

Direct numerical simulation of bedload transport using a local, dynamic boundary condition

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ABSTRACT

Temporally and spatially averaged models of bedload transport are inadequate to describe the highly variable nature of particle motion at low transport stages. The primary sources of this variability are the resisting forces to downstream motion resulting from the geometrical relation (pocket friction angle) of a bed grain to the grains that it rests upon, variability of the near-bed turbulent velocity field and the local modification of this velocity field by upstream, protruding grains. A model of bedload transport is presented that captures these sources of variability by directly integrating the equations of motion of each particle of a simulated mixed grain-size sediment bed. Experimental data from the velocity field downstream and below the tops of upstream, protruding grains are presented. From these data, an empirical relation for the velocity modification resulting from upstream grains is provided to the bedload model. The temporal variability of near-bed turbulence is provided by a measured near-bed time series of velocity over a gravel bed. The distribution of pocket friction angles results as a consequence of directly calculating the initiation and cessation of motion of each particle as a result of the combination of fluid forcing and interaction with other particles. Calculations of bedload flux in a uniform boundary and simulated pocket friction angles agree favourably with previous studies.

Keywords Bedload, equations of motion, numerical simulation, sediment transport, turbulence.

INTRODUCTION

The formation and evolution of sedimentary bed features, such as scours, bedload sheets, ripples, dunes, bars, armour, sorting and grading, are ubiquitous in both fluvial and marine environments. These features in the geological record are primarily classified by their characteristic geometric forms and size sorting. In nearly all cases, both form and sorting are produced by a spatially and temporally varying sediment transport field resulting from interaction of a turbulent flow with the sediment grains of an erodible bed. Although sedimentologists have been able to discern the conditions under which these features were formed by comparison with modern environmental analogues, a number

remain to be explained in a physical, quantitative manner.

The lack of accurate physical models of a number of sedimentary structures produced by bedload transport can be explained by the fact that the motion of mixed grain-size sediment, by rolling and saltation along the bed of a turbulent flow, is an inherently complex problem. Turbulence produces fluctuations in the near-bed velocity that give rise to fluctuations in the forces on, and resultant motion of, sediment grains. Some particles near the bed surface are surrounded by particles much higher in the flow, whereas others protrude above adjacent grains and therefore experience greater drag forces from the flow. The geometric arrangement of surrounding grains also leads to variability of forces required to begin

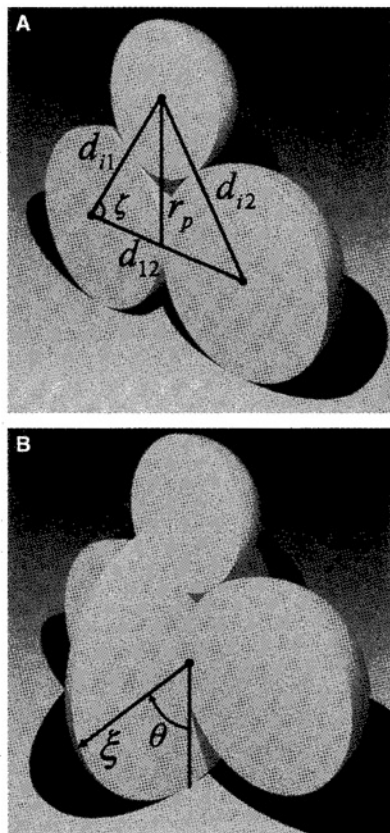


Fig. 5. Sketches showing the polar co-ordinate system formed by three spheres. (A) Distances and angle of Eqs 27–30. (B) The disc-shaped object depicts the plane of the polar co-ordinate system, and the two co-ordinate directions ξ and θ are shown on the disc.

the moving sphere back to the global Cartesian co-ordinate system at the end of each integration step. The position and orientation of the polar co-ordinate system is arbitrary relative to the global Cartesian co-ordinate system. The transformation from polar co-ordinates to the global co-ordinate system involves the conversion from polar co-ordinates to a local Cartesian co-ordinate system, and then rotation and translation of this local Cartesian system to the global co-ordinates. The x - y plane of the local Cartesian co-ordinate system, x_l , y_l , z_l , is chosen to coincide with the polar co-ordinate system so that:

$$x_l = \xi \cos \theta \quad (35)$$

$$y_l = \xi \sin \theta \quad (36)$$

$$z_l = 0 \quad (37)$$

Translation and rotation of the local Cartesian co-ordinates is accomplished by:

$$x = x_l(\hat{x}_l \cdot \hat{x}) + y_l(\hat{y}_l \cdot \hat{x}) + x_o \quad (38)$$

$$y = x_l(\hat{x}_l \cdot \hat{y}) + y_l(\hat{y}_l \cdot \hat{y}) + y_o \quad (39)$$

$$z = x_l(\hat{x}_l \cdot \hat{z}) + y_l(\hat{y}_l \cdot \hat{z}) + z_o \quad (40)$$

where the hatted characters represent unit vectors of the respective co-ordinate systems.

Carrying out this co-ordinate system transformation for the centre of mass of the moving particle gives:

$$x_i = x_o + (x_i - x_o) \cos \theta + \frac{\sin \theta}{d_{12}} ((y_2 - y_1) (z_i - z_o) - (z_2 - z_1) (y_i - y_o)) \quad (41)$$

$$y_i = y_o + (y_i - y_o) \cos \theta + \frac{\sin \theta}{d_{12}} ((z_2 - z_1) (x_i - x_o) - (x_2 - x_1) (z_i - z_o)) \quad (42)$$

$$z_i = z_o + (z_i - z_o) \cos \theta + \frac{\sin \theta}{d_{12}} ((x_2 - x_1) (y_i - y_o) - (y_2 - y_1) (x_i - x_o)) \quad (43)$$

The forces that are originally calculated in the global Cartesian co-ordinates are converted to the polar co-ordinate system with:

$$F_\theta = \frac{1}{r_p d_{12}} ([y_2 - y_1] (z_i - z_o) - (z_2 - z_1) (y_i - y_o)) F_x + [(z_2 - z_1) (x_i - x_o) - (x_2 - x_1) (z_i - z_o)] F_y + [(x_2 - x_1) (y_i - y_o) - (y_2 - y_1) (x_i - x_o)] F_z \quad (44)$$

Algorithm

In the previous section, the equations of motion for each individual particle making up a sediment bed were specified for situations in which a particle is moving and in contact with zero, one or two other particles. In order to integrate the motion of a single particle through time, it is then necessary to decide the number of particles that the particle of interest is in contact with. Owing to the variability of fluid forces, the changing geometry of the bed and the changing location of the particle, a moving particle may lose contact with one particle or two particles simultaneously. Additionally, a particle initially at rest in contact with a number of particles may experience sufficient driving forces to initiate motion in contact with zero, one or two other particles. Conversely, a moving particle can also collide with other particles, possibly resulting in motion in contact

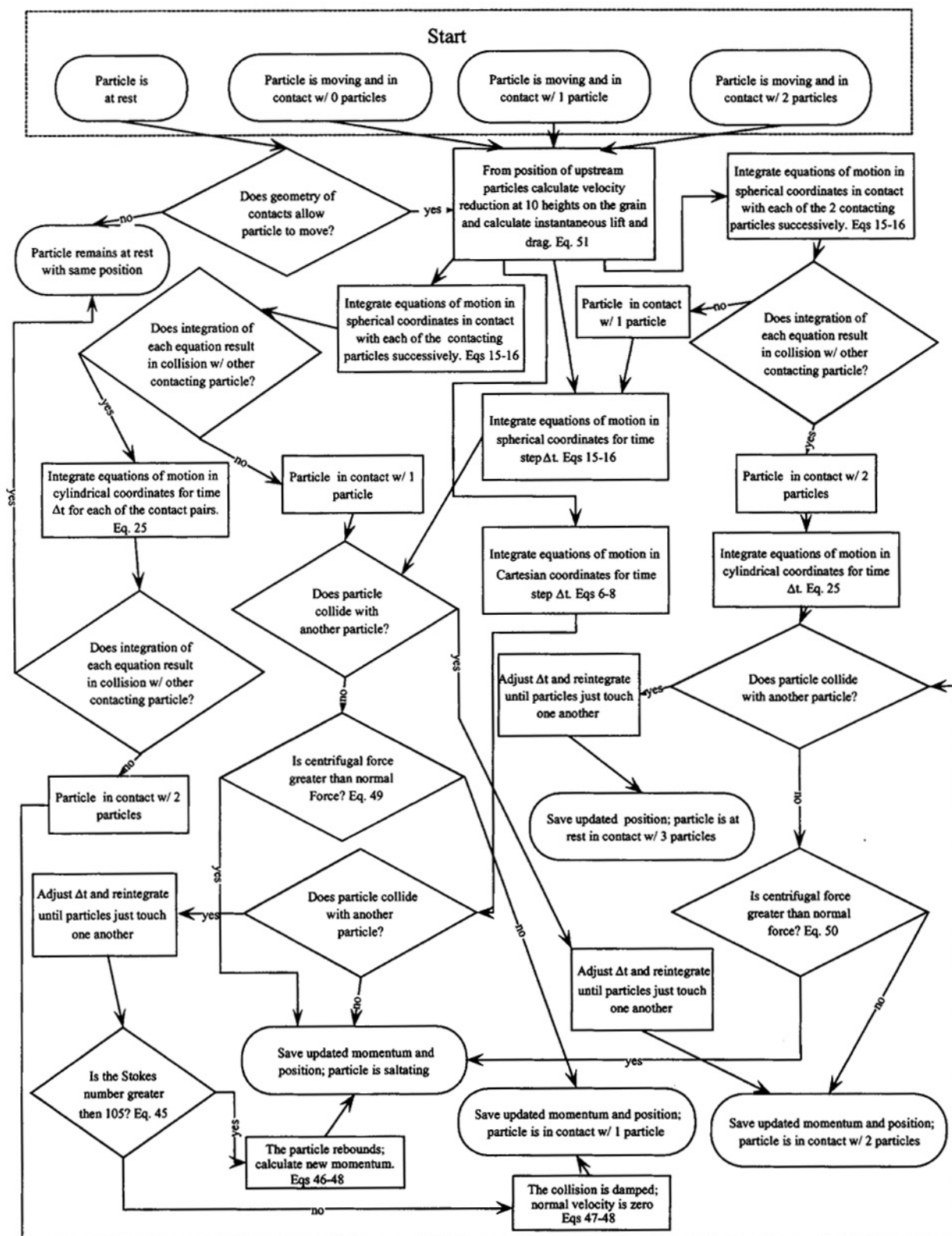


Fig. 6. Flow chart of algorithm for sediment transport simulations for a single time step and a single particle.

along the line from the centre of the moving sphere through the centre of the stationary sphere. If the Stokes number is greater than the

critical value of 105, the particle rebounds, and the new velocity is calculated in spherical co-ordinates as:

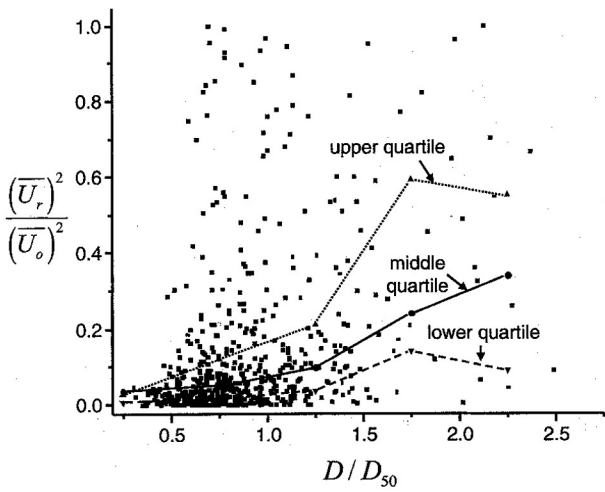


Fig. 10. Squared ratio of reduced velocity to fully exposed velocity, $(\overline{U}_r/\overline{U}_0)^2$, vs. relative grain size, D/D_{50} .

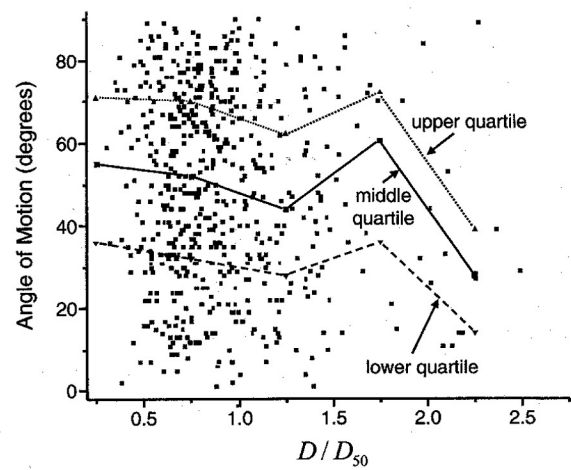


Fig. 11. Pocket friction angle vs. relative grain size, D/D_{50} .

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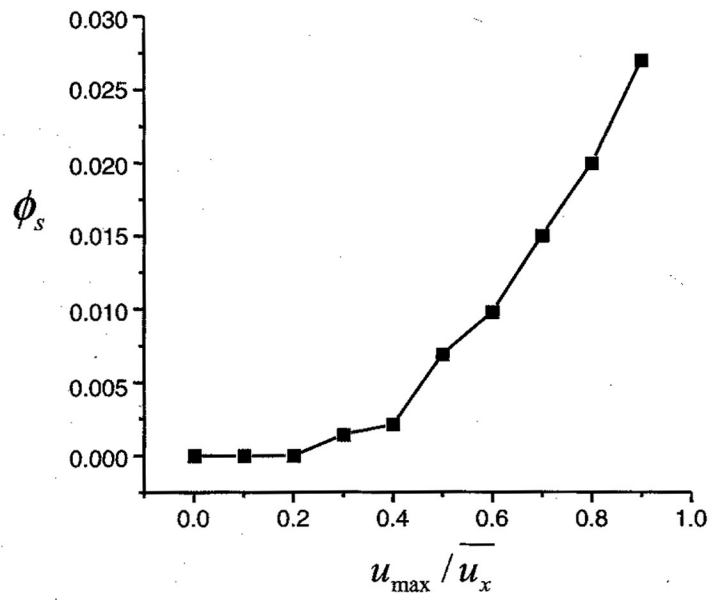


Fig. 13. Non-dimensional transport rate as a function of amplitude of near-bed velocity fluctuations.

