

2009 OREGON INVITATIONAL MATHEMATICS TOURNAMENT

GEOMETRY EXAM

PRINT _____
Last Name First Middle

SCHOOL _____

Please circle the grade you are in: 8 9 10 11 12

Problems 1-10 are either multiple choice or require only a correct answer but no justification (partial credit is not given for these problems).

Problems 11-15 require justification and partial credit is given for good and well written reasoning.

Problems 1 through 10 are 10 points each. Problems 11-15 are 20 points each, for a total of 200 points.

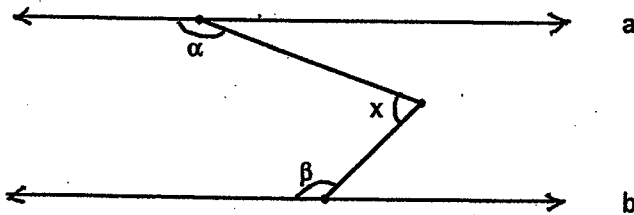
You do not have to do the problems in order. Do the easy ones first.

Question	Possible Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
11	20	
12	20	
13	20	
14	20	
15	20	
TOTAL	200	

1. Find the number of sides of a regular polygon each of whose interior angles is 179° .

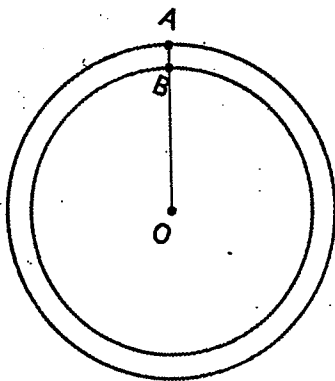
Answer: _____

2. If lines a and b are parallel and the measures of the marked angles are α and β as shown find x (the measure of $\angle BCA$) in terms of α and β .



Answer: _____

3. The two circles in the figure are concentric (share the same center O). If the circumference of the larger circle (with radius OB) is 100 ft longer than the circumference of the smaller circle (with radius OA), find AB (the gap between the circles).



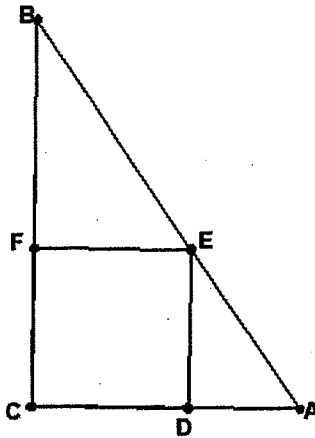
Answer: _____

4. Find the area of a triangle with sides a and b and the included angle of 60° (your answer should be in terms of a and b).

(a) $\frac{ab}{2}$ (b) $\frac{ab\sqrt{2}}{2}$ (c) $\frac{ab\sqrt{3}}{2}$ (d) $\frac{ab\sqrt{5}}{2}$ (e) None of the above.

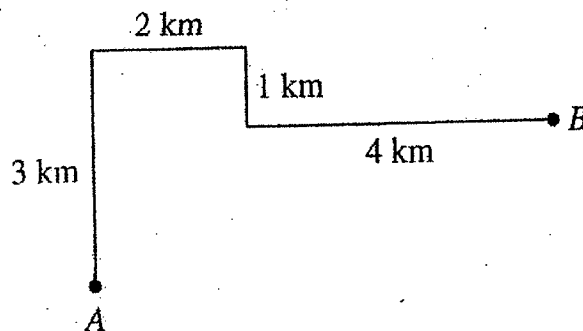
Answer: _____

5. Triangle ABC is a right triangle in which $AC = b$ and $BC = a$. If $DEFC$ is a square find the length of the side of the square in terms of a and b .



Answer: _____

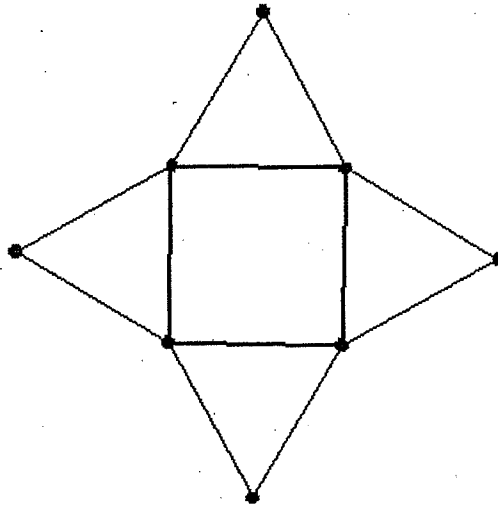
6. A boat starts at point A , moves 3 km due north, then 2 km due east, then 1 km due south, and then 4 km due east to point B . Find the distance AB .



Answer: _____

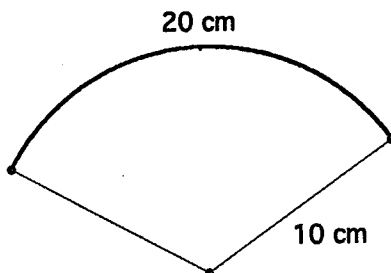
7. The following two-dimensional figure in which all segments are 10 cm long is assembled into a pyramid. Then the volume of the pyramid is (encircle one):

- (a) $\frac{1000}{3}$ cm² (b) $\frac{100}{3}\sqrt{75}$ cm³ (c) $\frac{100}{3}\sqrt{100}$ cm³ (d) $\frac{100}{3}\sqrt{50}$ cm³
(e) The answer cannot be found from the given information.



8. The area of the sector of a circle whose radius is 10 cm and whose corresponding arc is 20 cm long is (circle one):

- (a) 100 cm² (b) 50π cm² (c) $\frac{200}{3}\pi$ cm²
(d) The answer cannot be found from the given information.
(e) The answer is not given but can be found.



9. The equation $x^2 + y^2 + Bx + Cy + D = 0$ is an equation of a circle if and only if:
(circle one and justify your answer)

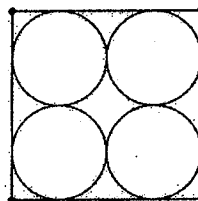
- (a) $B^2 + C^2 > 4D$ (b) $\frac{B^2 + C^2}{4} < D$ (c) $B > 0, C > 0$ and $D < 0$.
(d) $D < 0$ (e) $D > 0$ (f) None of the above.

10. A regular hexagon is inscribed in a circle with radius of length 1. Find the length of a side of the hexagon.

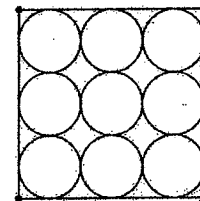
Answer: _____

11. Given a segment AB , the set of all points P in a plane such that $\frac{PA}{PB} = 2$ is one of the following: circle, line, ellipse or parabola. Which one? Justify your answer.

12. (a) Four congruent circles are cut out from a square sheet of tin as shown. What fraction of the tin is wasted?
- (b) What percentage of the tin is wasted if nine congruent circles are cut out of the sheet?
- (c) What percentage of the tin is wasted if n^2 congruent circles are cut out of the sheet? Prove your answer.



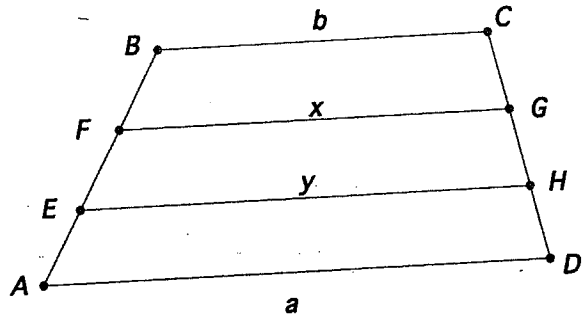
(a)



(b)

13. Consider the lines $x + y = -1$ and $x - y = 1$, and the lines $x + y + 1 + k(x - y - 1) = 0$, where k takes all possible real number values. What do all the lines $x + y + 1 + k(x - y - 1) = 0$ have in common? Justify your answer.

14. $ABCD$ is a trapezoid with bases a and b . If $AE = EF = BF$ and $CG = GH = HD$ find x and y in terms of a and b . Show your work.



15. $ABCD$ is a square inscribed in a circle and $EFGH$ is a square with E and H on side BC and F, G points on the circle. Find the side of the smaller square in terms of s . Justify your answer.

