PART I.

Name:________________________

School:________________________

Circle your qualifying level: Advanced I   Advanced 2

Answer all questions and write your answers in the boxes provided.

1. If John can fill a tank in 6 hours, Jane in 4 hours and Jeremy in 3 hours, how long will it take to fill the tank when they work together?

2. The surface of the globe is divided into sections by 17 parallels (lines parallel to the equator) and 24 meridians (lines connecting the poles). How many sections are there?

3. At a fast-food restaurant, the cost of 3 burgers, 5 drinks and 1 salad is \$23.50, while the cost of 5 burgers, 9 drinks and 1 salad is \$39.50. How much is 2 burgers, 2 drinks and 2 salads?

4. If x, y and z are positive integers such that $\frac{51}{11} = x - \frac{1}{y - \frac{1}{z}}$, find $x + y + z$.

5. If $f(x)$ is a function such that $2f(1/x) + f(x)/x = x$ for all $x \neq 0$, find $f(2)$.

6. How many integer numbers x satisfy $|20 - x| < 70$ and $|30 + x| > 40$?

7. Find a number between 1000 and 2000 which gives remainder 1 when divided by each of 2, 3, 4, 5, 6, 7 or 8.

8. Alice and Bob run on a circular track, starting at the same time at diametrically opposite points of the track. Alice runs clockwise and Bob counterclockwise. Each runner runs at a constant speed. They first meet when Alice has run 100 meters. Next they meet when Bob has run 150 meters from their previous meeting place. Find the length of the track (in meters).

☐

9. How many whole numbers between 1 and 999,999 (inclusive) are simultaneously perfect squares and perfect cubes?

☐

10. Let M and N be the midpoints of the sides BC and CD of a parallelogram $ABCD$. If the area of the triangle AMN is 15, what is the area of the parallelogram?

☐

11. Find $m+n$, where m and n are relatively prime positive integers such that

$$\frac{m}{n} = \left(1 - \frac{1}{2^2}\right)\left(1 - \frac{1}{3^2}\right)\left(1 - \frac{1}{4^2}\right) \cdot \ldots \cdot \left(1 - \frac{1}{24^2}\right).$$

☐

12. Let $p(x)$ be a cubic polynomial such that $p(1) = 1, p(2) = 4, p(3) = 9, p(4) = 13$. Find $p(5)$.

☐

PART II. Name: ______________________________

Answer all questions by writing your answers in the boxes provided.

1. Find a positive integer n such that $\frac{1}{n} < \sqrt{1002} - \sqrt{1001} < \frac{1}{n-1}$.

2. Find the last two digits of $7^{(7^7)}$.

3. Two circles of radius 10 are tangent to each other and to a line, such that they lie on one side of the line. Find the radius of the circle which is tangent to the line and both these circles.

4. How many different rectangles formed by the lines of the board are there on an 8×8 checkerboard? (For example, on the 2×2 board, there are 9 different rectangles.)

5. Consider a cube whose side length is 6. Let AB be the diagonal of the top face and let CD be the diagonal of the bottom face which is not parallel to AB. Find the volume of the pyramid ABCD.

6. Find the value of $\log_3 \tan 1° + \log_3 \tan 2° + \ldots + \log_3 \tan 89°$.

PART III. Name: ______________________

Solve as many problems as you can by writing your final answers in the boxes provided and giving FULL WRITTEN EXPLANATIONS with a complete justification/proof.

1. Find the closest integer to $(\sqrt{3}+\sqrt{2})^6$.

2. Inside a square 15 points are marked. Segments with endpoints either at these marked points or the vertices of the square are drawn so that they divide the square into triangular regions and the segments do not intersect each other (except at the endpoints). If each of the marked points is an endpoint of at least one of the segments, find the number of the triangular regions obtained.

□

3. The increasing sequence $1, 3, 4, 9, 10, 12, 13, \ldots$ consists of all those positive integers which are either powers of 3 or sums of distinct powers of 3. Find the 100th term of this sequence.

4. A boy is swimming at the center of a round pond in a park where swimming is not allowed. Suddenly, a guard comes up to the edge of the pond. The guard cannot swim, but he runs four times faster than the boy swims. The boy runs faster on land than the guard. Is it possible for the boy to escape (i.e. to reach the edge of the pond before the guard)?

☐