Final Exam
Raies - Math 111
Fall 2012

Monday, December 3, 2012

Name: ___________________________________________

Directions:

• You may use a calculator and a writing utensil (I strongly suggest that you use a pencil); you need to keep your desk clear of any other items. Such prohibited items include cell phones, notes, food or beverage, small animals, and weapons of any kind.

• There are 240 possible points on this exam, however the exam will be graded out of a total of 200 points. If you accumulate over 210 points the extra points will count as extra credit up to a maximum of 10 extra credit points.

• The exam will last one hour and fifty minutes. I will try to update the class as to the the time remaining in regular intervals but ultimately it is up to you to keep track of the time.

• Please ask questions if anything is unclear.

• Be sure to show all of your work.

• Scratch paper and extra graph paper is provided on the back of the exam. If possible try to contain your answer to the space provided within the exam but if necessary you may provide answers on the scratch paper and extra graph paper. If there is work on the scratch paper or extra graph paper that you wish to be graded be sure that you clearly indicate what should be graded and to which problem the work applies.

• Do not tear pages out of the exam if they contain work that you wish to be graded. I will grade them if I find them, but I take no responsibility for loose sheets that are handed in with the exam. You may tear out pages pages which don’t contain anything that should be graded such as pages which only contain scratch work.
1. (8pts) Consider the following graphs

Match the following types of functions with the graphs above:

- Exponential Function
- Logarithmic Function
- Power Function
- Polynomial Function

2. (8pts) The graph of the function $f(x)$ is given below. Assume all prominent features of the graph are shown.

Evaluate the following limits, if they exist. Otherwise write “does not exist.”

(a) $\lim_{x \to \infty} f(x)$

(b) $\lim_{x \to -\infty} f(x)$

(c) $\lim_{x \to 2} f(x)$

(d) $\lim_{x \to -2} f(x)$
3. (15pts) Consider the functions \( f(x) = \frac{1}{4}x + 3 \) and \( g(x) = 4x + 12 \).

(a) Find \((f \circ g)(x)\).

(b) Find \((g \circ f)(x)\).

(c) Are \(f\) and \(g\) inverses of one another? Why or why not?

4. (10 pts) Given the function \( f(x) = x^2 - 4x + 5 \), calculate

\[
f(x + h) - f(x)
\]

and simplify your answer.
5. (12pts) Find the equation of the exponential function $f(x)$ which goes through $(-1, 4)$ and $\left(2, \frac{27}{2}\right)$.

6. (12pts) The variable $y$ is directly proportional to the cube root of $x$. If $y = 12$ when $x = 8$ find a formula for $y$ in terms of $x$.

7. (12pts) Is the function $h(x) = \frac{x^3 + 5x}{x^3 - 16}$ even, odd, or neither? Justify your answer.
8. (24pts) Solve the following equations for $x$ (if a solution exists) and leave your answers in exact form:

(a) \[ \frac{x}{2x + 5} = 3 \]

(b) \[ -20 = 3x^3 + 4 \]

(c) \[ 5 - 6(3^x) = 7 \]

(d) \[ \log(x + 3) + \log(x + 4) = \log 2 \]
9. (20pts) Find the domain of the functions below. Write your answers in interval notation.

(a) \( f(t) = \frac{3t^2}{2t - 3} \)

(b) \( g(x) = \sqrt{-x + 4} \)

(c) \( p(z) = \begin{cases} 
3 & \text{if } z \geq 3 \\
|z| & \text{if } 2 \leq z < 3 \\
-z + 4 & \text{if } z < -2 
\end{cases} \)

(d) The function \( y = w(x) \) graphed below:
10. (12pts) The number of cells in a certain bacteria culture grows by 15% every hour. What is the doubling time of the population in this culture? Round your answer to two decimal places.

11. (20pts) A ball is thrown in the air. The height of the ball $t$ seconds after it is thrown is $h(t)$ meters where

$$h(t) = -5t^2 + 20t + 3.$$

(a) From what height was the ball initially thrown?

(b) What is the maximum height reached by the ball?

(c) How long does it take for the ball to hit the ground? Round your answer to two decimal places.
12. (20pts) Josiah Bartlett is riding his bike home from the grocery store to his house. After 10 minutes
he is 7 miles away from his house and after 15 minutes he is 6 miles away from his house. Assume that
he went at a constant speed throughout the entire trip.

(a) Write a function \( D(t) \) such that after \( t \) minutes Josiah is \( D(t) \) miles from his house. \( \text{Hint: His speed is constant through the entire trip. What kinds of functions have a constant rate?} \)

(b) How long does it take for him to get home?

(c) How far away is the store from his house?

13. (12pts) Exactly ten years ago there was $10,000 placed in an account on which interest was compounded
continuously. If the account is worth $15,000 today, what was the annual interest rate? Round your
answer to four decimal places.
14. (15pts) Consider the function \( f(x) = x^3 - 2x^2 \). The function \( g(x) \) is the result of performing the following transformations on \( f(x) \) (in the order that they’re given):

(i) horizontal reflection
(ii) vertical compression by a factor of \( \frac{1}{2} \)
(iii) horizontal shift to the right by 3 units

Find a formula for \( g(x) \). There is no need to simplify your answer.

15. (20pts) The graphs of two functions \( p(x) \) and \( q(x) \) are given below. Write \( q(x) \) as a translation of \( p(x) \).

![Figure 1: \( p(x) \)](image1)

![Figure 2: \( q(x) \)](image2)
16. (20pts) The graph of $f(x)$ is drawn below. Draw the graph of $g(x) = \frac{1}{2}f(-x + 3)$ on the same plot. You may draw intermediate steps if you wish but be sure to clearly identify your final answer. (Assume that the scale on the axes is in increments of one.)
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