

Psychology 512 Winter 1984: Analysis of Variance & Design of Experiments

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Class Meetings: (154 Straub)

Prerequisite: 511 or equivalent

Text: Edwards, Allen. (1984). Experimental design in psychological research. (5th ed.).

Reference texts:

- Afifi, A. A. & Azen, S. P. Statistical Analysis. A Computer Oriented Approach. (2nd Ed.) Academic Press 1979.
- Box, G. E. P., Hunter, W. G. & Hunter, J. S. Statistics for Experimenters. An Introduction to Design, Data Analysis, and Model Building, Wiley 1978.
- Brownlee, K. A. Statistical Theory and Methodology in Science and Engineering. Wiley 1960.
- Campbell, D. T. & Stanley, J. C. Experimental and Quasi-experimental Designs for Research. Rand McNally 1966.
- Cohen, J. Statistical Power Analysis for the Behavioral Sciences (Rev. ed.) Academic Press 1977.
- Cochran, W. G. & Cox, G. M. Experimental Designs (2nd Ed.) Wiley 1957.
- Draper, N. & Smith, H. Applied Regression Analysis (2nd Ed.) Wiley 1981.
- Kirk, R. E. Experimental Design. Procedures for the Behavioral Sciences (2nd Ed.) Brooks/Cole 1982.
- Loftus, G. R. & Loftus, E. F. Essence of Statistics, Brooks/Cole 1982 (Ch 11-15)
- Myers, J. L. Fundamentals of Experimental Design (3rd Ed.) Allyn & Bacon 1979.
- Snedecor, G. W. & Cochran, W. G. Statistical Methods (7th Ed.) Iowa State Press 1980.
- Tukey, J. W. Exploratory Data Analysis. Addison Wesley 1977.
- Winer, B. J. Statistical Principles in Experimental Design (2nd Ed.) McGraw-Hill 1971.
- Keppel, G. (1982) Design and analysis: A researcher's handbook (2nd ed.). Prentice-Hall

Topic Outline

- I. Introduction & Overview
- II. Single factor design
  - 1. Fixed effects model
  - 2. Basic data and notation
  - 3. Partitioning of total sum of squares (SS)
  - 4. Expected mean squares (EMS) and the F test
  - 5. Power of the F test
  - 6. Robustness of the F test

## Topic Outline (cont'd)

### III. Linear regression; ANOVA representation

1. Simple linear regression
2. Linear regression with several observations on Y for each X; lack of fit and pure error

### IV. Asking specific questions of data

1. Planned comparisons
2. Post hoc comparisons (data snooping)
3. Trend analysis: orthogonal polynomials

### V. Completely randomized multifactor design

1. Fixed and random factors; mixed models
2. Fixed effects model, 2-way layout
3. Expected means squares, multi-factor designs
4. Orthogonal contrasts and factorial designs
5. Power
6. Multiple comparisons

### VI. Designs for reduction of error

1. Randomized-block designs
2. Repeated measures designs

### VII. Incomplete factorial designs

1. Fractional designs
2. Latin-square designs
3. Nested designs

### VIII. Other topics (time permitting)

1. Analysis of covariance
2. Quasi-experimental designs

EVALUATION: Grade based on weekly assignments, two "midterm" exams and a final.

**FINAL EXAM--Open book (80 points)**

1. (10) An experiment was conducted to investigate the effects of different incentives on the solving of problems by fifth-grade children. The dependent variable was number of problems solved. The table below gives cell means based on  $n = 5$  observations in each of 6 conditions.

INCENTIVE CONDITION (A)

TASKS (B)	Verbal ( $A_1$ )	Monetary ( $A_2$ )	None ( $A_3$ )
Simple ( $B_1$ )	20	18	5
Complex ( $B_2$ )	5	5	5

Given that the AB interaction was significant, compute the sums of squares for appropriate partial interactions.

2. (10) Given the following table of cell means for a  $2 \times 3$  crossed design based on  $n = 10$  observations per cell. Compute the SS's for the trend components of the AB interaction, given B (only) numerical.

	$B_1$	$B_2$	$B_3$
$A_1$	1	2	12
$A_2$	2	10	12

3. (12) Given a  $A \times B \times C$  design. The following questions have to do with relations among population effects. State possible conclusions to be drawn about other population effects for each of the following cases:

- Simple main effects due to A are zero at all levels of B
- The AB interaction is zero at each level of C
- The main effect of A is zero, but simple main effects of A are not zero at all levels of B
- The AB interaction is zero, but simple AB effects are not zero at all levels of C
- The main effect of A and the AB interaction are zero
- The AB and ABC interaction effects are zero

4. (8) A conflict theorist has scaled TAT cards and chosen seven that are equally spaced along a (numerical) sexual content continuum (S). He predicts that low-guilt subjects will give increasing numbers of sexual responses as sexual content increases, and that high-guilt subjects will show the same number of responses (as low-guilt subjects) to the low-sex-content cards but will inhibit responses to the high-sex-content cards. We thus have a  $2 \times 7$  completely crossed design (with  $n$  different subjects in each cell) as follows:

SEXUAL CONTENT

GUILT	$S_1$	$S_2$	$S_3$	$S_4$	$S_5$	$S_6$	$S_7$
$G_L$							
$G_H$							

State explicitly what sources should be significant according to the predictions, and why.

5. (10) The purpose of this experiment is to test the effect of certain drugs on different categories of psychiatric patients from three different hospitals. There are thus three factors in the experiment: Hospitals ( $H_1, H_2, H_3$ ), Drugs ( $D_1, D_2, D_3$ ), and type of patient ( $C_1, C_2, C_3$ ). All drugs were used at each hospital, and all types of patients were tested at each hospital. The design to be used contains nine (9) treatment combinations, and 5 patients are assigned to each of the treatment combinations, for a total of 45 patients, 15 at each hospital.
- identify and write out the design to be used, exhibiting all treatment conditions
  - present a partial ANOVA table, listing each effect and corresponding df
  - specify possible confoundings in this design
6. (10) Suppose the scores from nested designs are incorrectly analyzed by the computer as completely crossed factorial designs, with the following results for your data:

<u>Source</u>	<u>df</u>	<u>SS</u>
A	2	2000
B	4	5000
C	3	3000
AB	8	1400
AC	6	1000
BC	12	2400
ABC	24	600

For each of the following nested designs (assumed to be the correct design erroneously analyzed as a crossed design by the computer) find the SS's and df for each effect (source) and state the number of cells (treatment combinations) in each design:

- $A \times B \times C(AB)$
- $A \times B(A) \times C(A)$

7. (12) Given the following nested design with 8 treatment combinations:

	$A_1$		$A_2$	
	$C_1$	$C_2$	$C_3$	$C_4$
$B_1$	2	4		
$B_2$	4	10		
$B_3$			7	5
$B_4$			4	12

The entries are cell means based on  $n = 5$  observations in each of the 8 cells. Compute SS and df for all sources, and summarize in ANOVA table (SS and df only).

8. (8) Find the expected mean squares for each source in an  $A \times B(A) \times C(A)$  nested design with A and B fixed and C random.

Psy 512. W84 Final KEY (80)

(10)

① I

	A <sub>1</sub>	A <sub>2</sub>
B <sub>1</sub>	1 (20)	-1 (18)
B <sub>2</sub>	-1 (5)	1 (5)

$$\hat{\psi}_{ABI} = 20 - 18 - 5 + 5 = 2$$

$$SS_{ABI}(\hat{\psi}) = \frac{(5)(2)^2}{4} = 5$$

$$SS_{I \times B \text{ comp}}(A \times B) = 5$$

$$SS_{II}(A_{\text{comp}} \times B) = 326.67$$

II

	A <sub>1</sub>	A <sub>2</sub>	A <sub>3</sub>
B <sub>1</sub>	1 (20)	1 (18)	-2 (5)
B <sub>2</sub>	-1 (5)	-1 (5)	2 (5)

$$\hat{\psi}_{ABII} = 20 + 18 - 10 - 5 - 5 + 10$$

$$\hat{\psi}_{ABII} = 28$$

$$SS_{ABII}(\hat{\psi}) = \frac{(5)(28)^2}{12} = 326.67$$

(10)

② A

	L	B <sub>1</sub>	B <sub>2</sub>	B <sub>3</sub>
A	-1	0	1	
A <sub>1</sub>	+1	-1 (1)	0	1 (12)
A <sub>2</sub>	-1	1 (12)	0	-1 (12)

$$\hat{\psi}_{L(B)} = -1 + 12 + 2 - 12 = 1$$

$$SS_{L(B)} = \frac{(10)(1)^2}{4} = 2.5$$

		B <sub>1</sub>	B <sub>2</sub>	B <sub>3</sub>
A	Q	1	-2	1
A <sub>1</sub>	+1	1 (1)	-2 (2)	1 (12)
A <sub>2</sub>	-1	-1 (2)	2 (10)	-1 (12)

$$\hat{\psi}_{Q(B)} = 1 - 4 + 12 - 2 + 20 - 12 = 15$$

$$SS_{Q(B)} = \frac{(10)(15)^2}{12} = 187.5$$

3. (12)

- A main effect zero and AB interaction effect zero.
- AB and ABC effects zero.
- AB effect not zero.
- ABC effect not zero.
- Simple main effects of A will be zero at each level of B.
- AB simple interaction effects will be zero at each level of C.

4. (8)

- Whether there is a sexual content main effect will depend upon whether or not high-guilt responses decline at the same rate as low-guilt responses increase as a function of sexual content.
- Should be simple linear trend of S at  $G_L$ , possibly quadratic trend at  $G_H$ .
- Should clearly be a guilt main effect, since the two groups have equal response frequency for low-frequency sexual content but the low group has a higher frequency at higher-content levels.
- There should be an interaction, since the two curves diverge. In particular,
- the linear trend component of the GS interaction should be significant, and perhaps other trend components.

5. (10)

a. Latin square design:

(c) Possible confoundings (if additivity not satisfied):

- H, DC, HDC
- D, HC, HDC
- C, HD, HDC

	$D_1$	$D_2$	$D_3$	(b) Source	df
$H_1$	$C_1$	$C_2$	$C_3$	H	2
$H_2$	$C_2$	$C_3$	$C_1$	D	2
$H_3$	$C_3$	$C_1$	$C_2$	C	2
				Res	2
				W	36
				T	44

6. (10) (a)  $A \times B \times C(AB)$

Source:	df	SS
A	2	2000
B	4	5000
C(AB)	45	7000
AB	8	1400

(b)  $A \times B(A) \times C(A)$ :

Source	df	SS
A	2	2000
B(A)	12	6400
C(A)	9	4000
BC(A)	36	3000

⑦ (12)  $A \times B(A) \times C(A)$  design

Cells (12): 111 112 121 122 233 234 243 244

Means :	2	4	4	10	7	5	4	12	$\sum C^2$	$\hat{\psi}$	SS
A	1	1	1	1	-1	-1	-1	-1	8	-8	40
B(A <sub>1</sub> )	1	1	-1	-1	0	0	0	0	4	-8	80
B(A <sub>2</sub> )	0	0	0	0	1	1	-1	-1	4	-4	20
C(A <sub>1</sub> )	1	-1	1	-1	0	0	0	0	4	-8	80
C(A <sub>2</sub> )	0	0	0	0	1	-1	1	-1	4	-6	45
BC(A <sub>1</sub> )	1	-1	-1	1	0	0	0	0	4	4	20
BC(A <sub>2</sub> )	0	0	0	0	1	-1	-1	1	4	10	125

Source	df	SS
A	1	40
B(A)	2	100
C(A)	2	125
BC(A)	2	145

⑧ (8)  $A \times B(A) \times C(A)$  design (C random)

Source	EMS
A	$\sigma_e^2 + bm\sigma_{C(A)}^2 + mbc\theta_A^2$
B(A)	$\sigma_e^2 + m\sigma_{BC(A)}^2 + mc\theta_{B(A)}^2$
C(A)	$\sigma_e^2 + bm\sigma_{C(A)}^2$
BC(A)	$\sigma_e^2 + m\sigma_{BC(A)}^2$
W	$\sigma_e^2$