Classical phase space structure in optical microcavities

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The study of optical processes in microcavities is a diverse and expanding field, extending from applied topics such as the design of novel microlasers and other optical devices to questions of more fundamental interest in nonlinear optics and cavity QED. For an overview see Ref. [1].

In all of these areas, the key element is an extremely small resonator cavity in the size range of 1 to 1000 μm, at wavelengths from 0.1 to 10 μm. One of the most effective designs relies on *total internal reflection* as a means of confining the light. This is achieved when light rays circulate near the perimeter of some convex cross section of a dielectric body surrounded by a lower-index medium. The resulting resonator modes have a characteristic spatial intensity distribution concentrated near the dielectric interface, which in recognition of an analogous acoustic effect is referred to as the "whispering-gallery" (WG) phenomenon. The longest measured optical resonance lifetimes for such WG modes in clean glass microspheres are three orders of magnitude longer than can be achieved in microwave resonators with superconducting mirrors.

However, for technological applications such extreme lifetimes are useless, and prescriptions for degrading the resonator quality to some desired level are sought. For dielectric bodies with rotational symmetry, there furthermore exists no preferred emission direction, leading to isotropic emission in microlasers based on such cavities. Both shortcomings can be addressed by turning to *asymmetric resonant cavities* (ARCs), convex but substantially deformed dielectric resonators for which the classical wave equation is nonseparable.

The significance of ARCs in the highly competitive field of microlaser development [2] has increased rapidly since their emission properties have been shown to be predictable largely relying on classical ray dynamics [3]. The ray dynamics of a generic ARC is partially chaotic and in fact represents a realization of a plane hard-wall billiard as long as no escape occurs. A semiclassical treatment is needed to connect the ray description with the discrete modes of the cavity. This makes ARCs a laboratory for the study of *quantum chaos*, similar to microwave resonators, however with an emphasis on novel questions arising due to the openness of the resonator, such as the lifetime and emission directionality of individual quasibound states.

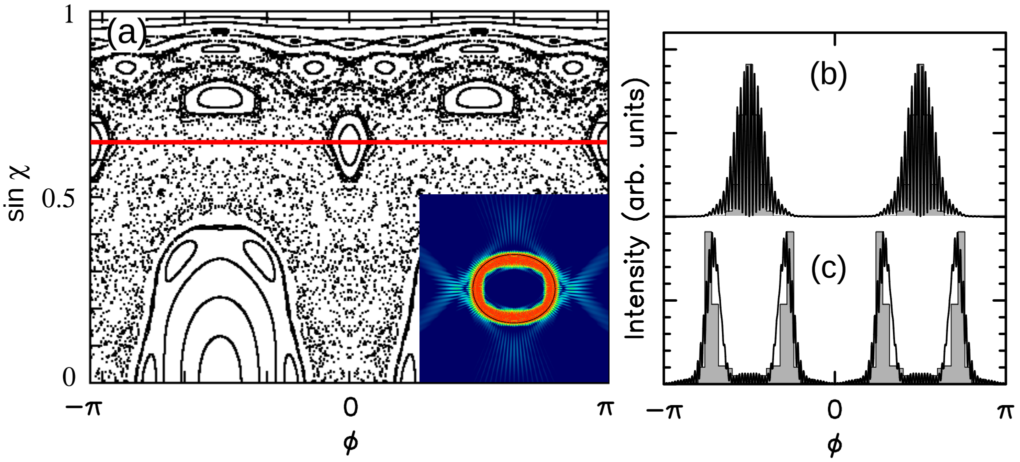

Figure 1: (a) Inset: TM polarized quasibound state wavefunction in a $\sim 10\%$ deformed ARC, $n = 1.54$. Main: Poincaré section for this cavity. Straight horizontal line indicates the classical escape condition. Farfield intensity patterns are shown in (b) for $n = 2$ and (c) for $n = 1.54$. Solid lines are wave results, histograms are ray simulations.

A WG mode in a glass rod with a deformed cross section is shown in the inset to Fig. 1(a). Contrary to the results of an adiabatic approximation that is known to be exact in the ellipse billiard, emission here does not emanate tangential to the points of highest curvature. This is due to the KAM transition to chaos occuring in this oval shape. The Poincaré section (SOS) shows islands due to a stable periodic orbit intersecting the total-internal-reflection condition, $\sin\chi = 1/n$, which represents the lower bound on the angle of incidence χ allowing total internal reflection at the interface for refractive index n. A chaotically diffusing ray cannot enter the island, and hence in this case is prevented from reaching the classical escape condition at the points $\phi = 0$ and π, which are the highest-curvature points. This *dynamic eclipsing* affects all WG modes for which classical ray diffusion as just described is the dominant escape mechanism, as opposed to the always-present tunneling. Ray and wave results agree very well at large deformations, whether the adiabatic approximation is valid [Fig. 1 (b)] or not [Fig. 1 (c)].

A cylindrical ARC shape has been combined with a quantum cascade

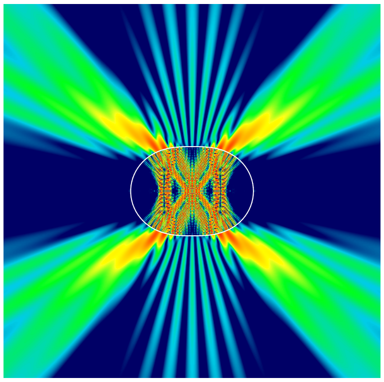

Figure 2: Bowtie mode in the 15% deformed "rounded-stadium" shaped microlaser.

heterostructure active region to create electrically pumped microlasers with an effective n of 3.3, a short diameter of $50\,\mu$ and long axes varying from $50\,\mu$ to $80\,\mu$, emitting at $\lambda = 5.2\,\mu$. The significance of phase space structure has been demonstrated in these devices [4]. At $\sim 15\%$ fractional deformation we find highly directional emission and an enhancement of the maximum emitted power by three orders of magnitude compared to circular resonators, but the dominant lasing modes are *not* of of the WG type. Instead, at $\sim 10\%$ to 12% deformation a crossover takes place from WG modes to bowtie-shaped bifurcations of the diametral "bouncing-ball" orbit, cf. Fig. 2. These orbits partially violate the total-internal-reflection condition but acquire higher reflectivity at higher deformation, until according to Fresnel's formula their lifetime in fact becomes long enough for lasing. This is substantiated by (1) excellent agreement of the laser mode spacings with semiclassical expectations, (2) a *decrease* of the laser threshold versus deformation above $\sim 12\%$, and (3) good agreement with numerically calculated intensity patterns.

References

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