2.3.1. Consider \( f \) and \( g \) in the table below.

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>2</td>
<td>3</td>
<td>5</td>
<td>7</td>
<td>0</td>
<td>9</td>
<td>1</td>
</tr>
<tr>
<td>( g(x) )</td>
<td>7</td>
<td>6</td>
<td>2</td>
<td>7</td>
<td>9</td>
<td>1</td>
<td>3</td>
</tr>
</tbody>
</table>

(a) Is \( f \) invertible?
Answer: Yes
(b) Is \( g \) invertible?
Answer: No

2.3.2. The functions \( p \) and \( q \) are defined such that
\[ p(x) = x^2 + 4x \quad \text{and} \quad q(x) = 6x + 1. \]

(a) Find and simplify \((p \circ q)(x)\).
Answer: \((p \circ q)(x) = 36x^2 + 36x + 5\)
(b) Find and simplify \((pq)(x)\).
Answer: \((pq)(x) = 6x^3 + 25x^2 - 4x\)
(c) The function \( q \) is invertible. Find its inverse.
Answer: \(q^{-1}(x) = \frac{1}{6}x - \frac{1}{6}\)
(d) Explain briefly why \( p \) is not invertible.
Answer: Since \( p(-5) = 5 \) and \( p(1) = 5 \), \( p \) cannot be invertible.

2.3.3. The following functions are invertible. Find their inverses:

(a) \( f(x) = -\frac{2}{3}x - 6 \)
Answer: \( f^{-1}(x) = -\frac{3}{2}x - 9 \)
(b) \( r(x) = \frac{1}{4}x - 1 \)
Answer: \( r^{-1}(x) = 3x + 3 \)
(c) \( t(x) = -\frac{4}{5}x + 3 \)
Answer: \( t^{-1}(x) = -\frac{7}{4}x + \frac{21}{4} \)
(d) \( w(x) = 3x - 1 \)
Answer: \( w^{-1}(x) = \frac{1}{3}x + \frac{1}{3} \)
(e) \( g(x) = x^3 + 4 \)
Answer: \( g^{-1}(x) = \sqrt[3]{x - 4} \)
(f) \( h(x) = \frac{2x + 3}{x + 1} \)
Answer: \( h^{-1}(x) = \frac{3 - x}{x - 2} \)
(g) \( p(x) = 1 - 2e^x \)
Answer: \( p^{-1}(x) = \ln\left(-\frac{1}{2}x + \frac{1}{2}\right) \)
(h) \( q(x) = \ln(x - 3) \)
Answer: \( q^{-1}(x) = 3 + e^x \)
(i) \( s(x) = 3 + \sqrt{x - 2} \)
Answer: \( s^{-1}(x) = x^2 - 6x + 11 \)
(j) \( v(x) = 1 - \sqrt{x + 4} \)
Answer: \( v^{-1}(x) = -\frac{1}{7}x^3 + \frac{3}{4}x^2 - \frac{3}{2}x - \frac{3}{7} \)

2.3.4. Sketch the inverse of the function \( y = f(x) \) graphed below:

(k) \( z(x) = 3 + 3\log(x + 5) \)
Answer: \( z^{-1}(x) = -5 + 10^{(y-3)/3} \)

2.3.5. A function \( y = h(x) \) is graphed below. Draw the graph of \( h^{-1} \).

Answer: On graph below
2.3.6. Is the function \( f(x) = \frac{2}{2x+3} \) invertible? If so, find its inverse.

**Answer:** Yes, \( f^{-1}(y) = \frac{2-3y}{2y} \)

2.3.7. Is the function \( g(x) = 3x - 1 \) invertible? If so, find its inverse.

**Answer:** Yes, \( g^{-1}(y) = \frac{1}{3}y + \frac{1}{3} \)

2.3.8. Is the function \( h(x) = |2x + 3| - 1 \) invertible? If so, find its inverse.

**Answer:** No

2.3.9. Is the function \( p(x) = 2\log(x) + 3 \) invertible? If so, find its inverse.

**Answer:** Yes, \( p^{-1}(y) = 10^{(y-3)/2} \)

2.3.10. Are the following functions invertible? If so, find their inverses.

(a) \( f(x) = 3x + 7 \)

**Answer:** Yes, \( f^{-1}(x) = \frac{1}{3}x - \frac{7}{3} \)

(b) \( g(x) = 1 - \frac{3}{2-x} \)

**Answer:** Yes, \( g^{-1}(x) = 2 - \frac{3}{1-x} \)

(c) \( h(x) = x^2 - 4 \)

**Answer:** No

(d) \( p(x) = \frac{2}{3}x - \frac{1}{2} \)

**Answer:** Yes, \( p^{-1}(x) = \frac{3}{2}x + \frac{3}{11} \)

(e) \( q(x) = \frac{5-3x}{2x+1} \)

**Answer:** Yes, \( q^{-1}(x) = \frac{5-y}{2y+3} \)

(f) The function \( y = r(x) \) is graphed below. You may draw the inverse if it is invertible.

**Answer:** Yes, the inverse is on the graph below.

2.3.11. True or False: The functions \( f(x) = 1 + 7x^3 \) and \( g(x) = \sqrt[3]{\frac{1}{x-1}} \) inverses of each other.

**Answer:** True

2.3.12. True or False: The functions \( f(x) = 1 - \frac{1}{x-1} \) and \( g(x) = 1 + \frac{1}{x} \) inverses of each other.

**Answer:** False

2.3.13. For each of the following pairs of functions, decide whether or not they are inverses for one another.
2.3.14. Find any two functions \( f \) and \( g \) such that \( (f \circ g)(x) = x \) for all \( x \) but there exists a number \( t \) such that \( (g \circ f)(t) \neq t \).
Answer: There are a lot of answers. I imagine the most popular answer will be \( f(x) = x^2 \) and \( g(x) = \sqrt{x} \).

2.3.15. Hint: The two parts of this question have different answers. Why is that?
(a) Are the functions \( f(x) = 3 + \sqrt{x - 1} \) and \( g(x) = (x - 3)^3 + 1 \) inverses of one another?
Answer: Yes
(b) Are the functions \( f(x) = 3 + \sqrt{x - 1} \) and \( g(x) = (x - 3)^2 + 1 \) inverses of one another?
Answer: No

2.3.16. The graphs of two functions, \( y = f(x) \) and \( y = g(x) \), are shown below.

(a) Is \( f \) an invertible function?
Answer: Yes
(b) Is \( g \) an invertible function?
Answer: No
(c) Sketch the graph of \( y = (f + g)(x) \).
Answer: Shown on graph

2.3.17. The airspeed velocity of a European swallow is proportional to its heart rate. That is, if a European swallow’s heart rate is \( h \) bpm then its airspeed is \( A(h) = 0.01h \) m/s.
(a) The average heart beat of a European swallow is 900 bpm. What is the airspeed velocity of such a swallow?
Answer: 9 m/s
(b) If a European swallow flies at 11.5 m/s, find its heart rate.
Answer: 1150 bpm

2.3.18. Cindy works at an hourly job where her pay is determined by a function \( P \). If Cindy works an average of \( t \) hours a week over the course of a year then she makes \( P(t) \) dollars in that year where
\[
P(t) = 600t + 800.
\]
Additionally, the amount that Cindy puts into savings depends on the amount of money that she makes in a year according to the function \( S \). That is, if she makes \( d \) dollars in a year then she will put \( S(d) \) dollars into her savings account that year where
\[
S(d) = \frac{3d - 8000}{20}.
\]
(a) She wants to know how many hours a week that she needs to work in order to make a given amount of money. Find a function \( f \) such that if she wants to make \( d \) dollars in a year then the average number of hours she needs to work per week during that year is \( f(d) \).
Answer: \( f = P^{-1} \) so \( f(d) = \frac{d - 800}{600} \)
(b) Cindy’s mom is worried about her and wants to know how much she will save depending on how many hours she works. Find a function \( g \) such
that if she works an average of \( t \) hours a week over the course of a year then she put \( g(t) \) dollars into savings in that year.

**Answer:** \( g = S \circ P \) so that \( g(t) = 90t - 280 \)

(c) Cindy also wants to know how many hours a week that she needs to work in order to save a given amount of money. Find a function \( h \) such that if she wants to save \( m \) dollars in a year then the average number of hours she needs to work per week during that year is \( h(m) \).

**Answer:** This can be done with either \( h = g^{-1} \) or \( h = P^{-1} \circ S^{-1} \) so that \( h(m) = \frac{m + 280}{90} \).