

**EC 423/523 Homework assignments, with hints.**

**Peter Lambert**

EC 432/523 Econometrics I

Peter Lambert

Homework #1 : Probability

**Questions in 6e:** 2.14, 2.16, 2.24, 2.26, 2.28, 2.42, 2.44, 2.48, 2.60, 2.70, 2.78, 2.84, 2.88, 2.90, 2.94

**Same ques. in 7e:** 2.16, 2.18, 2.32, 2.34, 2.36, 2.52, 2.54, 2.62, 2.76, 2.94, 2.102, 2.108, 2.112, 2.114, 2.118

To be handed in Tue week 2

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Homework #2 : Distributions

**Questions in 6e:** 3.4, 3.8, 3.16, 3.18, 3.28, 3.102, 3.116, 4.6, 4.8, 4.22, 4.30, 4.50, 4.60abc, 4.114

**Same ques. in 7e:** 3.4, 3.10, 3.22, 3.24, 3.40, 3.130, 3.146, 4.12, 4.14, 4.28, 4.40, 4.64a, 4.74abc, 4.146

To be handed in Tue week 3

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Homework #3 : Bivariate Dists.

**Questions in 6e:** 5.6, 5.8, 5.12, 5.20, 5.22, 5.24, 5.44, 5.46, 5.62, 5.64, 5.80, 5.98, 5.118

**Same ques. in 7e:** 5.8, 5.10, 5.14, 5.22, 5.26, 5.28, 5.52, 5.54, 5.72, 5.76, 5.94, 5.118, 5.138

To be handed in Tue week 4

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Homework #4 : Fns of RVs

**Questions in 6e:** 6.2, 6.6, 6.16, 6.24, 6.36, 6.44

**Same ques. in 7e:** 6.2, 6.6, 6.20, 6.28, 6.42, 6.52

To be handed in Tue week 5 (at end of quiz)

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Homework #5 : Sampling

**Questions in 6e:** 7.2, 7.6, 7.8, 7.18, 7.22, 7.24, 7.28

**Same ques. in 7e:** 7.10, 7.14, 7.16, 7.36, 7.42, 7.44, 7.48

To be handed in Tue week 6

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Homework #6 : Estimation

**Questions in 6e:** 8.2, 8.4, 8.8, 8.36, 8.44, 8.62, 8.70, 8.74, 8.88

**Same ques. in 7e:** 8.6, 8.8, 8.12, 8.40, 8.58, 8.74, 8.82, 8.88, 8.102

To be handed in Fri week 7 (at end of quiz)

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Homework #7 : more estimation, plus hypothesis testing

**Questions in 6e:** 9.2, 9.14, 9.32, 9.74, 10.2, 10.10, 10.28, 10.67\* 10.82

**Same ques. in 7e:** 9.2, 9.20, 9.40, 9.82, 10.2, 10.20, 10.38, 10.79(a,b)\*, 10.94

\* the answer to this odd-numbered question is in the back of the book: your GTF will *not* be satisfied without a *full explanation*

To be handed in Tue week 9

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# Hints for the homework questions

(using the numbering of the 6<sup>th</sup> edition, throughout)

EC 432/523 Econometrics I

Peter Lambert

Homework #1 : Probability

**Questions in 6e:** 2.14, 2.16, 2.24, 2.26, 2.28, 2.42, 2.44, 2.48, 2.60, 2.70, 2.78, 2.84, 2.88, 2.90, 2.94

**Same ques. in 7e:** 2.16, 2.18, 2.32, 2.34, 2.36, 2.52, 2.54, 2.62, 2.76, 2.94, 2.102, 2.108, 2.112, 2.114, 2.118

**2.14** straightforward

**2.16** begin with  $S = \{HH, TH, HT, TT\}$

**2.24** label the two styles 1 and 2, and complete the following list:  $S = \{1111, 1112, 1121, \dots\}$

**2.26** straightforward

**2.28** straightforward (is it about combinations or arrangements/permutations?)

**2.42** straightforward

**2.44** it is similar in structure to the example in lectures where  $Y = \#$  women when 2 persons chosen from these six:  $\{\text{♀♀♀♂♂♂}\}$ ....

**2.48** it is similar in structure to the example in lectures where 20 laborers are allocated to 4 construction jobs.

**2.60** Define two simple events,  $A = \{\text{plumber A does a consumer's job}\}$ ,  $U = \{\text{the consumer's job is done unsatisfactorily}\}$ . Relate the data you are given to *some* of the probabilities  $P(A)$ ,  $P(U)$ ,  $P(A \cap U)$ ,  $P(A \cup U)$ ,  $P(A | U)$ ,  $P(U | A)$ , and frame the two questions you are asked in terms of probabilities involving  $A$ ,  $U$ ,  $\sim U$ .

**2.70** Define events  $\alpha = A$  detects smoke and  $\beta = B$  detects smoke. From the probabilities that are given, are  $\alpha$  and  $\beta$  independent events or not?

**2.78** The following probability occurs somewhere in this question:  $P(I \cap II | I \cup II)$  (to use an obvious notation)...

**2.84** This question is asking for a **PROOF** of the result, not an **ILLUSTRATION** of its veracity. But try and think of an example to illustrate it, too... for the proof, start by defining two events  $\alpha = A$  and  $\beta = B|C$  and apply the result in 2.80 to the two events  $\alpha$  and  $\beta$ ..

**2.88** Let  $F_i$  be the event that radar set  $i$  fails to detect the aircraft,  $i = 1, 2, 3$ .

**2.90** You might find this helpful: define events  $P_T = \{\text{positive reading for truthful person}\}$ ,  $P_L = \{\text{positive reading for liar}\}$ , and let  $N_T = \sim P_T = \{\text{negative reading for truthful person}\}$ , and  $N_L = \sim P_L = \{\text{negative reading for liar}\}$ . Now list the points that make up the sample space :  $S = \{P_T P_L, P_T N_L, \dots\}$  and identify the events described in parts **a-d** of the question with subsets of  $S$ .

**2.94** Given that it takes 2 minutes to type the blood of each donor, how many donors can be typed *at most*, in the search for a match, before the patient is doomed?

**Questions in 6e:** 3.4, 3.8, 3.16, 3.18, 3.28, 3.102, 3.116, 4.6, 4.8, 4.22, 4.30, 4.50, 4.60abc, 4.114

**Same ques. in 7e:** 3.4, 3.10, 3.22, 3.24, 3.40, 3.130, 3.146, 4.12, 4.14, 4.28, 4.40, 4.64, 4.74abc, 4.146

**3.4** Express in terms of events, A, B and C : that valves 1,2,3 respectively fail to open

**3.8** If R is the simple event that a rental occurs on a given day and  $N = \sim R$ , the compound event you are interested in is  $\underbrace{RNNN..NR}_y$

**3.16** Straightforward

**3.18** Although this exercise precedes the presentation of the binomial probability distribution, you could certainly do it in your answer ... or, why not calculate  $P(0)$ ,  $P(1)$  and  $P(2)$  directly, and use them to calculate  $E(Y)$  and  $E(Y^2)$  ?

**3.28** This is binomial. If by “success” you mean recovery, then  $p = 0.8$ . If, on the other hand, you have a somewhat twisted view and think of “success” as being non-recovery (death?), then for you  $p = 0.2$ . Provided you frame the questions in each case, view, you will surely arrive at the same answers (though using different parts of the binomial table on p. 784 of the book). *Why not try both, it is good practice!*

**3.102** I suggest you draw up a little table whose three columns are headed  $y$ ,  $\lambda = 3$ ,  $\lambda = 4$ . In the first of those columns you put appropriate values for  $y$ . In columns 2 and 3, put the values of  $P(Y = y)$  for the appropriate Poisson distributions. It is straightforward to get those values from the table at the back of the book....

**3.116** Straightforward

**4.6**  $F(y)$  needs to be increasing from 0 to 1 along the domain of the random variable.

**4.8** Similar to 4.6

**4.22** You’ll need to use quite a bit of integration in this question

**4.30** This question mixes a continuous distribution [the uniform, for a single parachutist] and a discrete one [the binomial, for the 3 parachutists].

**4.50** Common type of problem: you are given that  $Y \sim N(\mu, \sigma^2)$ , and you are asked a question about  $Y$ . You need to convert this question into an equivalent question about  $Z = \frac{Y - \mu}{\sigma}$ , and then use the normal table at the back of the book.

**4.60abc** Ditto. In particular, for this one, you are asked to identify the “tail” containing the uppermost 28.1% of values. You will need to hunt in the body of the table for this...

**4.114** There are 2 ways you might tackle this. If you make an (unwarranted) assumption that the distribution is normal, you will obtain an interval which is quite narrow relative to the (failsafe) interval that the Tchebychev theorem will give you. Calculate both, compare and discuss!

**Questions in 6e:** 5.6, 5.8, 5.12, 5.20, 5.22, 5.24, 5.44, 5.46, 5.62, 5.64, 5.80, 5.98, 5.118

**Same ques. in 7e:** 5.8, 5.10, 5.14, 5.22, 5.26, 5.28, 5.52, 5.54, 5.72, 5.76, 5.94, 5.118, 5.138

5.6 standard application of definitions

5.8 we started this in class, the first thing you will have to do is sketch the triangle in  $(y_1, y_2)$ -space which the joint density is non-zero, and mark on it the sub-area over which you need to integrate for part (b). Use either horizontal or vertical shading, and figure out the appropriate bounds for the inner integral [in which one variable is given and determines the appropriate limits for the other variable, between which to integrate] and the outer integral [which is over the previously fixed variable's unrestricted range]. In fact, why not **do this both ways**? Excellent practice, and provides a **check that you got it right the first time**.

5.12 You'll need to check that  $\iint f(y_1, y_2) dy_1 dy_2 = 1$ , and this will involve sketching the region over which the density is non-zero and using horizontal or vertical shading... etc.

5.20 this has a table of joint probability values, just like the one I did on the board. Do you remember how to calculate the marginals and conditionals?

5.22 an excellent opportunity to use all those formula relating the joint density with the marginal and conditional densities! As a hint, consider this: if  $a$  and  $b$  are between 0 and 1, then

$$P(Y_2 \leq a \cap Y_1 \geq b) = \int_0^a \int_b^1 f(y_1, y_2) dy_2 dy_1$$

5.24 More smoking chimneys. To get  $f_2(y_2)$  you will need to integrate out over  $y_1$  for a given value of  $y_2$ , that is, along a "horizontal stripe" (assuming you plot  $y_2$  vertically).

5.44 See if you can factorize  $f(y_1, y_2)$  as  $f_1(y_1) \cdot f_2(y_2)$ ..... check that the domains of all three functions correspond...

5.46 smoking chimneys again

5.62 I'd like you to calculate each marginal, check that it is binomial with  $n=2$  and  $p=1/3$ , and then simply use standard binomial results..

5.64 results from 5.22 should be helpful with this

5.80 the formula for the correlation coefficient is  $\rho = \frac{\text{cov}(U_1, U_2)}{\sqrt{V[U_1]} \cdot \sqrt{V[U_2]}} \in [-1, +1]$  (as on an overhead); you will need to use moderately heavy algebra to compute all the values in this expression (in terms of the unstated variances  $\sigma_1^2$  and  $\sigma_2^2$  of  $Y_1$  and  $Y_2$ ). **Hint:** to check that you are getting the right expression for  $\rho$ , verify that your answer reduces to  $\rho = 1/2$  if  $\sigma_1^2 = 3$  and  $\sigma_2^2 = 1$ . If it doesn't, go back and check your algebra!

5.98 the only information you will need is that, if  $X$  is gamma-distributed with parameters  $\alpha$  and  $\beta$ , then  $E[X] = \alpha\beta$  and  $V[X] = \alpha\beta^2$ . There is no gamma distribution information that could help you with part b. And there is no need to plunge into the theory of  $\chi^2$  distributions either..... just use a very general theorem that puts bounds on central/tail areas, for any distribution...

**5.118** This may look somewhat forbidding but let's structure it a bit. First, you are told that  $p(y | \lambda) = \frac{\lambda^y e^{-\lambda}}{y!}$  (i.e. for a given  $\lambda$ ,  $y$  has a discrete Poisson distribution. So what are  $E[Y | \lambda]$  and  $V[Y | \lambda]$ ? Now use these transcriptions of the results linking conditional mean and variance to unconditional mean and variance:  $E[Y] = E_\lambda[E[Y | \lambda]]$  and  $V[Y] = E_\lambda[V[Y | \lambda]] + V_\lambda[E[Y | \lambda]]$  (where the subscript  $\lambda$  means "take the expectation (or variance) with respect to the (gamma) distribution of  $\lambda$ ". Recall that if  $X$  is gamma-distributed with parameters  $\alpha$  and  $\beta$ , then  $E[X] = \alpha\beta$  and  $V[X] = \alpha\beta^2$ .

EC 432/523 Econometrics I  
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 Homework #4 : Fns of RVs

**Questions in 6e:** 6.2, 6.6, 6.16, 6.24, 6.36, 6.44  
**Same ques. in 7e:** 6.2, 6.6, 6.20, 6.28, 6.42, 6.52

**6.2** The direct method is strongly recommended here. That is, first, obtain the distribution function for  $U$ ,  $G_U(u) = P(U \leq u)$ , in each case, and then, to get the density function  $g_U(u)$  from this distribution function .... what must you do? I strongly advise that, before embarking on this strategy, you first of all evaluate the distribution function for  $Y$ , namely  $F_Y(y) = P(Y \leq y) = \int_{-\infty}^y f(t)dt$  as a function of  $y$ . You should be able to use the formula for  $F_Y(y)$  to obtain  $G_U(u)$  in each of the three cases, without the need for any further integration. Be especially careful with part (c) ....

**6.6** You will need to calculate  $G_U(u) = P(U \leq u)$  and then differentiate it. The set of  $(Y_1, Y_2)$  such that  $U \leq u$  forms a sub-area of the triangular-shaped region in which  $f(y_1, y_2) \neq 0$  and you will need to integrate  $f(y_1, y_2)$  over that area to get  $G_U(u)$ . *Be careful: the area you need to identify depends on the value of  $u$  that you assume!* Consider the following 4 cases: (i)  $u \leq 0$ , (ii)  $0 \leq u \leq 1$ , (iii)  $1 \leq u \leq 2$  and (iv)  $u > 2$ . In case (iii), you may find it easier to first evaluate  $1 - G_U(u) = P(U \geq u)$ , rather than to evaluate  $G_U(u)$  directly.

**6.16** In each case, the question is asking for the density function  $g_W(w)$  - which you will get by differentiating the distribution function  $G_W(w) = P(W \leq w)$  - which you will get by turning the restriction  $W \leq w$  into a restriction on  $Y$  and then using the fact that  $f_Y(y) = 1$  for  $0 \leq y \leq 1$ , and is zero elsewhere....

**6.24** The aim is to show that  $g_u(u) = \frac{1}{2} e^{-u/2}$  when  $u > 0$ , and is zero otherwise. You could use either the direct method or the method of transformations for this...

**6.36** The question is asking for  $P(Y_2 > Y_1)$  i.e., setting  $U = Y_2 - Y_1$ , it wants  $P(U > 0)$ . The thing is that, because each of the  $Y$ 's is normally distributed, and they are independent,  $Y_2 - Y_1$  is also normally distributed: see Theorem 6.3 to get the mean and variance of  $Y_2 - Y_1$ . It should be straightforward from this point on.

**6.44** Use the fact that  $m_Y(t) = e^{\lambda(e^t - 1)}$  for  $i = 1, 2$  and the fact that the MGF for a sum of two independent random variables is the product of the two separate MGFs, to recognize (from its MGF) that  $Y_1 + Y_2$  must itself be Poisson-distributed. This means that you can write down a

formula for  $P(Y_1+Y_2=y)$  in the form  $\lambda^y e^{-\lambda} / y!$  for an appropriate value of  $\lambda$ . You should be able to obtain  $P(Y_1 = k \mid Y_1+Y_2 = m)$  now, using the usual rule for  $P(A \mid B)$ . See if you can recognize this probability function – it is binomial...

EC 432/523 Econometrics I

Peter Lambert  
Homework #5 : Sampling

**Questions in 6e:** 7.2, 7.6, 7.8, 7.18, 7.22, 7.24, 7.28  
**Same ques. in 7e:** 7.10, 7.14, 7.16, 7.36, 7.42, 7.44, 7.48

- 7.2** The point of this question is to explore the effect of (a) an increase in the population variance and (b) an increase in the sample size, on the probability that the error of estimation is at most 0.3. What does your intuition tell you? The corresponding probabilities in part (b) of the exercise referred to are as follows:  
 $n = 9 \Rightarrow p = .6318$ ,  $n = 16 \Rightarrow p = .7698$ ,  $n = 25 \Rightarrow p = .8664$ ,  $n = 36 \Rightarrow p = .9284$ ,  
 $n = 49 \Rightarrow p = .9642$ ,  $n = 64 \Rightarrow p = .9836$ .
- 7.6** Before doing this calculation for  $n$ , the size of the sample, check out the other question referred to, which asks for the probability that the error of estimation is at most 0.5 when  $n = 10$ . If you find that this probability is less than [exceeds] 95%, you would surely expect  $n$  to have to rise [fall] in order to achieve 95% probability ... wouldn't you?
- 7.8** Recall from the lecture materials that if  $Y_1 \sim N(\mu_1, \sigma_1^2)$  and  $Y_2 \sim N(\mu_2, \sigma_2^2)$  are independent random variables, then  $\bar{Y}_1 \pm \bar{Y}_2 \sim N(\mu_1 \pm \mu_2, \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2})$ . This property should enable you to adequately tackle the question.
- 7.18** This question relies on F-distributions. All you need to know about these is that, because of the independence of  $S_1^2$  and  $S_2^2$  and the relationship between population variances,  
 For part (a) :  $\frac{S_1^2}{2S_2^2}$  is F-distributed with n.d.f. = 9 and d.d.f. = 7  
 For part (b) :  $\frac{2S_2^2}{S_1^2}$  is F-distributed with n.d.f. = 7 and d.d.f. = 9  
*(ignore the hint in part (b)).* From tables, the 5% upper tail values for are 3.68 and 3.29 respectively (check this for yourself). **Now go for it!**
- 7.22** Standard application of the CLT
- 7.24** Do **7.23** as well. This pair of questions is rather like {Ex 7.5, Ex 7.6}, which you already dealt with, except that here you have to appeal to the CLT whereas there, you could assume normality.
- 7.28** You can't use  $s$  in place of  $\sigma$  in the CLT, but you *can* obtain a C.I. for  $\bar{Y} - \mu$  in terms of  $\frac{\sigma}{\sqrt{n}}$  from the CLT, and *then* approximate  $\frac{\sigma}{\sqrt{n}}$  by  $\frac{s}{\sqrt{n}}$  (where  $s$  is the "early estimate",  $s = 12$ ).

**Questions in 6e:** 8.2, 8.4, 8.8, 8.36, 8.44, 8.62, 8.70, 8.74, 8.88

**Same ques. in 7e:** 8.6, 8.8, 8.12, 8.40, 8.58, 8.74, 8.82, 8.88, 8.102

**8.2** Remember:  $X$  &  $Y$  independent  $\Rightarrow \text{cov}(X, Y) = 0$  &  $V[aX + bY] = a^2V[X] + b^2V[Y]$ . Part (b) will involve evaluating  $\frac{dV[\hat{\theta}_3]}{da}$  and checking the sign of  $\frac{d^2V[\hat{\theta}_3]}{da^2}$

**8.4** If  $Y$  has an exponential distribution with parameter  $\theta$ , as in this question, then  $E[Y] = \theta$ ,  $V[Y] = \theta^2$  and the distribution function for  $Y$  is  $F(y) = 1 - e^{-y/\theta}$  ( $y > 0$ ), 0 otherwise. The hint for question 8.2 above will serve you well in identifying the variances of  $\hat{\theta}_1$ ,  $\hat{\theta}_2$ ,  $\hat{\theta}_3$  and  $\hat{\theta}_5$ . For the mean and variance of  $\hat{\theta}_4$ , you will need to identify the distribution of  $U = \min \{Y_1, Y_2, Y_3\}$ . Ask yourself, how does  $P(U \geq u)$  relate to  $P(Y_i \geq u)$  for  $i = 1, 2, 3$ ? If you figure this out, you will realize that  $[1-G(u)] = [1-F(u)]^3$  and therefore  $G(u) = 1 - e^{-y/\tau}$  ( $y > 0$ ), 0 otherwise, where  $\tau$  is a certain fraction of  $\theta$ . At this point you will have identified  $U$  as exponentially distributed, with parameter  $\tau$ , so that  $E[U] = \tau$  and  $V[U] = \tau^2$ , enough information to complete the analysis. Go for it!

**8.8** To identify the MSE of  $\hat{\theta} = \bar{Y}$ , use  $\text{MSE}(\hat{\theta}) = V[\hat{\theta}] + B(\hat{\theta})^2$ . Recall also that a uniform distribution on  $[a, b]$  has mean is  $\frac{a+b}{2}$  and variance  $\frac{(b-a)^2}{12}$ .

**8.36** Completely straightforward!

**8.44** The interval you will derive comes down to  $\bar{Y} \pm 1.96s/\sqrt{n}$ , can you see why?

**8.62** The sample mean pH lies within  $\pm 1.96\sigma/\sqrt{n}$  of the true mean with probability 95%. So, how big should  $n$  be?

**8.70** You are not told that the population SAT scores are normally distributed, but you could assume it (having read the last paragraph of text on page 404 first).

**8.74** You could compute  $\bar{Y}$  and  $S^2$  directly from the raw data if you have a suitable calculator, or you could collapse the raw data into these two statistics,  $\sum Y_i = 108$  and  $\sum Y_i^2 = 1426$ , using mental arithmetic, and then use your intelligence....

**8.88** More mental arithmetic:  $\sum Y_i = 285$  and  $\sum Y_i^2 = 16823$ . But there is a shortcut for the "intelligence": you will need a value for  $(n-1)S^2$ , not  $S^2$ , and  $(n-1)S^2 = \sum Y_i^2 - \frac{1}{n}[\sum Y_i]^2$ . How do you turn a confidence interval for  $\frac{(n-1)S^2}{\sigma^2}$  into one for  $\sigma^2$  ?

EC 432/523 Econometrics I

Peter Lambert  
Homework #7 : More estimation, plus  
hypothesis testing

**Questions in 6e:** 9.2, 9.14, 9.32, 9.74, 10.2, 10.10,  
10.28, 10.67\* 10.82

**Same ques. in 7e:** 9.2, 9.20, 9.40, 9.82, 10.2,  
10.20, 10.38, 10.79(a,b)\*, 10.94

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**9.2** For this, you will need to apply the general result that if  $X_1 \dots X_n$  are independent then  $V(a_1X_1 + \dots + a_nX_n) = a_1^2 V(X_1) \dots + a_n^2 V(X_n)$ . [Incidentally, this result explains why  $V(\bar{Y}) = \sigma^2/n$ , where  $\bar{Y} = \sum Y_i / n$  and  $Y_1 \dots Y_n$  are independently drawn values from a distribution whose variance is  $\sigma^2$ : just put  $a_1 = a_2 = \dots = a_n = 1/n$ ].

**9.14** This can be inferred from the result that for large  $n$ ,  $\hat{p} \sim N(p, \frac{pq}{n})$ . You are asked: what is the reason that this property implies consistency of  $\hat{p}$ ?

**9.32** You *won't* need to solve Ex. 6.30 (on the Rayleigh distribution) before doing this question. Just write down the likelihood function and try to factorize it .....

**9.74** Obtain the likelihood function, factorize it to identify the sufficient statistic, take logs, and differentiate with respect to  $\theta$  in order to identify the MLE  $\hat{\theta}$  [it will be expressed in terms of the sufficient statistic you obtained]. If  $\hat{\theta}$  is unbiased, it will be an MVUE. If it is biased, but can be made unbiased, again you will have identified an MVUE. To check whether  $E[\hat{\theta}] = \theta$  or not, you will need to know that for this distribution,  $E[\mathbf{Y}^T] = \boldsymbol{\theta}$ .

**10.2** Remember that  $\alpha = P(\text{reject } H_0 | H_0 \text{ true})$  & (for a specific alternative  $H_a$ )  $\beta = P(\text{fail to reject } H_0 | H_a \text{ true})$ . Get these clear in your mind! Then do the question!

**10.10** In the terminology of my lecture, what is the test function  $d(\hat{\theta}, \theta)$  which is population-parameter-free? What is the critical value  $d_0$ , and what is the actual value? Should be straightforward. For the next question, express the rejection region in terms of a range of values for  $\bar{y}$ .

**10.28** (Following on from 10.10), with the specific  $H_a$ , for non-rejection you need to evaluate the probability of (non)-rejection in terms of the  $\bar{y}$ -value, but when  $\theta = \theta_a$ .

**10.67** This involves a  $\chi^2$  test statistic if \_\_\_\_\_ (complete the sentence, i.e. under what distributional assumption)? What is the possible range for the  $p$ -value?

**10.82** The UMP test is based on a criterion of the form  $LR = \frac{L(\sigma_0^2)}{L(\sigma_1^2)} < k$ . If  $H_0$  is true, why

is  $\frac{\sum (y_i - \mu)^2}{\sigma_0^2} \sim \chi_n^2$ ? Explain this as part of your answer.