# Death and Capital

Investment with and without Annuities*


SHANKHA CHAKRABORTY† — University of Oregon

MAUSUMI DAS‡ — Delhi School of Economics



First version: Sept. 2009

This version: Feb. 2019



**Abstract**

Two questions motivate this paper: (i) to what extent can dynastic altruism substitute for missing annuities in maintaining overall investment under lifetime uncertainty, and (ii) how does lifetime uncertainty affect the pattern of investment and economic development? In a life-cycle model, parents value the transfer of tangible assets to their offspring in the event of premature death which increases the subjective reward they obtain from investing in them. Consumption risk in the absence of annuities, on the other hand, discourages investment. We show that parental altruism can compensate for the absence of annuities under empirically plausible degrees of risk aversion. When people also invest in intangible human capital, lifetime uncertainty tilts portfolio choice in favor of tangible assets. This can translate into divergent growth paths, delayed transition from physical to human capital accumulation and a dampened response to mortality shock in developing countries.



KEYWORDS: Life Expectancy, Mortality, Health, Physical Capital, Human Capital, Parental Altruism, Growth

JEL CLASSIFICATION: I10, O10, O40



*For discussions and comments we thank Ania Aksan, Jeff Allen, David de la Croix, Oksana Leukhina and participants at various venues the paper was presented, especially the "Health and the Macroeconomy" conference at LAEF, UCSB (2009), SAET conference in Taiwan (2018) and $14^{th}$ Annual Conference at ISI, Delhi (2018). The usual caveat applies.



†Department of Economics, University of Oregon, Eugene, OR 97403-1285, USA. Email: shankhac@uoregon.edu

‡Department of Economics, Delhi School of Economics, University of Delhi, Delhi 110007, India. Email: mausumi@econdse.org

# 1 Introduction

This paper studies the effect of adult mortality in a lifecycle economy. Specifically, we ask: (i) to what extent can dynastic altruism substitute for missing annuities in maintaining investment under lifetime uncertainty, and (ii) how does lifetime uncertainty affect the pattern of investment and economic development?

Mortality affects households' consumption and savings behavior through multiple channels. A commonly studied one is its negative effect on future consumption possibilities that causes households to prioritize present consumption and invest less. This is seen to lower growth as in Ram and Schultz (1979), Gersovitz (1983), Shastry and Weil (2003), Chakraborty (2004), Lorentzen *et al.* (2005) and Jayachandran and Lleras-Muney (2009) among others. Actuarially fair annuity markets can, of course, mitigate this problem and, if annuity markets are absent or imperfect, sufficiently strong bequest motives can take on their role.

Yet, not all assets can be readily annuitized or bequeathed. We differentiate between physical assets and human capital as alternative sources of future income, a key difference being the latter's inalienability. Physical assets such as capital, land and livestock are readily transferable across people in a way that human capital is not. This difference is particularly salient when an investor faces lifetime uncertainty that can cut short her amortization period. Transferability of physical assets implies that well-functioning annuity markets can deliver a risk-free return on it but not on human capital, and tilt portfolio choice towards tangible investment.[1] This is true in the absence of annuities too because altruistic parents value the transferability of their wealth to their progeny in the event of premature death.

The first focus of this paper is to understand to what extent this altruistic motive is able to replicate investment behavior under the ideal benchmark of perfect annuitization. Our interest in this issue is motivated by the frequent attribution of economic underdevelopment to market imperfections. We show that as far as transferable assets are concerned, parental altruism can substitute for imperfect annuity markets in a particular sense: for empirically plausible values of risk-aversion, the investment rate through the altruism channel is at least as large as that under full annuities as long as parents are sufficiently altruistic.

This conclusion mirrors that in other works on annuities and intra-family altruism. Kotlikoff and Spivak (1981) show that resource sharing between household members with

[1] The premise that human capital investment has an inherently non-diversifiable idiosyncratic risk component is not new in the literature; see, for example, Levhari and Weiss (1974), Eaton and Rosen (1980), Krebs (2003) and Gottardi *et al.* (2015). Much of this literature identifies the non-diversifiable risk with unemployment risk, whereas here it comes from lifecycle uncertainty and the non-transferability of human capital. The latter has deeper consequences for household decisions beyond the usual risk-return trade-off on the production side.

independent mortality risk can substantially compensate for missing annuities. A more recent literature, see for example Davidoff *et al.* (2005) and Lockwood (2012), identifies the bequest motive as the main reason why households do not fully annuitize their wealth under lifetime uncertainty.[2] Our work complements this literature by switching the focus from consumption and welfare to investment and growth.

The second theme of this paper follows from the differential effect of mortality on tangible versus intangible investment. In developing countries where mortality risks are high, the model predicts that the predominant form of asset accumulation will naturally be in physical capital. It then follows that patterns of investment and production will shift towards human capital only when lifecycle uncertainty falls along with economic development.

Two stylized facts are relevant for this result. First, mortality declined sharply in the late nineteenth and early twentieth centuries in the West due, in large measure, to exogenous improvements in public health and medicine (Wrigley and Schofield (1981), Szreter (1988), Dobson (1997), Cutler and Miller (2005)). Secondly, as documented by Abramovitz and David (2000), Goldin and Katz (2001), and Galor and Moav (2004), there was a concomitant transition from physical capital to human capital as the primary engine of growth. These two transitions become related in our model: during the initial stages of development, high mortality is accompanied by investment in transferable assets (physical capital, land) while in later stages, lower mortality from (possibly exogenous) health improvements is accompanied by investment in human capital.[3]

Evidence of the differential effect of mortality is discernible even in contemporary experiences. For example, Fortson (2011) argues that while the growth effect of the HIV epidemic in sub-Saharan Africa has been ambiguous, it had a definite negative effect on schooling and human capital formation. This would suggest that the loss of output from lower human capital formation was attenuated by other effects; a shift towards physical assets is one possibility. Put differently, the tendency of altruistic families to over-accumulate physical assets under lifecycle uncertainty implies that the cost of epidemic shocks would be relatively lower in developing countries that face already-high mortality risks. Preliminary quantitative evidence in Chakraborty and Perez-Sebastian (2018) point to the relevance of this self-insurance mechanism.

Indeed adjusting the portfolio of asset stocks for consumption smoothing purposes in the face of idiosyncratic income shocks (not necessarily mortality shock) is not uncommon

[2]A different explanation of this 'annuity puzzle' is Brown *et al.* (2008) who attribute it to less-than-fully-rational households' aversion to annuities.

[3]For example, 2010 life expectancy (at birth) in Swaziland was 53.6 years while that in Iceland was 81.8 years (`http://www.who.int/gho/mortality_burden_disease/life_tables/situation_trends/en/`). More relevant to our work is working-age mortality. In 2010, the mortality rate of 15 year old men dying before reaching the age 60 was 76.5% in Swaziland, highest in the world, compared to 6.5 % in Iceland, lowest in the world (Rajaratnam *et al.*, 2010).

in developing countries where insurance mechanisms are weak. Examining data from rural India, Jacoby and Skoufias (1997) find that seasonal fluctuations in income were accompanied by seasonal fluctuations is children's school attendance where child labor was used as a mechanism to smooth consumption instead of borrowing. Based on a study of consumption and investment behavior of Indian farmers, Rosenzweig and Wolpin (1993) conclude that when hit by adverse weather conditions, farmers are more likely to sell their livestock than jewelry or land. Similar self-insurance mechanism for consumption smoothing are reported by Janzen and Carter (2013) in the context of Kenya.

That adult mortality affects the returns associated with capital formation (physical or human) and thereby overall investment and growth is well known in the literature.[4] Many of these studies look at either the relationship between mortality and the effective rate of time preference or a single productive asset. Razin (1976) is an early contribution that recognizes how mortality risk distinguishes human capital investment from other types over the lifecycle. But his analysis is restricted to partial equilibrium where rates of return are exogenous. In a dynamic general equilibrium framework, asset returns respond to factor accumulation, and incentives change over time. By identifying more clearly the portfolio choice margin in a dynamic setting, we highlight its relative importance at various stages of development and ascertain its robustness to the availability of insurance.[5]

The structure of the paper is as follows. The following section presents the overall framework. Section 3 analyzes the extent to which intergenerational altruism can compensate for missing annuity markets when individuals invest in a single tangible asset. In section 4, we introduce human capital and analyze the differential impact of mortality on human capital investment vis-a-vis investment in the tangible asset. A general equilibrium version in section 5 incorporates pecuniary externalities and life insurance. Section 6 concludes.

# 2 Structure of the Economy

In a discrete-time overlapping-generations economy individuals potentially live for two periods that we label "youth" and "middle-age". Individuals live in youth for sure but their survival into middle-age is dictated by a constant (exogenous) survival probability $p \in [0, 1]$. At the end of youth, each individual gives birth to a single offspring towards

[4]See, for example, Blackburn and Cipriani (1998), Kalemli-Ozcan *et al.* (2000), Murphy and Topel (2006), Bhattacharya and Qiao (2007), Zhang *et al.* (2013), Andersen and Bhattacharya (2014) and Bembrilla (2016) for various theoretical mechanisms.

[5]The broader literature on adult survival and economic development in lifecycle models is also relevant here, e.g., Lancia and Prarolo (2012), Ricci and Zachariadis (2013), Gori and Sodini (2014) and Prettner and Canning (2014).

whom she is altruistic.

Individuals are endowed with a share of the family income in youth. They also inherit the tangible asset stock of the family upon the death of the parent. First period income is used for consumption, investment in tangible capital and (in some cases) acquiring intangible human capital. The latter two determine future income. If an agent survives into middle-age she consumes a part of her second period income and transfers the remainder to her offspring as intended bequest. When she does not survive, her share of second period income either goes to the annuity issuer (in the case of perfect annuities) or to her offspring as "unintended" bequest (when annuities markets are absent). Parents derive utility from both types of bequests.

Agents have identical preferences. The expected lifetime utility $V_t$ of a young adult at $t$ with income endowment $y_t$ received either as intended or unintended bequest is

$$V_t = u(c_{1t}) + \beta p u(c_{2t+1}) + \gamma E_t V_{t+1}. \tag{1}$$

Here $\beta \in (0, 1)$ is the subjective discount rate, $\gamma > 0$ represents the intensity of parental altruism and utility from death has been normalized to zero. Even though altruism is pure in that parents care about their offsprings' lifetime welfare, they do not necessarily discount their offsprings' lifetime utility at the same rate as they discount their own future consumption. It may be plausibly assumed that $\gamma \leq \beta$.

# 3 Investment in a Single Tangible Asset

We start by studying how the bequest motive affects investment choice for a single asset and conditions under which it substitutes for missing annuities. As has already been noted, mortality can potentially lower the level of investment and alter the portfolio choice of agents. We focus here only on the first margin.

Suppose individuals can acquire physical capital $k$, in the form of livestock, farm tools and machinery, that generates income $q(k)$. The production function satisfies $q(0) = 0$, $q' > 0$ and $q'' < 0$. Let $1 - \theta$ denote the fraction of output that a parent intends to share with her offspring. $\theta \in (0, 1)$ is exogenously given by social customs and convention. If the parent is alive in middle-age, she consumes $\theta q(k)$, leaving the rest to her offspring. If she does not survive, that $1 - \theta$ share goes to the offspring (when annuities are unavailable) or the annuity issuer (under perfect annuities).[6]

[6]It is worth mentioning here that the assumption of an exogenous income sharing rule is a convenient assumption that simplifies the algebra, but is by no means essential. Indeed in the absence of any first period income of the child (other than bequests), convex preferences with Inada conditions will always ensure positive optimal bequests. The Appendix shows that model implications are robust to the optimal choice of $\theta$ for log preferences.

Under **missing annuity markets**, the offspring's initial endowment $y_t$ depends on parental survival whose realization we denote by $z_t \in \{a, d\}$ corresponding to "alive" and "deceased" respectively:

$$y_t = y(k_t, z_t) = \begin{cases} (1-\delta)k_t + (1-\theta)q(k_t), & \text{if } z_t = a, \\ (1-\delta)k_t + q(k_t), & \text{if } z_t = d. \end{cases}$$

Her decision problem is

$$V(k_t, z_t) = \max \{u(c_{1t}) + \beta p u(c_{2t+1}) + \gamma E_t V(k_{t+1}, z_{t+1})\}$$

subject to

$$\begin{aligned} c_{1t} + x_t &= y(k_t, z_t) \\ c_{2t+1} &= \theta q(k_{t+1}) \\ k_{t+1} &= (1-\delta)k_t + x_t \\ z_{t+1} &\sim iid \end{aligned}$$

where expectations are taken with respect to $z_{t+1}$, $x_t$ is investment in physical capital and $\delta \in [0, 1]$ its depreciation rate. Expected lifetime consumption for this household is increasing in own $p$ through expected lifetime wealth $W_t^{NA}$:

$$c_{1t} + \frac{pc_{2t+1}}{R_{t+1}} = y_t + (1-\delta)k_t - k_{t+1} + \frac{p\theta q(k_{t+1})}{R_{t+1}} \equiv W_t^{NA} \tag{2}$$

assuming an implicit interest factor $R_{t+1} \equiv 1 + r_{t+1}$ that is pinned down by arbitrage, $R_{t+1} = 1 + \theta q'(k_{t+1}) - \delta$. Consumption smoothing leads to the intertemporal condition

$$u'(c_{1t}) = \beta p \theta u'(c_{2t+1}) q'(k_{t+1}) + \gamma E_t \left[u'(c_{1t+1}) y_1(k_{t+1}, z_{t+1})\right] \tag{3}$$

where the second term on the right is the expected marginal psychic return from (intended and unintended) bequests.

Under **actuarially fair annuities**, on the other hand, a young adult's initial endowment is non-stochastic

$$y_t = (1-\theta)q(k_t)$$

and the annuitized middle-age budget constraint is

$$c_{2t+1} = \theta q(k_{t+1})/p.$$

Since the parent is committed to sharing $1-\theta$ fraction of family income with her offspring, she can pledge only $\theta q(k_{t+1})$ to the annuity issuer. Zero expected profits in actuarially fair annuities yields the return $\theta q(k_{t+1})/p$ in the event of survival while the annuity issuer keeps $\theta q(k_{t+1})$ in the event of death. Expected return on investment is hence independent of the survival probability.[7]

Premature parental death has no effect on the offspring's budget set (given $k_t$) who faces the optimization problem

$$V(k_t) = \max_{\{k_{t+1}\}} \{u(c_{1t}) + \beta p u(c_{2t+1}) + \gamma V(k_{t+1})\}$$

subject to

$$\begin{aligned} c_{1t} &= (1-\theta)q(k_t) + (1-\delta)k_t - k_{t+1}, \\ c_{2t+1} &= \theta q(k_{t+1})/p. \end{aligned}$$

In this case, expected lifetime consumption is independent of $p$ since lifetime wealth $W_t^A$ is annuitized (Barro and Friedman, 1977):

$$c_{1t} + \frac{pc_{2t+1}}{R_{t+1}} = y_t + (1-\delta)k_t - k_{t+1} + \frac{\theta q(k_{t+1})}{R_{t+1}} \equiv W_t^A \tag{4}$$

The standard Euler equation follows

$$u'(c_{1t}) = \beta\theta u'(c_{2t+1})q'(k_{t+1}) + \gamma[1-\delta+(1-\theta)q'(k_{t+1})]u'(c_{1t+1}). \tag{5}$$

To make further progress, suppose that $q(k) = Ak^\alpha$ with $\alpha \in (0,1)$ and $\delta = 1$. To analyse how the risk-return trade-off under missing annuities may alter the optimal consumption and investment decisions vis-a-vis the benchmark case of actuarially fair annuities, we obviously have to specify the risk preference of the household. Below we entertain three common specifications, starting with the simplest case of risk neutrality.

**Linear Utility**

Suppose $u(c) = c$. In the absence of annuities, the first order condition (3) becomes

$$1 = \beta p\theta q'(k_{t+1}) + \gamma\{p(1-\theta) + (1-p)\}q'(k_{t+1})$$

and future wealth is given by

$$k_{t+1} = [\alpha A\{p\beta\theta + \gamma(1-p\theta)\}]^{1/(1-\alpha)}$$

[7]Implicitly, the annuity issuer can recover the investment returns upon the individual's death. To be more specific, suppose the entire investment $k_{t+1}$ is intermediated through a mutual fund that is in charge of managing both the parent's and offspring's portions. On the parent's portion, it returns $\theta q(k_{t+1})/p$ to each surviving members of that generation, earning zero profits via the law of large numbers. On the offspring's portion, it simply returns the entire capital income $(1-\theta)q(k_{t+1})$ to each, again earning zero profits from the operation. This is in line with the perfect annuities assumption in the literature, e.g., Yaari (1965).

if $k_t \geq \left[ [\alpha A \{ p\beta\theta + \gamma(1-p\theta) \}]^{1/(1-\alpha)} / A(1-\theta) \right]^{1/\alpha} \equiv \bar{k}$, and

$$k_{t+1} = (1-\theta) A k_t^{\alpha}.$$

otherwise.

Under perfect annuities, on the other hand, equation (5) leads to

$$k_{t+1} = [\alpha A \{ \beta\theta + \gamma(1-\theta) \}]^{1/(1-\alpha)},$$

as long as $k_t \geq \left[ [\alpha A \{ \beta\theta + \gamma(1-\theta) \}]^{1/(1-\alpha)} / [A(1-\theta)] \right]^{1/\alpha} \equiv \hat{k}$. Otherwise the individual is at a constrained optimum where she invests her entire first period income

$$k_{t+1} = (1-\theta) A k_t^{\alpha}$$

and consumes only in middle-age.

A simple comparison of no-annuities to annuities tells us that unconstrained investment is lower under missing markets as long as $\beta > \gamma$. It is only when $\beta = \gamma$ (which also ensures $\bar{k} = \hat{k}$) that the altruistic motive fully replicates the annuitized return on investment. How mortality affects investment in the absence of annuities relative to their presence depends on two margins. One is parental altruism, specifically the value parents place on their unintended bequest should they die prematurely. For example, under annuities, a parent knows that their offspring gets the bequest amount $(1-\theta)q(k_{t+1})$ for sure, whereas without annuities, there is a chance the offspring gets the additional amount $\theta q(k_{t+1})$ with probability $1-p$. This encourages the parent to invest more relative to the annuities case. Working against it is the second margin: expected lifetime wealth is lower in the absence of annuities ($W_t^{NA} < W_t^{A}$ as long as $p < 1$), which discourages investment.

Some intuition for the importance of $\gamma$ and $\beta$ can be had by comparing these expected payoff differences under risk neutrality. From above, the difference in expected lifetime wealth between no-annuities and annuities is

$$W_t^{NA} - W_t^{A} = -(1-p)\frac{\theta q(k_{t+1})}{R_{t+1}} < 0$$

which is costly in terms of own-consumption. On the other hand, the difference in bequest is

$$[p(1-\theta)q(k_{t+1}) + (1-p)q(k_{t+1})] - (1-\theta)q(k_{t+1}) = (1-p)\theta q(k_{t+1}) > 0$$

which increases parental utility discounted at the rate $\gamma$. The household is no-worse-off without annuities as long as:

$$\gamma(1-p)\theta q(k_{t+1}) \geq (1-p)\frac{\theta q(k_{t+1})}{R_{t+1}} \Leftrightarrow \gamma \geq 1/R_{t+1} = \beta$$

where the last step follows from the Euler equation of a risk-neutral household smoothing consumption via an asset that pays the guaranteed gross return $R_{t+1}$.

Of course risk-averse households will not base decisions on expected payoff comparisons: the degree of risk aversion also determines which of the two margins dominates.

**Log Utility**

Let's start with the simplest specification of risk aversion, $u(c) = \ln c$. We have,

$$y_t(k_t, z_t) = \begin{cases} (1-\theta)\alpha A k_t^{\alpha-1}, & \text{if } z_t = a \\ \alpha A k_t^{\alpha-1}, & \text{if } z_t = d \end{cases}$$

In the case of no-annuities, optimal investment depends on parental survival via the policy function $k_{t+1} = T(k_t, z_t)$. Since $z$ takes discrete values and $y$ depends on $z$ only through a scaling constant, $z$ affects investment through a scaling constant alone. Given $k_t$, suppose we denote future assets as $k_{a,t+1}$ and $k_{d,t+1}$ for the two realizations of parental survival. From (3)

$$k_{a,t+1} = \alpha(1-\theta-\mu)\left[\beta p + \frac{\gamma p(1-\theta)}{1-\theta-\mu} + \frac{\gamma(1-p)}{1-\nu}\right] A k_t^{\alpha},$$

and

$$k_{d,t+1} = \alpha(1-\nu)\left[\beta p + \frac{\gamma p(1-\theta)}{1-\theta-\mu} + \frac{\gamma(1-p)}{1-\nu}\right] A k_t^{\alpha}.$$

where

$$\nu = \frac{\alpha(\beta p + \gamma)}{1+\alpha\beta p} \text{ and } \mu = \frac{\alpha(\beta p + \gamma)}{1+\alpha\beta p}(1-\theta).$$

Conditional on the level of unintended bequest exceeding intended bequest, the saving propensity is $\alpha(\beta p + \gamma)/(1+\alpha\beta p)$, increasing in the survival probability since $\alpha\gamma < 1$.

Similarly, under annuities,

$$k_{t+1} = \frac{\alpha(\beta p + \gamma)}{1+\alpha\beta p}(1-\theta) A k_t^{\alpha}. \tag{6}$$

Evidently the investment propensity is identical to no-annuities and increasing in $p$. Even though actuarially fair annuity markets ensure that expected returns on investment do not depend on lifetime uncertainty, $p$ affects the savings rate since annuities offer a lower level of second period consumption when $p$ is higher (for the same level of investment). Risk-averse consumers who want to bring the consumption levels across the two periods closer together respond to this future consumption loss by increasing their savings rate.

What is surprising, however, is that $p$ affects investment behavior the same way as under missing annuities. We examine this more closely below. For now note that, despite similar investment propensities, since unintended bequest is higher than intended bequest, $p$ has a general equilibrium effect on aggregate investment when annuities are unavailable.

**CRRA Utility**

A more general specification of risk aversion is the CRRA utility function, $u(c) = c^{1-\sigma}/(1-\sigma)$, where $\sigma > 0$. It is useful to keep in mind the empirical plausible range of values $\sigma$ can take. Since it corresponds to risk-aversion in the model, Chetty's (2006) recent work on the coefficient of relative risk aversion is relevant. He provides a mean value close to 1 with values below that quite common but values above rare. If we broaden the interpretation of $\sigma$ to include (the inverse of) the elasticity of intertemporal substitution, the evidence surveyed by Browning *et al*. (1999) points to values of $\sigma$ slightly below 1.

Let $k$ denote the parent's assets and $k'$ the offspring's assets under parental death. Consider the effect of a sudden disappearance of actuarially fair annuities.[8] Assuming $\delta = 1$, the expected marginal utility loss an individual suffers from this, discounted appropriately, is

$$\begin{aligned} \Gamma &\equiv \beta\theta\left[u'\left(\frac{\theta q(k)}{p}\right) - pu'\left(\theta q(k)\right)\right] q'(k) \\ &= \left[\beta\theta[\theta q(k)]^{-\sigma} p^{\sigma}(1-p^{1-\sigma})\right] q'(k). \end{aligned}$$

The marginal benefit, on the other hand, comes from the offspring enjoying higher endowment under parental death (unintended bequest) which the parent takes into consideration. Weighted by the degree of parental altruism, this benefit is

$$\Psi = \gamma(1-p)\left[\{q(k)-k'\}^{-\sigma} - (1-\theta)\{(1-\theta)q(k)-k'\}^{-\sigma}\right] q'(k).$$

Let $\tilde{\phi}$ denote the investment propensity out of first period income. Under missing annuities this income is different (higher) for an offspring whose parent dies prematurely. But suppose the individual maintains his savings propensity when annuity markets "disappear". This allows us to identify in which direction optimal investment moves and why.

Given the assumptions above, the **net marginal benefit of missing markets** (ignoring the common terms) is

$$\Delta(p) \equiv \Psi - \Gamma = \gamma(1-p)(1-\tilde{\phi})^{-\sigma}[1-(1-\theta)^{1-\sigma}] - \beta\theta^{1-\sigma}p^{\sigma}(1-p^{1-\sigma}).$$

For linear utility ($\sigma = 0$), this simplifies to $\Delta = -(\beta-\gamma)(1-p)\theta$. As long as $\beta > \gamma$, at the margin, the consumption loss from missing markets cannot compensate for the utility gain the offspring enjoys. The household would lower investment as we saw above.

[8] Primes on functions continue to denote the first derivative. The expressions that follow are derived from piece-wise comparison of the right-hand sides of

$$\begin{aligned} u'(c_{1t}) &= \beta\theta u'(c_{2t+1})q'(k_{t+1}) + \gamma(1-\theta)u'(c_{1t+1})q'(k_{t+1}), \\ u'(c_{1t}) &= p\beta\theta u'(c_{2t+1})q'(k_{t+1}) + \gamma\left[p(1-\theta)u'(c^{a}_{1t+1}) + (1-p)u'(c^{d}_{1t+1})\right] q'(k_{t+1}), \end{aligned}$$

the Euler equations with and without annuities.

When $\beta = \gamma$, that is when parents value an extra unit of their offspring's consumption exactly as they would their own, altruism is fully compensatory and investment is unaffected.

For logarithmic utility ($\sigma = 1$), in contrast, $\Delta = 0$ irrespective of $p$, $\beta$ and $\gamma$. This happens because $\Psi = \Gamma = 0$ and is exactly why, despite consumption risk under missing annuities, the investment propensity did not differentially depend on $p$ above. Suppose the household is given a choice between a lottery that pays the consumption good $\omega$ with probability $p$ and nothing otherwise, and a guaranteed consumption of $\omega/p$. The marginal expected valuation of consumption for the first is the same as the marginal valuation of consumption for the latter when preferences are logarithmic: $pu'(\omega) = u'(\omega/p)$. The endowment $\omega$ is analogous to future consumption in our model and, because the household does not, *at the margin*, value one over the other, its investment allocation towards future consumption is insensitive to whether or not annuities are available (although its utility from the latter is unambiguously higher).

For more general values of $\sigma$, the response of $\Delta$ to $p$ is shown numerically in Figure 1. The net marginal benefit is positive for all values of $p$ and decreasing in $p$ as long as $\sigma < 1$ and $\gamma$ is not too low relative to $\beta$, as in the left panel. Missing annuities here makes the household strictly better off at the margin (at the same investment rate) relative to under annuities. The second panel, for a higher degree of risk aversion $\sigma = 1.1$, shows the opposite: here the riskiness of lifetime consumption dominates any utility gains from leaving more to the offspring in the form of unintended bequests. The lower panel of Figure 1 shows the importance of $\gamma$: when parents are not sufficiently altruistic, at relatively high values of survival, the altruistic motive for asset accumulation is dominated by the riskiness of consumption in the absence of annuities as the offspring does not get much of a windfall from unintended bequests.[9] Since most plausible estimates of risk aversion place the value of $\sigma$ at or below 1, it follows that the absence annuities would generally encourage investment relative to actuarially fair annuities unless altruism is severely limited in the sense $\gamma \ll \beta$.

Now turn to **optimal investment**.[10] We further assume that the production function is linear ($\alpha = 1$) because it lets us make considerable progress without having to solve the dynamic path of investment.[11] Let $\phi$ denote the investment rate under annuity markets.

[9] The monotonicity of the net marginal benefit function also depends on $\theta$. For low values, the net benefit at first decreases with $p$ and then increases. The offspring gets a relatively large share of output when $\theta$ is low, which decreases by a lot the marginal utility of her consumption under parental death. This reduces the attractiveness of the accidental bequest motive to the parent, unless $p$ is relatively small too in which case the individual's expected marginal utility from self-consumption is small.

[10] Since $\sigma > 1$ generates negative utility from being alive, we require a concomitantly larger negative utility from death to counterbalance it. This scaling issue is ignored as it has no bearing on optimal choices.

[11] Qualitative results do not depend on this assumption. In fact, the log case from above (for $\alpha = 1$) will

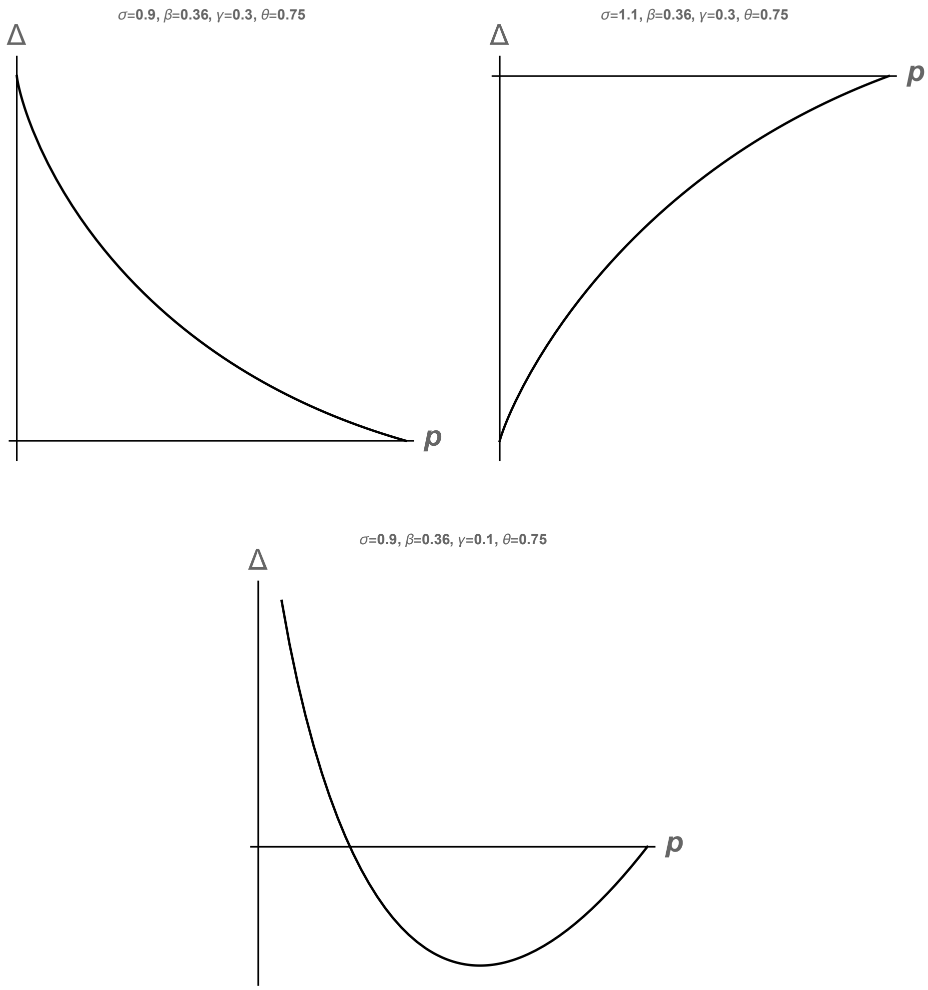


Figure 1: The Net Marginal Benefit of Missing Annuities

Under actuarially fair annuities, consumption levels for a given $k$ are

$$\begin{aligned} c_1 &= y - k', \\ c_2 &= \theta A k'/p, \end{aligned}$$

where $y = (1-\theta)Ak$, $k'$ denotes the physical capital stock one period ahead (i.e., $k_{t+1}$) and $k''$ two periods ahead (i.e., $k_{t+2}$). The Euler equation

$$[(1-\theta)Ak - k']^{-\sigma} = \theta\beta A[\theta Ak'/p]^{-\sigma} + \gamma(1-\theta)A[(1-\theta)Ak' - k'']^{-\sigma}$$

still be nested. But the linear case will not be because of corner solutions. For linear utility and $q(k) = Ak$, tangible investment is independent of $p$ under annuity markets as long as $[\beta\theta + \gamma(1-\theta)]A \geq 1$. When annuities are missing, investment is positive and independent of $p$ iff $p \geq [1-\gamma(1-p\theta)]/(\beta\theta A)$, zero otherwise. Investment is now a weakly increasing function of the survival probability.

under CRRA implicitly defines $\phi$ as a function of the survival probability $p$

$$\left(\frac{1-\phi}{\phi}\right)^{-\sigma} = A^{1-\sigma}\left[\beta p^{\sigma}\theta^{1-\sigma} + \gamma(1-\theta)^{1-\sigma}(1-\phi)^{-\sigma}\right].$$

When annuities are missing, consumption levels and investment choices depend on parental survival. But optimal investment *rates* under parental survival and death are identical for a linear production function. Let $\psi$ denote the investment rates in this case and denote by $k'_a$ and $k'_d$ investments under parental survival and death respectively. The Euler equation, given an income endowment $y$, is now

$$\begin{aligned}[y-k'_a]^{-\sigma} &= \theta\beta p A[\theta A k'_a]^{-\sigma} + \gamma A[p(1-\theta)\{(1-\theta)Ak'_a - k''_a\}^{-\sigma} \\ &\quad +(1-p)\{Ak'_a - k''_d\}^{-\sigma}]\end{aligned}$$

where without loss of generality we have specified the problem for an adult whose parent survives in middle-age. Simplifying, the investment rate $\psi(p)$ solves[12]

$$\left(\frac{1-\psi}{\psi}\right)^{-\sigma} = A^{1-\sigma}\left[\beta p\theta^{1-\sigma} + \gamma p(1-\theta)^{1-\sigma}(1-\psi)^{-\sigma} + \gamma(1-p)(1-\psi)^{-\sigma}\right].$$

Figure 2 compares $\phi(p)$ to $\psi(p)$ for various values of $p$. Clearly $\phi(0) = [\gamma A^{1-\sigma}(1-\theta)^{1-\sigma}]^{1/\sigma} > [\gamma A^{1-\sigma}]^{1/\sigma} = \psi(0)$ whenever $\sigma < 1$, with the sign reversed for $\sigma > 1$.[13] At $p = 1$, the two rates are equal since parental bequests are same. As the first panel of Figure 2 shows, investment is higher under missing annuities under $\sigma = 0.9$; this is true for all values of $\sigma < 1$. The second panel of Figure 2 compares the investment rates for $\sigma = 1.1$. In this instance, investment under missing markets no longer dominates, with the investment loss being higher at higher mortality rates. The third panel shows, for a very low value of $\gamma$, that investment under annuities typically exceeds that under no-annuities.

We conclude that the ability of the altruistic motive to compensate for missing annuities when it comes to tangible investment hinges on the degree of risk aversion as long as households are altruistic enough. Under empirically plausible degrees of risk aversion ($0 < \sigma \leq 1$), it does so quite well. If lifecycle uncertainty extracts an economic cost it is not through the non-availability of market-based insurance mechanisms. Our result corroborates the findings in Greenwood and Smith (1997) who show that in the presence of idiosyncratic preference shocks, depending on the degree of risk aversion, households may save more under financial autarky than under banking institutions which provide insurance against such risks.

[12]Substituting $\sigma = 1$ gives us the investment rates $\phi = \psi = (\gamma + \beta p)/(1 + \beta p)$, same as in (4), (11) and (12) under $\alpha = 1$.

[13]There is a discontinuity in $\phi$ at $p = 0$. For arbitrarily small $p$, annuity purchases are positive but zero at $p = 0$.

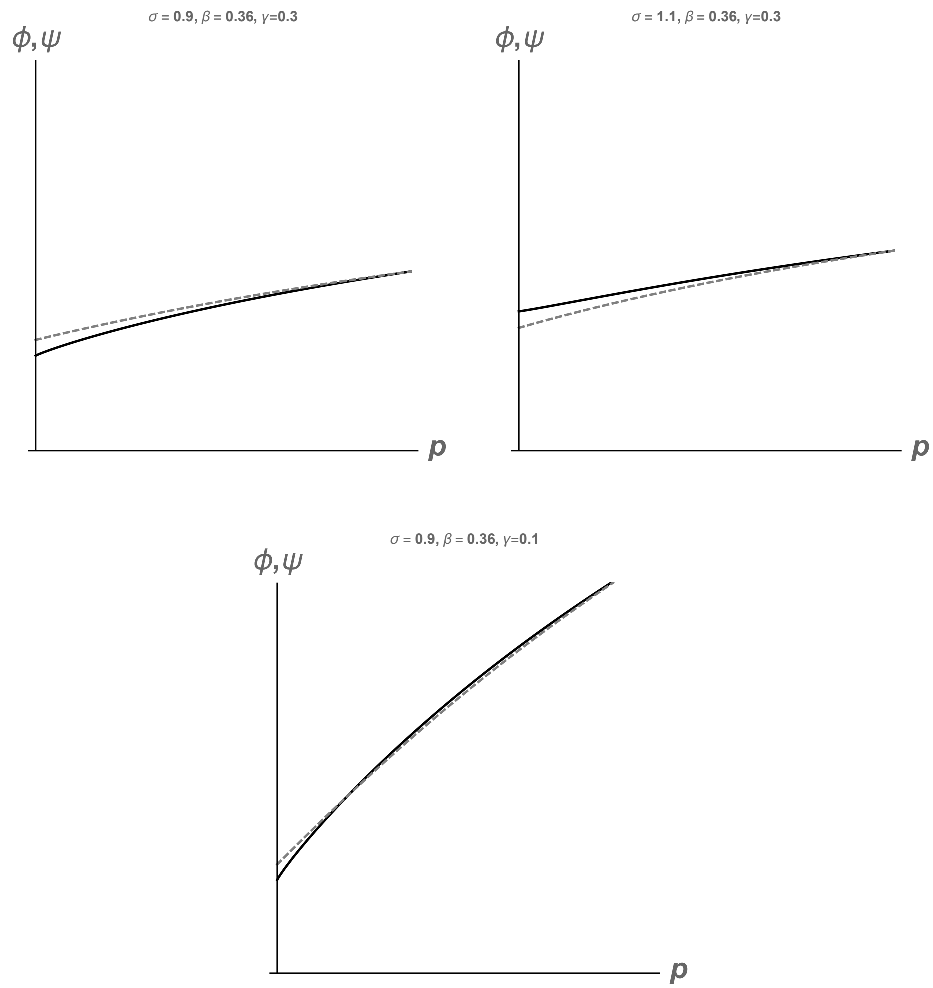


Figure 2: Tangible Investment without ($\psi$, dashed gray) and with ($\phi$, solid black) annuities

# 4 Mortality, Altruism and the Pattern of Investment

Suppose now that people have access to a second investment vehicle, human capital, and the return to it is independent of physical capital. The latter assumption is relaxed in section 5.

The family shares its income from both physical and human capital. Specifically a middle-aged parent shares $1 - \theta_1$ fraction of the family's capital income and $1 - \theta_2$ fraction of his labor income with the child.[14] All individuals are born with the same level of

[14]If all that is being shared is family income, it is natural to assume $\theta_1 = \theta_2$. Human capital opportunities, however, can be geographically removed from farming or small manufacturing that utilizes physical capital (with raw labor). If sharing of labor earnings is relatively more difficult, $\theta_2 < \theta_1$. It is also conceivable that asset ownership in developing countries is not as well defined as human capital ownership (which is embodied in a person in any case). Consequently physical assets are family properties with each member

innate skills (normalized to zero) that is additively separable from acquired skills.[15] Let $e_t$ denote the young agent's investment in human capital in the first period; resulting labor earnings in the second period of life is given by $h_{t+1} = g(e_t)$ where $g$ is an increasing concave function satisfying $g(0) = 0$. We first illustrate, using linear preferences, how the non-transferability of human capital across generations tilts investment in favor of tangible assets.

Under **perfect annuities** and given the endowment $y_t$, an adult in period $t$ now maximizes his expected lifetime utility

$$V_t = u(c_{1t}) + \beta p u(c_{2t+1}) + \gamma E_t V_{t+1}$$

subject to

$$\begin{aligned} c_{1t} + x_t + e_t &= y_t, \\ c_{t+1} &= \theta_1 q(k_{t+1})/p + \theta_2 g(e_t), \\ k_{t+1} &= (1-\delta)k_t + x_t. \end{aligned}$$

Now there is consumption risk as long as $\theta_2 > 0$ since the inalienability of parental human capital deprives the offspring of its returns should the parent die prematurely:

$$y_{t+1} = \begin{cases} (1-\theta_1)q(k_{t+1}) + (1-\theta_2)g(e_t), & \text{with prob. } p, \\ (1-\theta_1)q(k_{t+1}), & \text{with prob. } 1-p. \end{cases}$$

Under $\delta = 1$, we have the pair of Euler equations:

$$u'(c_{1t}) = p\left[\theta_2 \beta u'(c_{2t+1}) + \gamma(1-\theta_2)u'(c^a_{1t+1})\right] g'(e_t)$$

and

$$u'(c_{1t}) = \left[\theta_1 \beta u'(c_{2t+1}) + \gamma(1-\theta_1)\left[pu'(c^a_{1t}) + (1-p)u'(c^d_{1t})\right]\right] q'(k_{t+1})$$

for human capital and physical capital investment respectively.

Suppose now $u(c) = c$. In a corner equilibrium where $c_{1t} = 0$, the portfolio choice problem leads to

$$\frac{g'(e_t)}{q'(k_{t+1})} = \frac{\theta_1 \beta + \gamma(1-\theta_1)}{p\left[\theta_2 \beta + \gamma(1-\theta_2)\right]}$$

which under $q(k) = Ak^\alpha$ and $g(e) = Be^\alpha$ gives the optimal ratio of investment in human vis-a-vis physical capital as

$$\rho \equiv \frac{e_t}{k_{t+1}} = \left[\frac{pB\left[\theta_2 \beta + \gamma(1-\theta_2)\right]}{A\left[\theta_1 \beta + \gamma(1-\theta_1)\right]}\right]^{1/(1-\alpha)}.$$

having some right over its produce: $\theta_1 > 0$, $\theta_2 = 0$. This, of course, implies the young can contribute to family-based activities without seriously hampering their learning process.

[15]This is a simplification. Adding a constant non-zero labor income in the first period would not significantly change results.

Higher survival ($p$) evidently tilts investment in favor human capital. Moreover, if the assets are treated symmetrically for income sharing, $\theta_1 = \theta_2$, the optimal ratio depends only on relative expected returns.

In an interior equilibrium, on the other hand, the Euler equations for human and physical capital investments can be solved independently

$$e_t = [Bp\,[\theta_2\beta + \gamma(1-\theta_2)]]^{1/(1-\alpha)}, \;\; k_{t+1} = [A\,[\theta_1\beta + (1-\theta_1)\gamma]]^{1/(1-\alpha)}.$$

Physical capital investment is now insensitive to mortality as one would expect. However, the investment ratio $\rho$ is the same as above, with higher $p$ tilting investment towards human capital.

Compare these to decisions under **missing annuities**. Given $y_t$, an adult maximizes his expected lifetime utility

$$V_t = u(c_{1t}) + \beta p u(c_{2t+1}) + \gamma E_t V_{t+1}$$

subject to

$$\begin{aligned} c_{1t} + x_t + e_t &= y_t \\ c_{t+1} &= \theta_1 q(k_{t+1}) + \theta_2 g(e_t) \\ k_{t+1} &= (1-\delta)k_t + x_t \end{aligned}$$

and taking into account the stochastic nature of his child's first period income

$$y_{t+1} = \begin{cases} (1-\theta_1)q(k_{t+1}) + (1-\theta_2)g(e_t), & \text{w.p. } p \\ q(k_{t+1}), & \text{w.p.} 1-p \end{cases}$$

Optimality for investment requires

$$u'(c_{1t}) = p\left[\theta_2\beta u'(c_{2t+1}) + \gamma(1-\theta_2)u'(c^a_{1t+1})\right] g'(e_{t+1})$$

for $e_t$ and

$$u'(c_{1t}) = \left[\theta_1\beta p u'(c_{2t+1}) + \gamma\left[p(1-\theta_1)u'(c^a_{1t+1}) + (1-p)u'(c^d_{1t+1})\right]\right] q'(k_{t+1})$$

for $k_{t+1}$. Under linear preference and the same technologies as above, the optimal ratio of physical capital vis-a-vis human capital investment in a corner equilibrium is now

$$\rho = \left[\frac{B}{A}\frac{p\,[\theta_2\beta + \gamma(1-\theta_2)]}{[\theta_1\beta + \gamma(1-p\theta_1)]}\right]^{1/(1-\alpha)},$$

lower than under perfect annuities. This is specifically due to physical capital investment being higher. In the interior equilibrium, on the other hand, the ratio is exactly the same

$$\rho = \left[\frac{B}{A}\frac{p\{\theta_2\beta + \gamma(1-\theta_2)\}}{\gamma + \theta_1 p(\beta-\gamma)}\right]^{1/(1-\alpha)}.$$

We generalize to **CRRA preferences**, full depreciation of physical capital and linear production functions for the two assets, $q(k) = Ak$ and $g(e) = Be$, with $B \geq A$. Without loss of generality we impose $\theta_1 \equiv \theta$ and $\theta_2 = 0$.[16] Let $\phi$ and $\eta$ denote the investment propensities in physical capital and human capital under annuities. The investment rates $(\phi, \eta)$ solve the pair of equations

$$(1-\phi-\eta)^{-\sigma} = \theta\beta p B\left(\frac{\theta A\phi}{p} + B\eta\right)^{-\sigma} \tag{7}$$

$$\left(1-\frac{A}{pB}\right) = \gamma A^{1-\sigma}(1-\theta)^{1-\sigma}\phi^{-\sigma}. \tag{8}$$

Similarly let $\psi$ and $\nu$ denote the investment propensities in physical capital and human capital when annuities are missing. These solve the pair of equations[17]

$$(1-\psi-\nu)^{-\sigma} = \theta\beta p B(\theta A\psi + B\nu)^{-\sigma} \tag{9}$$

$$\left(1-\frac{A}{B}\right) = \gamma A^{1-\sigma}\psi^{-\sigma}[p(1-\theta)^{1-\sigma} + (1-p)]. \tag{10}$$

In both cases we are interested in how the relative investment rates $\eta/\phi$ and $\nu/\psi$ respond to $p$.

Figure 3 illustrates this responsiveness. The solid (black) lines correspond to $\eta/\phi$ and the dashed (gray) lines to $\nu/\psi$. In the upper left panel of Figure 3, both relative investment rates are increasing in survival: households may or may not diversify away the mortality risk on tangible investment through the altruism channel, but $p$ has a differentially higher effect on intangible investment. In other words, higher $p$ incentivizes human capital investment over physical capital investment whether or not annuities are available. The upper right panel of Figure 3 shows that investment in human capital rises less with $p$ under higher degrees of risk aversion because the parent has to "compensate for" strongly diminishing marginal utility of the offspring by investing more in the tangible asset. This effect is more pronounced in the absence of annuities. The lower panel of Figure 3 illustrates the relevance of $\gamma$. Recall from Figure 2 that tangible investment is weaker without annuities when altruism is weak and this is what determines the higher rate of substitution towards human capital when $p$ goes up in the no-annuities case. Finally note the curvature of the relative investment rates. In the absence of annuities, the switch from physical assets to human capital occurs at a faster rate. This is because, as Figures 1–2 foreshadowed, physical capital investment is higher for the parameter values used in Figure 3 so that human capital investment is lower relative to the annuities case and responds more strongly to a change in $p$.

[16] $\theta_2 > 0$ would only accentuate the effect of $p$ on human capital investment since premature parental

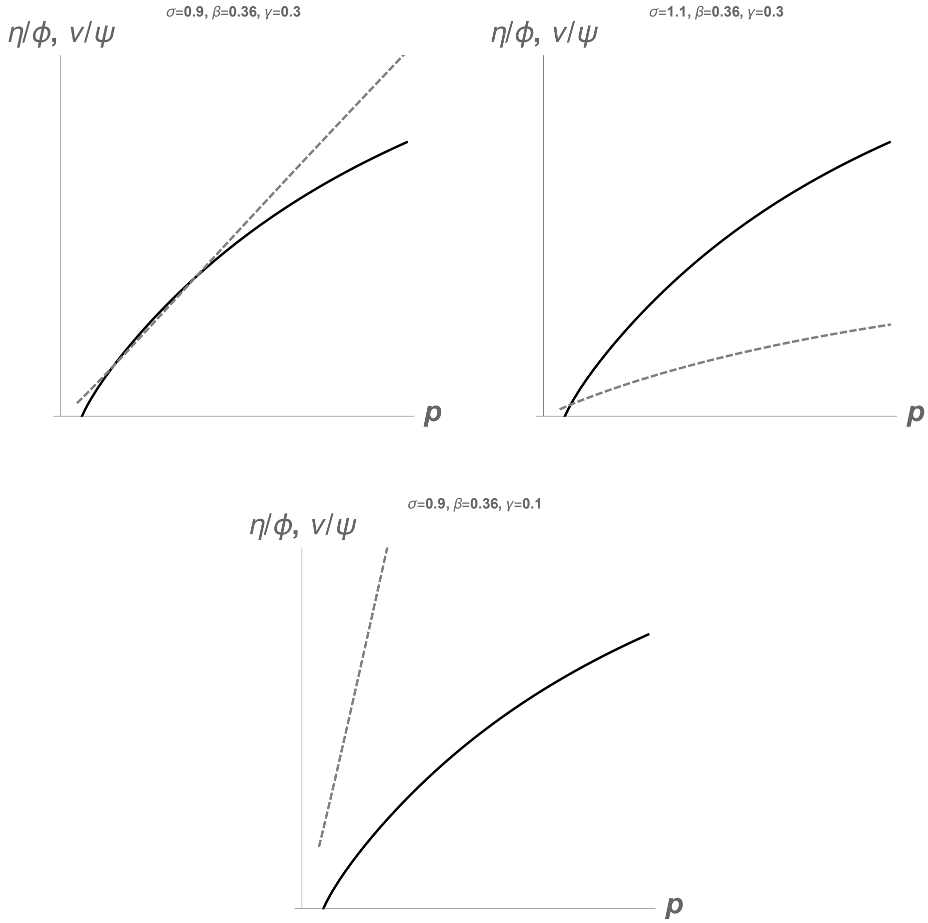


Figure 3: Relative Investment in Human-to-Physical Capital in the presence ($\eta/\phi$, solid black) and absence of annuities ($\nu/\psi$, dashed gray)

# 5 Pecuniary Externalities

The general equilibrium effects of tangible versus intangible investment under lifetime uncertainty is what we turn to next. We assume annuities are *not* available hereon and allow for complementarity in the returns of the two assets. The core intuition from above generalizes. An increase in $p$ now has a more pronounced effect on human capital: it increases the supply of human capital (for the same level of investment) and induces further investment. This raises the return on the complementary input, physical capital, encouraging its accumulation too. The net effect however is to tilt investment *and* production towards human capital.

death would eliminate the ability to enjoy part of parental labor income.

[17] Suppose $B = \tau A$ where $\tau > 1$. For very low values of $p$, the left hand side of equation (8) can turn negative as returns to human capital are not high enough to compensate for mortality risk. To avoid that we restrict to $p \in [1/\tau, 1]$. $\tau > 1$ also ensures that the left hand side of equation (10) is positive.

A unique final good is produced from aggregate stocks of physical ($K$) and human capital ($H$) using

$$Y_t = F(K_t, H_t) = AK_t^{\alpha} H_t^{1-\alpha}$$

where $\alpha \in (0,1)$. In perfectly competitive factor and goods markets, wage per efficiency unit of labor and rental on capital (assume $\delta = 1$) are

$$\begin{aligned} w_t &= (1-\alpha)A(K_t/H_t)^{\alpha}, \\ r_t &= \alpha A(H_t/K_t)^{1-\alpha}. \end{aligned} \tag{11}$$

We assume a unit measure of households born at each date with a $p$ fraction of them surviving into middle age. Denote household holdings of the two assets by $k$ and $h$. The aggregate stocks are then $K_t = k_t$ and $H_t = ph_t$ where $h$ is the human capital of each middle-aged person before experiencing their survival shock. The young consume out of their shares $(1-\theta_1)$ and $(1-\theta_2)$ respectively of the parental capital and labor income. As with the CRRA case of the previous section, suppose $\theta_1 \equiv \theta$ and $\theta_2 = 0$ without loss of generality.

Youth households invest $x_t$ in physical capital and $e_t$ in human capital that yields asset levels

$$k_{t+1} = f(x_t), \;\; h_{t+1} = g(e_t)$$

the following period. The production functions[18] $f$ and $g$ are concave and satisfy $f(0) = 0 = g(0)$. The rate of transformation of the consumption good into physical capital is not constant to allow for relative price effects on $k$ since individuals are risk neutral (see below).

In the absence of annuities, the decision problem is to maximize expected lifetime utility

$$V_t \equiv u(c_{1t}) + \beta p u(c_{2t+1}) + \gamma E_t V_{t+1}$$

subject to

$$\begin{aligned} c_{1t} &= y_t - x_t - e_t, \\ c_{2t+1} &= \theta r_{t+1} k_{t+1} + w_{t+1} h_{t+1}, \end{aligned}$$

and stochastic bequests

$$y_{t+1} = \begin{cases} (1-\theta) r_{t+1} k_{t+1}, & \text{w.p. } p \\ r_{t+1} k_{t+1}, & \text{w.p. } 1-p \end{cases}$$

The middle-age budget constraint embodies the assumption that returns to physical capital are shared with the offspring and ownership of that asset is costlessly passed on to her if the parent dies prematurely.

[18] $f$ here denotes the transformation of current consumption into future consumption. In previous sections we used $q$ to denote the return on physical assets which here is linear and exogenous to the individual.

Assume linear utility,

$$f(x) = ax^{\chi}, \;\; g(e) = be^{\chi}, \quad \chi \in (0,1) \tag{12}$$

and that all families start with a relatively high initial endowment of physical capital $k_0$ so that they are at their unconstrained optima where $c_1 > 0$. The optimality conditions then lead to investment decisions

$$\begin{aligned} x_t &= \left[a\chi \left\{\gamma + p\theta(\beta - \gamma)\right\} r_{t+1}\right]^{1/(1-\chi)}, \\ e_t &= \left[b\beta p\chi w_{t+1}\right]^{1/(1-\chi)}, \end{aligned}$$

household stocks of tangible and intangible capital

$$\begin{aligned} k_{t+1} &= a^{1/(1-\chi)} \left[\chi\{\gamma + p\theta(\beta - \gamma)\}\right]^{\chi/(1-\chi)} r_{t+1}^{\chi/(1-\chi)}, \\ h_{t+1} &= b^{1/(1-\chi)} (\chi\beta p)^{\chi/(1-\chi)} w_{t+1}^{\chi/(1-\chi)}, \end{aligned}$$

and the relative aggregate capital stock[19]

$$\frac{K_t}{H_t} = \frac{k_t}{ph_t} = \left(\frac{a}{b}\right)^{1/(1-\chi)} \left[\frac{\gamma + p\theta(\beta - \gamma)}{\beta p}\right]^{\chi/(1-\chi)} \frac{1}{p} \left(\frac{r_t}{w_t}\right)^{\chi/(1-\chi)}. \tag{13}$$

From (11), on the other hand,

$$\frac{r_t}{w_t} = \frac{\alpha}{1-\alpha}\frac{H_t}{K_t} \tag{14}$$

Using (13) and (14), we can solve for the equilibrium factor ratio

$$\frac{K_t}{H_t} = \frac{a}{b}\left(\frac{\alpha}{1-\alpha}\right)^{\chi} \frac{1}{p^{1-\chi}} \left[\frac{\gamma + p\theta(\beta - \gamma)}{\beta p}\right]^{\chi} \tag{15}$$

which is a decreasing function of $p$ under $\beta \geq \gamma$.

Investment in physical capital depends positively on its return, $r$. Since $K$ and $H$ are complementary inputs, an increase in the supply of human capital induced by $p$, would raise returns to physical capital and encourage its investment. Equilibrium supply of physical capital now depends positively on $p$. But as equation (15) shows, this second-round effect is not enough to bias the overall response away from human capital.[20]

It is easy to show that aggregate output

$$Y = \Gamma p^{\frac{1-\alpha}{1-\chi}} \left[\frac{1}{\beta}\left\{\gamma + \theta p(\beta - \gamma)\right\}\right]^{\frac{\alpha\chi}{1-\chi}}$$

[19] The ratio of aggregate stocks would be the same if individuals were at a constrained optima.

[20] There are two effects of $p$ in this equation. The first term involving $p$ on the right-hand side is the direct supply effect: changes in $p$ shift aggregate labor supply for any level of $h$. The second term is the effect on individual portfolio choice.

depends positively on longevity as long as $\beta \geq \gamma$. Since both physical and human capital depreciate fully, the economy will jump straight to this steady-state output level assuming a high enough $k_0$. If capital did not fully depreciate, however, the transition path would also depend on $p$. Not only would low-$p$ countries converge to a lower steady-state, their transition would be slower too. These high mortality economies would rely more intensively on physical capital, the switch from physical to human capitals as engines of development occurring later and remaining incomplete.

There are two implications. First, from the expression above, aggregate output is a convex function of $p$ as long as the return to human capital is not too low relative to the return to physical capital as in Figure 4 left panel. This means, a reduction in $p$ at high levels of survival, lead to proportionately higher output loss than an equivalent reduction in $p$ at low levels of survival. In effect, a low-$p$ society self-insures against mortality shocks by allocating more towards transferable assets. Of course, the more relevant effect is on output per worker, $y = Y/(1+p)$, and for the same parameter values, $y$ too is convex in $p$. This is readily seen in the right panel of Figure 4 where the marginal effect of $p$ on $y$ rises with the survival rate. Hence, a mortality shock that lowers $p$, such as an HIV or ebola outbreak affecting the adult population, will cost less output per worker in already-high mortality environments than in low-mortality ones.[21]

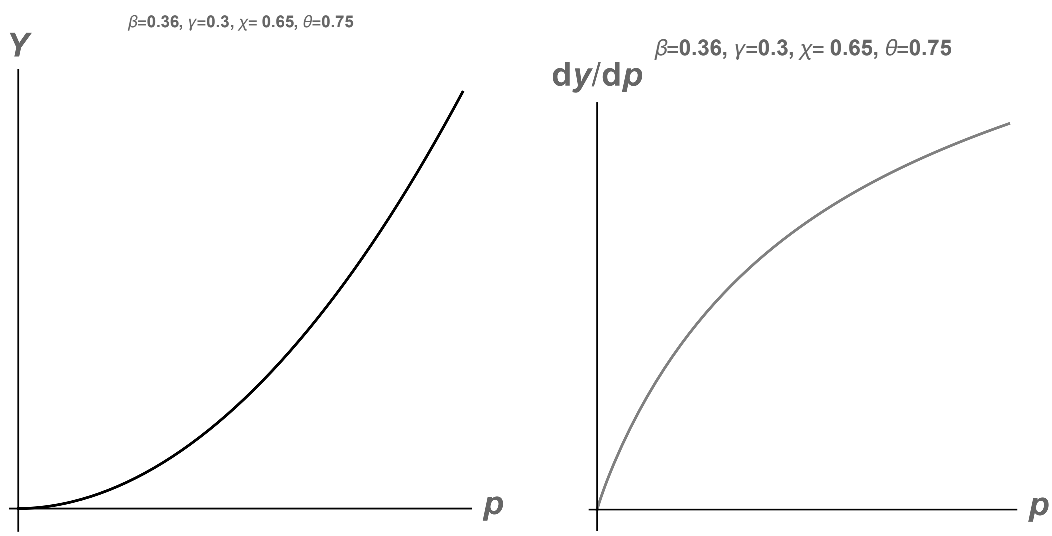


Figure 4: Effect of $p$ on aggregate output $Y$ and marginal output per worker $dy/dp$

A second implication is that a transition from physical to human capital based production can be facilitated by health and mortality improvements. The widespread mortality reductions (not limited to child and infant survival) in late nineteenth century West-

[21] In other words, diminishing marginal product of a factor input does not necessarily imply a proportionately higher output loss from lower $p$ in developing countries – one has to take into account the portfolio effect.

ern Europe may have spurred accumulation and innovation towards newer generations of technologies that were biased towards human capital.[22] If newer technologies in the twentieth century have been skill oriented, as a body of work now argues, it has implications for developing countries. For instance an increase in the return to human capital $B$ in a low-$p$ country would see a more muted response in skill acquisition compared to a high-$p$ country. High mortality, in other words, biases the response away from newer technologies. The lack of catch-up in parts of the developing world plagued by epidemics and health challenges may be as much to do with the low return from adopting modern technologies as with institutional constraints that prevent such adoption.[23]

## 5.1 Availability of Life Insurance

We have so far ignored the availability of a life insurance policy that provides guaranteed income to survivors in the event of premature parental death. The appeal of such a policy is that it allows an altruistic parent to circumvent the problem of non-transferability of human capital (Fischer, 1973).

Suppose an agent has the option of investing a part of her first period income in life insurance with the objective of transferring a part of his total earnings (from physical as well as human capital) to her child even in the event of premature death. Life insurance firms operate on a no-profit no-loss basis and invest the funds in the market. The returns from this are transferred to offsprings whose parents have died prematurely. Children whose parents are alive to make an end-of-the-period bequest get nothing. Since human capital is inalienable, the only investment vehicle available to life insurance companies is physical capital.

A life insurance market allows parents to diversify bequest risks arising from premature death. We ignore annuities since whether or not they are available for the diversification of consumption risk is peripheral for this part of the analysis and results derived below can be compared directly to those immediately above.

The aggregate technology is as above and factor payments given by (11). The aggregate human capital stock is $ph$ while the aggregate physical capital now consists of individual holdings of physical capital (denoted by $k$) and holdings of capital by the life

[22]See Cutler *et al* (2006) on mortality reduction. On technological change, Abramovitch (1993) writes, as quoted in Galor and Moav (2004): "In the nineteenth century, technological progress was heavily biased in a physical capital-using direction. ... In the twentieth century, however, the physical capital-using bias weakened; it may have disappeared altogether. The bias shifted in an intangible (human and knowledge) capital-using direction and produced the substantial contribution of education and other intangible capital accumulation to this century's productivity growth..."

[23]Similar distributional implications are possible if households differed in their survival rates: low-$p$ households would exhibit a preference towards tangible assets and benefit less from skill-biased technological change.

insurance firms ($\kappa$ per policy holder). Hence

$$K_t = k_t + \kappa_t, \ \ H_t = ph_t.$$

Suppose in their youth individuals invest $x_t$ in physical capital, $e_t$ in human capital and $z_t$ in buying life insurance that yields asset levels in the following period,

$$k_{t+1} = f(x_t), \ \ \kappa_{t+1} = f(z_t), \ \ h_{t+1} = g(e_t),$$

where the production functions $f$ and $g$ are concave as specified in (12). The decision problem of a young adult at time $t$ is to maximize expected lifetime utility

$$V_t \equiv u(c_{1t}) + \beta p u(c_{2t+1}) + \gamma E_t V_{t+1}$$

subject to

$$\begin{aligned} c_{1t} &= y_t - x_t - e_t - z_t, \\ c_{2t+1} &= \theta r_{t+1} k_{t+1} + w_{t+1} h_{t+1}, \end{aligned}$$

and

$$y_{t+1} = \begin{cases} (1-\theta) r_{t+1} k_{t+1}, & \text{w.p. } p \\ r_{t+1}\left(k_{t+1} + \dfrac{\kappa_{t+1}}{1-p}\right), & \text{w.p. } 1-p \end{cases}$$

Bequests include life insurance policy payouts to children of prematurely deceased parents.

The first order conditions for optimal investment are

$$x_t : u'(c_{1t}) = [\theta\beta p u'(c_{2t+1}) + \gamma \left\{ p(1-\theta) u'(c^a_{1t+1}) + (1-p) u'(c^d_{1t+1}) \right\}] r_{t+1} f'(x_t) \quad (16)$$

$$e_t : u'(c_{1t}) = \beta p u'(c_{2t+1}) w_{t+1} g'(e_t) \quad (17)$$

and

$$z_t : u'(c_{1t}) = [\gamma u'(c^d_{1t+1})] r_{t+1} f'(z_t) \quad (18)$$

For comparability assume linear utility and that all dynasties start with a relatively high initial endowment of physical capital $k_0$ so that members are at their unconstrained optima ($c_1 > 0$). Equations (16) , (17) and (18) then lead to optimal investment decisions of

$$\begin{aligned} x_t &= \left[a\chi \left\{\gamma + p\theta(\beta - \gamma)\right\} r_{t+1}\right]^{1/(1-\chi)} \\ e_t &= \left[b\beta p \chi w_{t+1}\right]^{1/(1-\chi)} \\ z_t &= \left[a\chi\gamma r_{t+1}\right]^{1/(1-\chi)}. \end{aligned}$$

Consequently household stocks of the assets are

$$\begin{aligned} k_{t+1} &= a^{1/(1-\chi)} \left[ \chi\{\gamma + p\theta(\beta-\gamma)\} \right]^{\chi/(1-\chi)} r_{t+1}^{\chi/(1-\chi)} \\ h_{t+1} &= b^{1/(1-\chi)} (\chi\beta p)^{\chi/(1-\chi)} w_{t+1}^{\chi/(1-\chi)} \\ \kappa_{t+1} &= a^{1/(1-\chi)} \left[ \chi\gamma \right]^{\chi/(1-\chi)} r_{t+1}^{\chi/(1-\chi)} \end{aligned}$$

and the ratio of aggregate capital stocks is

$$\frac{K_t}{H_t} = \frac{k_t + \kappa_{t+1}}{ph_t} = \left(\frac{a}{b}\right)^{1/(1-\chi)} \left[\frac{2\gamma + p\theta(\beta-\gamma)}{\beta p}\right]^{\chi/(1-\chi)} \frac{1}{p} \left(\frac{r_t}{w_t}\right)^{\chi/(1-\chi)}. \tag{19}$$

From (11), on the other hand,

$$\frac{r_t}{w_t} = \frac{\alpha}{1-\alpha}\frac{H_t}{K_t} \tag{20}$$

Using (13) and (14), we can solve for the equilibrium factor ratio

$$\frac{K_t}{H_t} = \frac{a}{b}\left(\frac{\alpha}{1-\alpha}\right)^{\chi} \frac{1}{p^{1-\chi}} \left[\frac{2\gamma + p\theta(\beta-\gamma)}{\beta p}\right]^{\chi} \tag{21}$$

which once again is a decreasing function of $p$ for $\beta \geq \gamma$. The corresponding output per capita is given by

$$Y = \Gamma p^{\frac{1-\alpha}{1-\chi}} \left[\frac{1}{\beta}\left\{2\gamma + \theta p(\beta-\gamma)\right\}\right]^{\frac{\alpha\chi}{1-\chi}}$$

which also depends positively on longevity as long as $\beta \geq \gamma$.

A direct comparison with the results derived without life insurance tells us, for any $p$, the equilibrium $K/H$ ratio is *higher* with life insurance than without. This is partly because of the assumptions of linear utility and individuals having sufficient first period income to achieve their unconstrained optima. Together they imply that investment in each available asset is pushed to its maximum possible limit (where its marginal return equals unity). Availability of a third asset (life insurance) does therefore affect household investment in other assets.[24] Thus, while households' investments in physical and human capital remain unchanged, life insurance firms now invest in physical capital alone, which increases the aggregate $K/H$ ratio relative to before.

But aggregate physical capital would be higher even if we relax the assumption of linear utility and/or tighten the budget constraint of the household to force them to corner solutions. In that case, the household would maintain a constant ratio of all the assets (such that their marginal returns are all equal), but availability of a third asset would lower the household's investment allocation towards physical capital. However, at the

[24]Observe from the respective first order conditions that optimal investments in $x_t$ and $e_t$ are exactly the same with and without life insurance.

aggregate level, $K/H$ ratio would still go up under life insurance as life insurance investments are channeled towards physical capital formation which more than compensates for the fall in household physical capital investment. Consequently the overall balance tilts towards physical capital vis-a-vis human capital under lifetime uncertainty.

We conclude that the availability of life insurance does not make a qualitative difference to the basic result of sections 4 and 5, that mortality has a differential effect on tangible versus intangible investment because of the latter's non-transferability.

# 6 Conclusion

Our study of the effect of mortality on economic development makes two contributions. First we show that intergenerational wealth transfer by altruistic households can mitigate the investment loss that can occur from future consumption uncertainty when annuity markets are missing. Secondly, when people face uncertain lifetimes, they are shown to invest more intensively in tangible assets that can be passed on to their survivors. High mortality societies would therefore rely on physical capital accumulation more intensively than low mortality ones. Together these results have implications for long-run growth, convergence, and technology adoption.

There are several avenues for future work. In ongoing research, Chakraborty and Das (2019) explore the long-run implications of mortality when parental altruism develops in response to the environment. For example, if altruism requires parental time investment, developing a sufficiently high altruism comes at the opportunity cost of time devoted to building human capital. Thus altruism is likely to be high when parents are engaged in occupations that are less skill intensive, like primary production. At the same time, high mortality itself makes investment in physical capital more profitable than human capital following the logic of this paper. In the initial stages of development these two mechanisms work in tandem to generate a scenario where high mortality leads to concentration of production in the primary sector, which via endogenous altruism produces a high altruism coefficient that in turn sustains this scenario for a long period of time until some exogenous improvement in mortality breaks the vicious circle.

Another extension would be to identify how the intergenerational transmission of wealth under lifetime uncertainty affects fertility choice and willingness to invest in child human capital. When parents expect their children to live short lives and face the same non-transferability problem of intangible assets, they would be unwilling to invest in child quality, amplifying the human capital margin identified above. Under the usual quantity-quality tradeoff, fertility rates would be higher too which, conditional on child survival, further incentivizes tangible investment. The demographic transition, in this story, becomes tightly linked to the adult health transition through a mechanism different from the ones emphasized in the literature such as Kalemli-Ozcan *et al.* (2000), Soares (2005) and Aksan and Chakraborty (2014).

# Appendix: Endogenous $\theta$

Here we allow $\theta$ to be endogenously determined in the single-asset case and show that for the case of logarithmic utility it is constant over time, depends positively on the survival rate $p$ and there is no differential effect on the investment rate of the availability of annuities.

### Perfect Annuities

An agent born at the beginning of period $t$ has an endowment of $(1-\theta_t)q(k_t)+(1-\delta)k_t$, where both $\theta_t$ and $k_t$ are chosen by the parent. Hence $c_{1t}=(1-\theta_t)q(k_t)+(1-\delta)k_t-k_{t+1}$. At the beginning of $t+1$, before the mortality shock is realized, she gets an income of $q(k_{t+1})$ from the augmented asset and decides to leave $(1-\theta_{t+1})$ proportion of its income to her progeny. The rest is pledged to the annuity provider. In the event she survives, $c_{2t+1}=\theta_{t+1}q(k_{t+1})/p$.

The first order conditions for investment and bequest are

$$u'(c_{1t})=\beta p u'(c_{2t+1})\frac{\theta_{t+1}q'(k_{t+1})}{p}+\gamma V_1(k_{t+1},\theta_{t+1})$$

$$\beta p u'(c_{2t+1})\frac{q(k_{t+1})}{p}=\gamma V_2(k_{t+1},\theta_{t+1})$$

and the envelope conditions:

$$V_1(k_{t+1},\theta_{t+1})=u'(c_{1t+1})\left[(1-\theta_{t+1})q'(k_{t+1})+(1-\delta)\right]$$

$$V_2(k_{t+1},\theta_{t+1})=u'(c_{1t+1})[-q(k_{t+1})]$$

Suppose now $u(c)=\ln c$, $q(k)=Ak^\alpha$, and $\delta=1$ under which the two optimality conditions are

$$\frac{1}{(1-\theta_t)Ak_t^\alpha-k_{t+1}}=\beta p\alpha\frac{1}{k_{t+1}}+\gamma\left[\frac{1}{(1-\theta_{t+1})Ak_{t+1}^\alpha-k_{t+2}}\right](1-\theta_{t+1})\alpha Ak_{t+1}^{\alpha-1} \quad (22)$$

$$\frac{\beta p}{\theta_{t+1}}=\gamma\left[\frac{1}{(1-\theta_{t+1})Ak_{t+1}^\alpha-k_{t+2}}\right]Ak_{t+1}^\alpha \quad (23)$$

From (23), simplifying,

$$\theta_{t+1}=\frac{\beta p}{(\gamma+\beta p)}\frac{(Ak_{t+1}^\alpha-k_{t+2})}{Ak_{t+1}^\alpha} \quad (24)$$

Conjecture that the solution takes the form: $k_{t+1}=\mu q(k_t)=\mu Ak_t^\alpha$, where $\mu$ is an unknown constant. Leading the solution one period forward and substituting in equation (24) gives us

$$\theta_{t+1}=\frac{\beta p}{\gamma+\beta p}(1-\mu)$$

Since this is time-invariant, setting $\theta_{t+1} = \theta_t = \theta$ and substituting the solution for $k_{t+1}$ and $k_{t+2}$ in equation (22) and simplifying, we have

$$\begin{aligned} \frac{1}{(1-\theta-\mu)Ak_t^{\alpha}} &= \frac{\beta p\alpha(1-\theta-\mu)+\gamma\alpha(1-\theta)}{(1-\theta-\mu)k_{t+1}} \\ \Rightarrow k_{t+1} &= \left[\beta p\alpha(1-\theta-\mu)+\gamma\alpha(1-\theta)\right] Ak_t^{\alpha} \end{aligned}$$

Finally by the method of undetermined coefficients, $\mu = \alpha\gamma$ which means the policy rule is $k_{t+1} = \alpha\gamma Ak_t^{\alpha}$. Next note that first period income is $(1-\theta)Ak_t^{\alpha}$. Accordingly,

$$k_{t+1} = \frac{\alpha\gamma}{(1-\theta)}(1-\theta)Ak_t^{\alpha},$$

implies the investment propensity is $\alpha\gamma/(1-\theta)$. Using the equilibrium value of $\theta$, the propensity becomes $\alpha(\gamma+\beta p)/(\gamma+\beta p\alpha\gamma)$. For this to lie between zero and one, we need $\alpha\gamma < 1$ and under this parametric restriction, the propensity is positively related to $p$.

Also note that $\theta$ itself is a positive function of $p$, meaning as the probability of survival increases, households retain a higher proportion of their middle-age income for own consumption, and less (in proportional terms) to the offspring. The offspring's initial income may still go up, since middle-age income will be higher from higher investment.

**Missing Annuities**

When annuities are unavailable, the agent keeps $\theta$ proportion of her first-period income for herself, and leaves $(1-\theta)$ proportion to the offspring before she realizes her mortality shock. If she is alive at the end of the period, she consumes her share. Otherwise that share is automatically passed on to the offspring. As before denote the offspring's initial endowment as

$$y_t = y(k_t, \theta_t, z_t) = \begin{cases} (1-\delta)k_t + (1-\theta_t)q(k_t), & \text{if } z_t = a \\ (1-\delta)k_t + q(k_t), & \text{if } z_t = d \end{cases}$$

For the optimization problem

$$V(k_t, \theta_t, z_t) = \max_{\{k_{t+1}, \theta_{t+1}\}} \left[u(c_{1t}) + \beta p u(c_{2t+1}) + \gamma E_t V(k_{t+1}, \theta_{t+1}, z_{t+1})\right]$$

subject to

$$\begin{aligned} c_{1t} &= y(k_t, \theta_t, z_t) - T_{t+1} \\ c_{2t+1} &= \theta_{t+1} q(k_{t+1}) \\ z_{t+1} &\sim iid \end{aligned}$$

$$\begin{aligned} u'(c_{1t}) &= \beta p u'(c_{2t+1})\theta_{t+1}q'(k_{t+1}) + \gamma E_t\left[u'(c_{1t+1})y_1(k_{t+1},\theta_{t+1},z_{t+1})\right] \\ \beta p u'(c_{2t+1})q(k_{t+1}) &= -\gamma E_t\left[u'(c_{1t+1})y_2(k_{t+1},\theta_{t+1},z_{t+1})\right] \end{aligned}$$

As before, suppose $u(c) = \ln c$, $q(k) = Ak^\alpha$ and $\delta = 1$. Then, following similar steps as in the text, the guessed policy rules $k_{a,t+1} = \mu A k_t^\alpha$ and $k_{d,t+1} = \nu A k_t^\alpha$ lead to

$$\theta_{t+1} = \frac{(1-\mu)\beta}{(\beta+\gamma)} = \theta \ \forall t \tag{25}$$

$$\nu = \frac{\alpha(\beta p + \gamma)}{1+\alpha\beta p} \tag{26}$$

$$\mu = \frac{\alpha(\beta p + \gamma)}{1+\alpha\beta p}(1-\theta). \tag{27}$$

and hence,

$$\mu = \frac{\alpha\gamma(\beta p + \gamma)}{\beta + \gamma - \alpha\beta\gamma(1-p)}$$

The investment propensity is the same whether or not the parent survives or dies, since in the former case it is $\mu/(1-\theta)$. The restriction $\alpha\gamma < 1$ ensures it lies in $[0,1]$. Finally note that it is exactly the same as under annuities.

This result can be generalized to the CRRA case with additional assumptions, including what the current generation expects of future generations' choice of $\theta$. The log case shows that the very fact parents can choose within-family income sharing does not overturn the main tradeoffs they face regarding investment. In particular, as long as the Inada condition is satisfied for preferences, households prefer to leave positive intended and unintended (under missing annuities) bequests *and* invest in the asset.

# References


[1] Abramovitz, M. (1993), "The Search of the Sources of Growth: Areas of Ignorance, Old and New", *Journal of Economic History*, **53**, 217-243.

[2] Abramovitz, M. and P. A. David (2000), "American Macroeconomic Growth in the Era of Knowledge-Based Progress: The Long-Run Perspective", in S. L. Engerman and R. E. Gallman (eds.): *The Cambridge Economic History of the United States*; Cambridge University Press, Cambridge; New York.

[3] Aksan, Anna-Maria and Shankha Chakraborty (2014), "Mortality versus Morbidity in the Demographic Transition," *European Economic Review*, **70**, 470-492.

[4] Andersen, Torben M. and Joydeep Bhattacharya (2014), "A dynamic-efficiency rationale for public investment in the health of the young," *Canadian Journal of Economics*, **47**(3), 697-719.

[5] Barro, Robert and James Friedman (1977), “On Uncertain Lifetimes”, *Journal of Political Economy*, **85**, 843–849.

[6] Bhattacharya, Joydeep and X. Qiao (2007), “Public and Private Expenditures on Health in a Growth Model”, *Journal of Economic Dynamics and Control*, **31**(8), 2519-2535.

[7] Blackburn, K. and G. P. Cipriani (1998), **“**Endogenous Fertility, Mortality and Growth”, *Journal of Population Economics*, **11**, 517-34.

[8] Bembrilla, Laurent (2016), “Endogenous lifetime and economic growth: the role of the tax rate”, *Economic Theory Bulletin*, forthcoming.

[9] Brown, Jeffrey R., Jeffrey R. Kling, Sendhil Mullainathan, and Marian V. Wrobel (2008), ”Why Don’t People Insure Late-Life Consumption? A Framing Explanation of the Under-Annuitization Puzzle.” *American Economic Review*, 98 (2): 304-09.

[10] Browning, Martin, Lars Peter Hansen and James J. Heckman (1999), “Micro Data and General Equilibrium Models”, chapter 8 in *Handbook of Macroeconomics*, vol. 1A.

[11] Chakraborty, Shankha (2004), “Endogenous Lifetime and Economic Growth”, *Journal of Economic Theory*, **116**, 119-137.

[12] Chakraborty, Shankha and Mausumi Das (2019), “Mortality and Structural Change under Directed Altruism,” work in progress.

[13] Chakraborty, Shankha and Fidel Pérez-Sebastián (2019), “Age-Specific Effects of Mortality Shocks and Economic Development”, work in progress.

[14] Chetty, Raj (2006), “A New Method of Estimating Risk Aversion”, *American Economic Review*, 96 (5): 1821-1834.

[15] Cutler, David, Angus Deaton and Adriana Lleras-Muney (2006), “The Determinants of Mortality”, *Journal of Economic Perspectives*, **20**, 97-120.

[16] Davidoff, Thomas, Jeffrey R. Brown and Peter Diamond (2005), “Annuities and Individual Welfare”, *American Economic Review*, **95**(5), 1573-1590.

[17] Dobson M. (1997), *Contours of Death and Disease in Early-Modern England*, Cambridge University Press, Cambridge.

[18] Eaton, Jonathan and Harvey S. Rosen (1980), “Taxation, Human Capital, and Uncertainty”, *American Economic Review*, **70**(4), 705-715.

[19] Fischer, Stanley (1973), “A Life Cycle Model of Life Insurance Purchases”, *International Economic Review*, **14**(1), 132-152.

[20] Fortson, Jane G (2011), "Mortality Risk and Human Capital Investment: The Impact of HIV/AIDS in Sub-Saharan Africa", *Review of Economics and Statistics*, **93**(1), 1-15.

[21] Galor, Oded and Omer Moav (2004), "From Physical to Human Capital Accumulation: Inequality and the Process of Development", *Review of Economic Studies*, **71**, 1001-1026.

[22] Gori, Luca and Mauro Sodini (2014), "Endogenous Labour Supply, Endogenous Lifetime and Economic Development", *mimeo*, University of Genoa.

[23] Gottardi, Piero, Atsushi Kajii and Tomoyuki Nakajima (2015), "Optimal Taxation and Debt with Uninsurable Risks to Human Capital Accumulation." *American Economic Review*, **105**(11), 3443-70.

[24] Greenwood, Jeremy and Bruce Smith (1997), "Financial Markets in Development and the Development of Financial Markets", *Journal of Economic Dynamics and Control*, **21**(1), 145-181.

[25] Jacoby, Hanan G. and Emmanuel Skoufias (1997), "Risk, Financial Markets, and Human Capital in a Developing Country", *Review of Economic Studies*, **64**(3), 311–335.

[26] Janzen, Sarah A. and Michael R. Carter (2013), "The Impact of Microinsurance on Consumption Smoothing and Asset Protection: Evidence from a Drought in Kenya", Research Paper # 31, ILO MicroInsurance Innovation Facility.

[27] Jayachandran, Seema and Adriana Lleras-Muney (2009), "Life Expectancy and Human Capital Investments: Evidence from Maternal Mortality Declines", *Quarterly Journal of Economics*, **124 (**1**)**, 349–397.

[28] Kalemli-Ozcan, Sebnem, Ryder, Harl E. and Weil, David (2000), "Mortality decline, human capital investment, and economic growth," *Journal of Development Economics*, **62 (1)**, 1-23.

[29] Kotlikoff, Lawrence J. and Avia Spivak (1981), "The Family as an Incomplete Annuities Market", *Journal of Political Economy*, **89**(2), 372-91.

[30] Krebs, Tom (2003), "Human Capital Risk and Economic Growth", *Quarterly Journal of Economics,* **118**(2), 709-744.

[31] Lancia, Francesco and Giovanni Prarolo (2012), "A politico-economic model of aging, technology adoption and growth," *Journal of Population Economics*, **25**(3), 989-1018.

[32] Levhari , David and Yoram Weiss (1974), "The Effect of Risk on the Investment in Human Capital", *American Economic Review*, **64**(6), 950-963.

[33] Lockwood, Lee M. (2012): "Bequest motives and the annuity puzzle", *Review of Economic Dynamics,* **15**, 226–243.

[34] Mckeown, T. , R. G. Record & R. D. Turner (1975), ”An interpretation of the decline of mortality in England and Wales during the twentieth century”, *Population Studies*, **29(3)**,

[35] Prettner, Klaus and David Canning (2014), “Increasing life expectancy and optimal retirement in general equilibrium”, *Economic Theory*, **56**(1), 191-217.

[36] Rajaratnam, Julie Knoll, Jake R Marcus, Alison Levin-Rector, Andrew N Chalupka, Haidong Wang, Laura Dwyer, Megan Costa, Alan D Lopez, Christopher J L Murray (2010), “Worldwide mortality in men and women aged 15-59 years from 1970 to 2010: a systematic analysis,” *The Lancet*, Vol. 375, No. 9727, p1704-1720.

[37] Razin, Assaf (1976): “Lifetime Uncertainty, Human Capital and Physical Capital”, *Economic Inquiry*, **14**(3), 439-448.

[38] Ricci, Francesco and Marios Zachariadis (2013), “Education Externalities on Longevity,” *Economica*, **80**(319), 404-440.

[39] Rosenzweig, Mark R. and Kenneth I. Wolpin (1993), “Credit Market Constraints, Consumption Smoothing, and the Accumulation of DurableProduction Assets in Low-Income Countries: Investments in Bullocks in India”, *Journal of Political Economy*, **101**(2), 223-244.

[40] Soares, Rodrigo R. (2005), “Mortality Reductions, Educational Attainment, and Fertility Choice,” *American Economic Review*, **95**(3), 580-601.

[41] Szreter, Simon (1988), “The importance of social intervention in Britain’s mortality decline c. 1850-1914: A reinterpretation of the role of public health,” *Social History of Medicine*, 1, 1–37.

[42] Zhang, Jie, Junsen Zhang and Ronald Lee (2013), “Rising longevity, education, savings, and growth”, *Journal of Development Economics*, **70**, 83-101.

[43] Wrigley, E.A. and R.S. Schofield (1981): *The Population History of England,1541-1871: A Reconstruction*, Cambridge University Press, Cambridge.