line 17: e^{id} should be e^{i\theta d}.
line 17: The theta in r_\theta is not the same as the theta in
-id/d\theta. The former is an element of the group that acts, while
the latter is the coordinate on S^1.
line -11: Note that most of the time, the expressions e_j(m)^* and
e_i(m+n) anticommute. The expression for E_{ij}(n) can then be
rewritten as a sum over Z.
line -5: The second term should be +m(X,Y)\delta_{n+m,0} I.
lines -8 to -5: The equations in (a), (b) and (c) depend linearly on
a_{ij}. So they don't depend on the condition \sum a_{ij}E_{ij}\in Lie
U(N), i.e., on the condition a_{ij} = -\bar a_{ji}.

Page 487
line 6. There's an extra comma after X.
line 12: It's in the equation two lines below that m is assumed to be
non-negative.
line 12: "since \lambda(X,Y)(m,n) = -\lambda(Y,X)(n,m) by
antisymmetry" is a better explanation.

Page 488
line 8: This estimate is not optimal. The optimal estimate would have
an s+1/2 instead of the s+1. This is due to the fact that the estimate
on line 15 is not optimal.
line 15: The optimal estimate involves a factor of the form O(|n|) +
O(\sqrt \mu).
line -10: The first of the three X's should be lowercase.
lines -3 to -1: That doesn't seem to get used in the argument.

Page 489
line 12: One may safely remove "H^0 \subset".
line 15: Recall that by definition (top of p. 485), a "level \ell"
positive energy representation is a summand of the \ell-th tensor
power of the Fock representation.
This is an annoying convention, since we'd like to know that *any*

projective (LSU(N)\rtimes S^1)-rep differentiates to a projective (L^0 \g \rtimes \RR)-rep.
The latter is actually easy to show: use that LSU(N)\rtimes S^1 contains lots of copies of SU(2) and SU(3), and that any representation of a compact group decompose into finite dimensional pieces.
line 19: The last summand should be n\ell\delta_{n+m,0}(X,Y).

Page 490
line 3: The expression for H^0 might be clearer with two extra pairs of parentheses.
line 18: H_f and H_g are isomorphic iff there is an a such that f_i=g_i+a for all i from 1 to N.
line -20: The relevant lemma is on page 489.

Page 491

Page 492
ine 15: non-zero subrepresentations,...
line -10: The first occurrence of H_2 should be an H_1.

Page 493
line 1: Recall that in order to talk about self-adjoint operators, it is enough for the Hilbert space to be defined over the reals. Therefore it makes sense to talk about conjugate-linear self-adjoint operators.
line 15. For \xi in i K, the function f(t) = \delta^{it} \xi also extends to the same same strip. But this time, it satisfies f(t-i/2) = -jf(t).
line 18: The statement includes a few unnecessary assumptions: The fact that j_1 and u_t commute follows from the equation on line 23. And the fact that g(t) is bounded on the whole strip is a consequence of the fact that it's bounded on the compact interval [0,-i/2], and that u_t is unitary.
line 23: The two f's should be g's.
line -14: The argument on the bottom of page 497 provides an explanation of the words "By uniqueness of analytic extension".
line -14 The two f's should be g's.

Page 494
line 10: M_sa denotes the self-adjoint part of M (and not its skew-adjoint adjoint part).
line -12: The assumption JMJ\subseteq M' is always true.

Page 495
line 4: "the lemma" refers to lemma 2.
line 17: The argument goes a bit fast here: one needs to argue that \Delta^{it} fixes the closure of N_{sa}\Omega.

Page 496
line 8: Here, Wassermann really means \Lambda (not \tilde \Lambda).
line 16: Note that M is not the same as the von Neumann algebra generated by the a(\xi)'s, for \xi in K. The latter is simply B(H_0).

Page 497
line 5: The statement of the KMS condition used here is somewhat different than the one stated on page 493. It reads:
For \xi in K + iK, the function f(t):=\delta^{it}(\xi) extends to the strip -1/2 \le Im(t) \le 1/2, and satisfies f(t-i/2) = jsf(t), where s is the (unbounded) involution
whose +1 and -1 eigenspaces are K and iK respectively.
line -9: The correct formula is j=-i(2P-1)F

Page 498
line 7: Remove "the restriction of".
line 7: The fact that polynomials are dense is actually somewhat non-trivial (try to approximate z^{-1} on the upper semi-circle by a

polynomial in z).
line 9: Holomorphic with respect to the new complex structure.
line 14: <1/2 should be <1/4.
line 15: <1/2 should be <1/4.
line 16: Parentheses are missing in the argument of p.
line 17: <1/2 should be <1/4.
line 21: <1/2 should be <1/4.
line 23: The maximum modulus principe cannot be applied in this situation. The argument is therefore wrong. To see that the function f is bounded, one first uses the compactness of [0,-i/2] to show that it is bounded on [0,-i/2]. One then shows that llf(t-is)ll = llf(-is)ll, since the former can be obtained from the latter by applying the unitary u_t.
line 25: Ff(t) = +iPQFp_t +...
line 26: f_1(t-i/2) = +iPQFp_t
line -16: U is not unitary unless one inserts the factor \sqrt{2} in front of the right hand side of Uf(x)=... . This omission has no further consequences.
line -12: "it is easy to check..." The way one computes the Fourier transform of f_+ is by approximating the integral by a complex contour integral, and then writing it as 1/(2\pi i) times the sum of the residues in the upper half plane.

Page 499
line 1: The expressions for b(x) and c(x) are off by a minus sign.
line 7: In the equation (I-r)^{it}r^{-it}=(I-A)^{it}A^{-it}, the left hand side uses the new complex structure, while the right hand side uses the old one.
line 10: In the expression for (e-f)/2, the right hand side is off by a sign.
line 11: The useful formula is W(PQ-QP)W^* = (0 & m(c) \\ m(c) & 0).
lines -4 to -1: The creation operators a(\xi) used on page 496 are defined with respect to the new complex structure, whereas the formulas used here are written with respect to the old one.

Page 500
line 15: An extra factor of sqrt(2) should be inserted to make the Cayley transform unitary.
line 17: Note that W is not the same as the W from page 498.

Page 501
line 19: The "exponential lemma" is on page 487.
line -3: W = \C ^{N\ell} = (\C^N)^{\oplus \ell}.

Page 502
line 1: The meaning of "compatible" is explained in the lemma on page 483.
line -6: One should remove the \otimes I.
line -5: In two places I should be I^c.
line -2: p_j should be replaced by its adjoint p_j: H_j --> F_W.

Page 503
line 16: N = pi_0(L_I G)''

Page 504

Page 505
line -8: remove the (1-z).
line -6: see page 508 for an explanation of the rational canonical form.
line -4: see page 509 for an explanation of the symmetry property.
line -1: This formula is proved on page 513.

Page 506
line 2: Q is not the same as the Q on the previous page!
line 6: A = P.
line 9: the indices should be j instead of i.
line 9: the \mu's are not the same as the \mu's appearing in equation (1).
line 12: despite the appearance, the quotient of gamma functions is the same as the one appearing on the bottom of the previous page:

indeed, the new \mu_j's are equal to \alpha plus the old \mu_j's.
line 13: \gamma should be \alpha.
line -7: Note that having equal eigenvalues is allowed.
line -1: the extra n's are unnecessary but not wrong.

Page 507
lines -8 and -5: v_i should be \xi_i.

Page 508
line 6: It should be c(t) = a(t) + b(t).
line 11: similar mistake: b(t) = c(t) - a(t).
lines -14 to -7: This part of the argument is completly upside down.
The matrices P and Q constructed in that paragraph are the transposed
of the ones appearing on lines 1 and 2. To fix the argument, replace
\phi with v everywhere, and read all formulas from right to left.

Page 509
line 2: R should be -R.
line 3: c(t) = a(t) + b(t).
line -12: \sum \lambda_i - \mu_i is automotically non-zero, see the
corollary on page 511.
line -4 and -3: The normalizations are not compatible with the
normalizations introduced on page 505. To fix this, one should add
e^{i\pi\lambda_i} in front of z^{\lambda_i}g(...)... and e^{i\pi\mu_j}
in front of (z_1)^...h(...)...
line -3: the exponent of (z-1) should be -\mu_j.
line -1: One should add e^{i\pi\lambda_i} on the left hand side, and
e^{i\pi\mu_j} in front of (z_1)^... on the right hand side.

Page 510
line 4: One should add e^{i\pi(\lambda_i-\mu_j)} to the right hand side.
line -8: \alpha_n-\alpha_{n-1} = (\chi_P(\lambda+n)-1)\alpha_{n-1}.

Page 511
line 1: A = t + P.
lines 1 to 6. This proof is easier to understand if one starts reading
it from line 5.
lines 8 to 10: The letter chi should not be capitalized.
line -8: the second occurrence of \lambda_1 should be a \lambda_j.
lines -10 to -8: The exact set of conditions used in the argument on
page 512 is:
\mu_1 > \mu_j for all j not equal to 1;
\mu_j > \lambda_j for all j not equal to 1;
\lambda_1+1 > \mu_j for all j not equal to 1;
line -8: The above mentionned set of conditions does not imply \delta
< 0 (unless we also impose \mu_1 > \lambda_1).
line -6: remove (1-z).
line -4 remove \phi(x)\eta.
line -1: the exponent -\lambda_i should be -\lambda_1.
line -1: one should add an extra z^n just after the summation symbol.

Page 512
line 1: The beta function identity can be proved by a change of
variables (x,y):=(zt,z(1-t)) in the double integral defining
\Gamma(a)\Gamma(b).
line 7: the inequalities should hold for i and j ranging from 2 to N.
line 9: The important case is are when z is large and negative.
line 13: remove (\zeta_j,\eta).
line -6: The product should run over all j \not = 1.

Page 513
An important part of the argument is missing: one needs to should that
U_0 is non-empty.
This can be done by picking a(t) and b(t) as on page 508, such that
the roots \lambda_1 > \lambda_2 > ... > \lambda_N of a(t) are
interspersed with the roots of b(t).
line -19: z^n should be z^{-n}.

line -4: \xi \otimes f should be replaced by v \otimes \xi, and f should be replaced by v.

Page 514
line -8: (p_\square \otimes p_f) should be (p_\square \otimes p_f)^*.

Page 515
The constant A can be safely omitted since it is equal to 1.
line 15: p_j should be p_j^*.
line 18: P_j should be P_j^*.
line -4: The formula for c_{kh} appears on page 506, modulo a correction to be found on line 11 of page 518.
line -4: \mu_{kh} is given on line -16 of page 521 (see also the proposition on page 520).
line -2: e^{i \mu \theta} should be replaced by e^{-i \mu \theta}.

Page 516
line 3: in the formula on the top of the page, u, v, \xi, and \eta should be taken to be free variables.
line 8: The two occurrences of F on this line refer to different things!
line 9: One may safely omit the support conditions: they are not used in this proposition.
line 10: the last e^{i\theta} should be e^{-i\theta}.
line 12: f_n g_{-n} should be replaced by f_{-n}g_n.
line 13: Once more, the last e^{i\theta} should be an e^{-i\theta}.
line -14: note that exchanging the two support conditions doesn't affect the argument.
line -12: the last e^{i\theta} should be e^{-i\theta}.

Page 517
line 1: This follows from the proposition on page 520 (see also line -16 on page 521).
line 2: extra comma.
line 3: With the support conditions as stated above, the correct formula has an extra e^{2i\mu_{kh}\pi} after the summation symbol. Another way of fixing the formula is to take supp(g) before supp(f), going counterclockwise from 1.
line 4: e^{i\mu\theta} should be e^{-i\mu\theta}.
line 8: \tilde g \star f should be replaced by \tilde f \star g.
line 8: G_k should be G_h.
line 9: The useful formulas are e_\mu(\tilde f * g) = \tilde{e_{-\mu}} * e_\mu g if supp(f) is before supp(g)
and e_\mu(\tilde f * g) = e^{2\pi i \mu} \tilde{e_{-\mu}} * e_\mu g is supp(g) is before supp(f).
line 16: the proposition is correct assuming that supp(g) is before supp(f).

Page 518
line 4: Replace "after" by "before".
lines 6,7: The corollary on page 516 only used the fact that the supports of f and g are disjoint.
Similarly, the argument on the previous page only used the fact that the supports of f and g are disjoint and don't include 1.
line -13: The inner product is defined on the bottom of page 486.
line -5: This is twice the dual Coxeter number of SU(N), a fact that holds for all Lie algebras.
lines -1 and -2: The first summation symbol also applies to the second part of the formula.

Page 519
line 1: N \ell X(1) should be -\ell X(1).
lines 1,2: Once again, the first summation symbol also applies to the second part of the formula.
line 3: In two places, N \ell X(1) should be replaced by -\ell X(1).
line 3: The last (0) should be (1).
line 4: A minus sign is missing from the last formula.
line 6: The minus sign should be removed from the first formula.

Page 520
line 13: The 1/2 should be removed.
lines -12 and -11: n should be -n, and -n should be n (twice).

Page 521
line 2: The X should be X_i (twice), and the pi_q should be pi_v.
line 2: The last X_i should be X_i \otimes 1.
line 3: The X_i should be 1 \otimes X_i.
line 6: add "is the".
line -8: With the notation from page 518, "after" leads to \nu_{ij} = d_{ij}, and "before" leads to \nu_{ij} = c_{ij}.
line -5: The sentence "and h permissible" should be added after "h>g" (recall that the notation h>g means that h is obtained by adding one single box to g).
line -3: The subscript f should be a g. (here f is a function on S^1, while g is a Young tableau!!).

Page 522
line 7: the subscript f should be a g.
line 8: the subscript \square should be \bar\square.
line 11: the subscript f should be a g; the subscript g should be an f; and the subscript \square should be \bar\square.
line 12: the subscript g should be an f.
line 16: The N is irrelevant and can be omitted.
line 16: is decreasing.
lines -17, -16: The h_k correspond to places where one can add a box, while the f_j correspond to places where one can remove a box. So Wassemann got his north-west and south-east reversed.
lines -9, -8: the i and j should be interchanged.
line -5: the i and j should be interchanged.
line -4: it gives +1.
line -4: The correct sentence is "... that if h is non-permissible and f is permissible, ..."

Page 523
line 5: Note that the result remains true if "after" is replaced by "before".
line 6: The sum is indexed over all g_1 sitting between f and h.
line 6: The coefficient \mu_{g,g_1} (which is equal to the quantity d_{kh} from pagre 518, line 11) is non-zero except if g is permissible while g_1 is not permissible.
line 8: The fact that dim(W) = 2 is responsible for the label "hypergeometric", as opposed to "generalized hypergeometric".
line 9: The indices of \Omega should be two squares (as opposed to a 2x1 rectangle).
line 12: The indices of \Omega should be two squares.
line 12: \beta = 2/(N + \ell).
line 14: The second and third g's should be g_1.
line 17: add "0 <" before the absolute values.
line 19 The indices of \Omega should be two squares.
line 20: The formula is maybe clearer is one inserts a \circ between T and (S\otimes I).
line -12: The product over all g_1' \not = g_1 consists of only one factor.
line -12: The g_1' in the argument of the second gamma function should be a g_1.

Page 524
line 4: Note that Theorem C is very similar to Theorem A, but now g \not = g_1.
line 11: Similarly, Theorem D is very similar to Theorem B, but now g = g_1.
line -20: \alpha = (\Delta_h + \Delta_f - 2\Delta_g) / 2(N + \ell).
line -16: once again, the two squares in the index of \Omega should not be touching each other.
line -12: the transport coeficient is e^{-i\pi\nu_{\pm}}.
line -11: the index should be \alpha instead of -\alpha.
line -10: the index should be -\alpha instead of \alpha.
line -3: All the g_1's should be g's (and the very last one can be removed from the inequality).
line -3: the sum is over all permissible h.
line -2: The "1" is an index of "f".

line -2: The condition at the end is h < f,f_1 < g (which means h < f < g and h < f_1 < g).
line -2: the sum is over all permissible f_1.

Page 525
line 9: The typos of the previous theorem reappear unchanged in the corollary.

Page 526
line 13: The last expressions should be yx\Omega.
line 17: The word "natural" is misleading. The unitary U_\phi is well defined up to phase, and becomes well defined given a choice of lift of \phi to the universal cover of SU(1,1).

Page 527
line 3: while x and z are elements of curly X and Z, y is an element of straight Y.
line 3: the lemma referred to is the "Hilbert space continuity lemma".
line 8: "where the sum runs over all permissible h satisfying h > g,g_1" is maybe more clear (and recall that h>g means that h is obtained by adding one single box to h).
line 9: the sum runs over all permissible f_1 satisfying h < f_1 < g.
line 15: the sum is indexed over all permissible g such that g > f.

Page 528
line 3: The first sum is over all (permissible) g_1 subject to g_1 > f. The second sum is indexed over all (permissible) h,k, subject to f < g,g_1,k < h, and the big parentheses are misplaced.
line 4: c.f. page 503.
line 8: The sum is over g_1,h,k.
line 9: "taking all but one \eta_{g_1} equal to zero, and the remaining one not in the kernel of a_{h,g_1}"
line 10: Here, there is a mistake. Following the argument, we get that \sum_{h:h>g,g_1}\lambda_{g_1}\nu_h\mu_{g_1}||a_{h,g_1}\eta_{g_1}||^2 \ge 0.
We are free to pick \eta_{g_1} in H_{g_1} and a in L^2 of the upper half circle. So by von Neuman density, we're free to pick a_{h,g_1}\eta_{g_1} in the direct sum of H_h, indexed over all h > g,g_1.
Picking it so that all but one component vanishes, we get \lambda_{g_1}\nu_h\mu_{g_1} \ge 0. We know that \lambda_{g_1}>0 and that \nu_h and \mu_{g_1} are non-zero, hence \nu_h\mu_{g_1}>0.
line 10: Despite the notation, \nu_h also depends on g, g_1 and f. Similarly, h_k also depends on h, g and f.
line 11: Once again, despite the notation, \nu_h and \mu_{g_1} depend on all of f, g, g_1 and h.
line 13: sum over g:g>f
line 14: sum over g,h,k: f<g,k<h (by which I mean f<g<h and f<k<h).
line 17: First sum over i,j. Second sum over i.
line 18: direct sum over k:k>f.
line 20: sum over g,h,k: f<g,k<h.
line -9: sum over g,h:f<g,g_1<h.
line -8: sum over h > g_1.

Page 529
line 17: The title of the section is misleading: we're doing Connes fusion with the positive energy representation whose lowest energy subspace is an exterior power of C^N (i.e. the vector representation of SU(N)).
line 21: The alpha doesn't havea minus.
line 7: It will turn out that \lambda(g) is never null.

Page 530
line 8: the notation ">_k f" is wrong since we don't want to include yet the condition that the blocks are in different rows.
line 11: "non-negative" in the sense that (\mu\xi,\xi)\ge 0 for all vectors \xi.
line -14: "vectors in"
line -12: The first equation should be understood as an equality

between operators from \oplus_{f_1:f_1>f}H_{f_1} to H_{f'}.
line -5: sum over paths P,Q.
line -2: k is a subscript of >.

Page 531
line 2: The star should be on the b.
line 12: The sum is over paths P and Q. The index of a' should be a Q.
line 14: The very first occurrence of Q_1 should be a Q.

Page 532
line 1: This non-strict inclusion will soon turn out be an equality.

Page 533
line -10: \Lambda_0 is (N+\ell) times the lattice {(m_1) | sum_i m_i = 0}.

Page 534
line 13: In Lie theory, it is very standard to denote this element \delta by the letter \rho.
line -12: The X notation comes from page 478/479. We always have X = \chi.
line -12: The notation is a little bit abusive since \sigma(f+\delta)-\delta is a signature (= not a positive weight for SU(N)).

Page 535
line 3: These characters are the elementary symmetric functions.
line 4: A priori, it is not clear that the map S \otimes \C ---> \C^T is injective. One first needs to show that that map is surjective, and then count dimensions.
line 7: "coincides" has not been proven yet: only "maps onto".
line -20: The notation \R has been introduced on page 474.
line -18: "character" is an unfortunate name. "character of the lowest energy subspace" is a better description.

Page 536