1. The graph of a periodic function \( y = f(x) \) is shown below.

   \[
   \begin{align*}
   y &= f(x) \\
   \end{align*}
   \]

   (a) [2pt] Find the period of \( f \).
   
   \textbf{Answer:} 8

   (b) [2pt] Find the midline of \( f \).
   
   \textbf{Answer:} \( y = -2 \)

   (c) [2pt] Find the amplitude of \( f \).
   
   \textbf{Answer:} 3

   (d) [2pt] Find \( f(21) \).
   
   \textbf{Answer:} 1

2. Suppose that \( \theta \) is an angle such that \( \cos(\theta) = 0.56 \) and \( \sin(\theta) < 0 \).

   (a) [1pt] True or False: \( \theta = \cos^{-1}(0.56) \). \textit{Hint: What do you know about the inscription point corresponding to } \theta?\textit{?}
   
   \textbf{Answer:} False

   (b) [5pt] Find \( \sin(\theta) \). Round to two decimal places.
   
   \textbf{Answer:} -0.83

   (c) [2pt] Find \( \tan(\theta) \). Round to two decimal places.
   
   \textbf{Answer:} -1.48

3. Consider the figure shown below: \textit{(Note: It is not drawn exactly to scale.)}

   \[
   \begin{align*}
   \end{align*}
   \]

   (a) [3pt] Find \( h \). Round to two decimal places.
   
   \textbf{Answer:} 7.42

   (b) [3pt] Find \( \theta \). Round to two decimal places.
   
   \textbf{Answer:} 47.90°

   (c) [3pt] Find \( \ell \). Round to two decimal places.
   
   \textbf{Answer:} 6.70
4. [5pt] A child is spinning on a carousel in the playground of a school. This carousel has a diameter of 12 ft and its center is 11 ft from the school. Find a function $d$ such that when the child is at an angle of $\theta$ as shown in the diagram below, the child is $d(\theta)$ feet from the school.

![Diagram of a carousel with a child at an angle $\theta$ from the school] 

Answer: $d(\theta) = 6 \sin(\theta) + 11$

5. State street runs north and south while Main street runs east and west. These two streets intersect in the city of Green. 19 mi north of their intersection there is a train station on State street and 13 mi east of their intersection there is a second train station on Main street. There is a train which runs in a straight line between these two train stations.

(a) [3pt] How far is the train ride between the two train stations? Round to two places.

Answer: 23.02 mi

(b) [3pt] Find the acute angle between State street and the train tracks. Round to two places.

Answer: 34.38°

6. Let $f(x) = 2x - 1$ and let $g(x) = x^2 + x - 2$.

(a) [3pt] Find and simplify $(f - g)(x)$.

Answer: $(f - g)(x) = -x^2 + x + 1$

(b) [3pt] Find and simplify $(g \circ f)(x)$.

Answer: $(g \circ f) = 4x^2 - 2x - 2$

(c) [3pt] Let $h(x) = \frac{1}{2}x + \frac{1}{2}$. Are $f$ and $h$ inverses of one another? Verify your answer.

Answer: Yes

7. [5pt] A city planner is studying the population of the city of Green. He finds a function $P$ which describes this population. That is, $t$ years from today the population will be $P(t)$ people where

$$P(t) = 2000e^{0.015t}.$$ 

Find (and simplify) a function $Y$ such that $Y(n)$ is the amount of time (from today) before the city’s population reaches $n$ people.
Answer: \( Y(n) = \frac{200}{3} \ln(0.0005n) \)

8. [5pt] An engineer is doing an analysis on the efficiency of a certain car. She finds two relationships. First, she finds that if this car is going at a speed of \( r \) miles per hour then its gas mileage is \( M(r) \) miles per gallon where

\[
M(r) = -0.01r^2 + 0.5r + 30.
\]

Additionally, she finds that if this car’s gas mileage is \( m \) miles per gallon then it emits \( C(m) \) grams of carbon per mile where

\[
C(m) = 5m + 220.
\]

Find (and simplify) a function \( f \) such that when the car is going at a speed of \( r \) miles per hour then it emits \( f(r) \) grams of carbon per mile.

Answer: \( f(r) = -0.05r^2 + 2.5r + 370 \)

9. [1pt] True or False: If \( f \) and \( g \) are any functions such that \( (f \circ g)(x) = x \) for all values of \( x \) in the domain of \( f \circ g \) then \( f \) and \( g \) must be inverses of each other.

Answer: False

10. [1pt] True or False: \( \sin^{-1}(\sin(\theta)) = \theta \) for all angles \( \theta \).

Answer: False

11. [1pt] True or False: The function \( y = f(x) \) graphed below is an invertible function.

Answer: True

12. [1pt] True or False: The function \( y = f(x) \) graphed below is a periodic function.

Answer: False

13. [1pt] True or False: If \( \theta \) is an angle such that \( \cos(\theta) = \frac{\sqrt{3}}{2} \) then it must be the case that \( \theta = 30^\circ \).

Answer: False