The substructure of jets

Davison E. Soper
University of Oregon

with help from Zoltan Nagy and Michael Spannowsky

KITPC, Beijing, June 2012

- There are several methods for finding LHC signals using the substructure of jets.
- This talk is not a review of these analysis methods.
- Rather it is about the structure in nature that these methods try to take advantage of.
- I present mostly information that everybody knows.
- At the end, as time allows, I present one method: "shower deconstruction."

Signal and background

- We seek new physics signals in a standard model background.

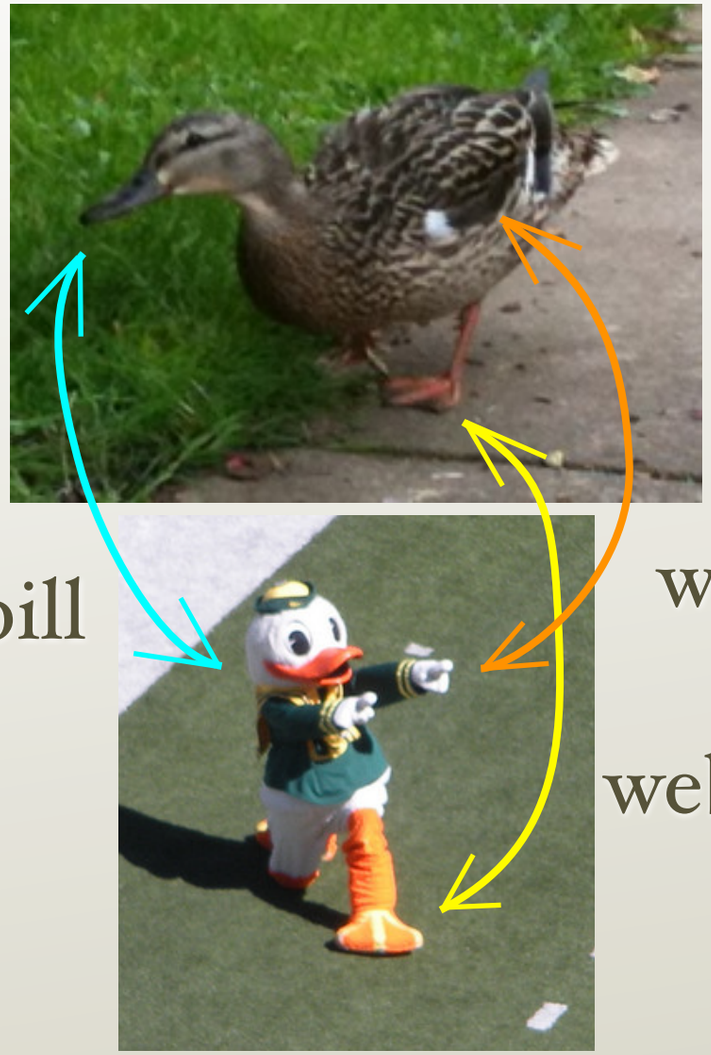
- In Eugene Oregon, there are many background creatures.

- There are signal creatures that look a somewhat like background creatures, but there are very few of them.

- A signal creature.

- There are many features in common between signal and background creatures.

- We need to find the differences. (Eg. one of these can't swim.)



- Unfortunately, background creatures come with a range of mutations that make them sometimes look like a signal creatures.

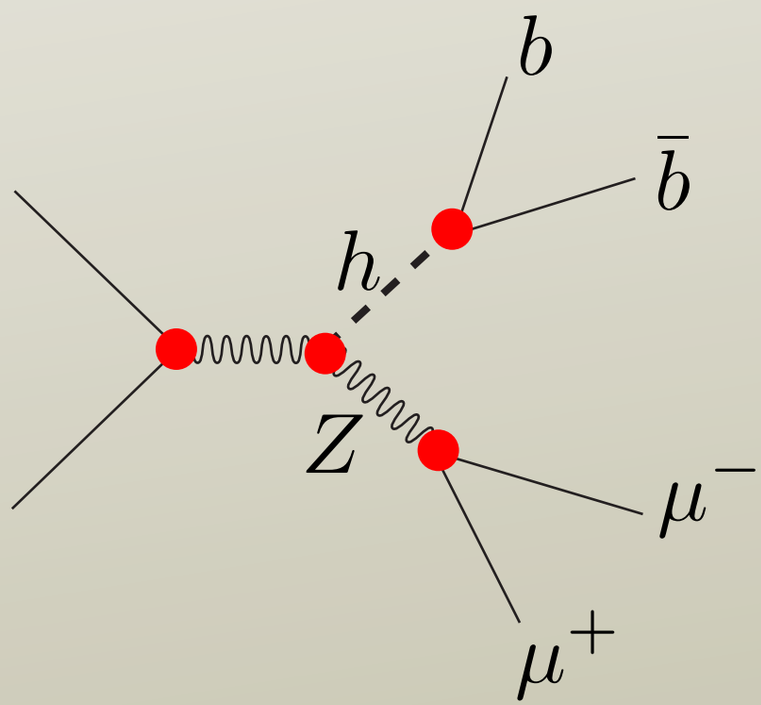
- Thus we look need a statistical method to tell if there are signal creatures.

Signals and backgrounds at the LHC

Signal events

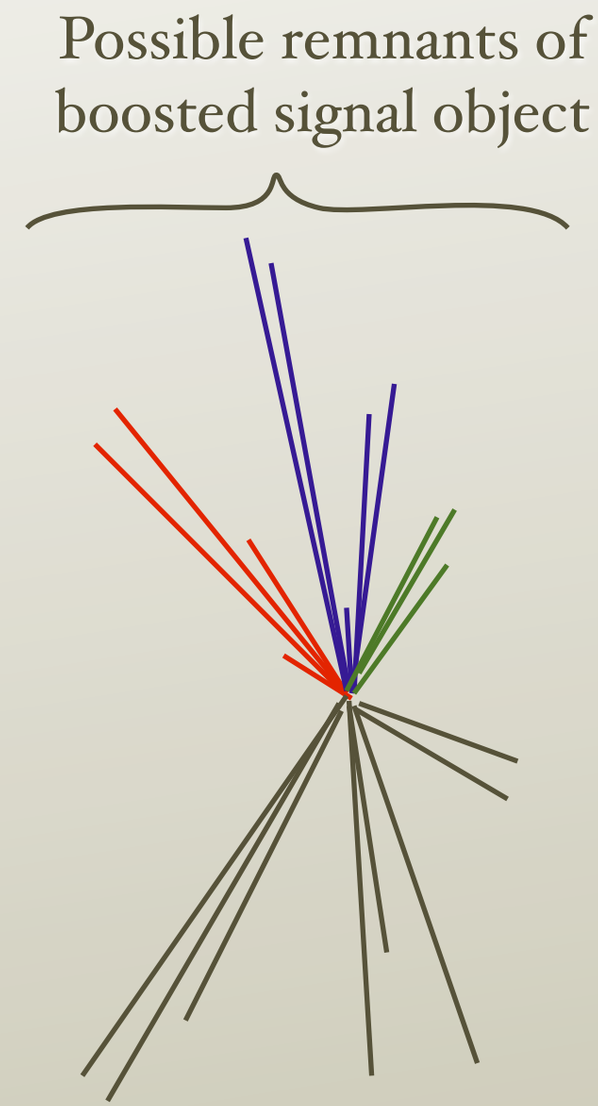
- I take a signal event to be one in which one or more new heavy particles is created and decays.
- Sometimes there is a chain of decays.
- Also, top quarks could be part of both signal and background events for some signals.

A Higgs boson signal event



Signal and background with jets

- Both signal and background events have jets.
- There are several techniques for separating signal from background using the structure of the jets.
- This is especially effective for highly boosted heavy objects.

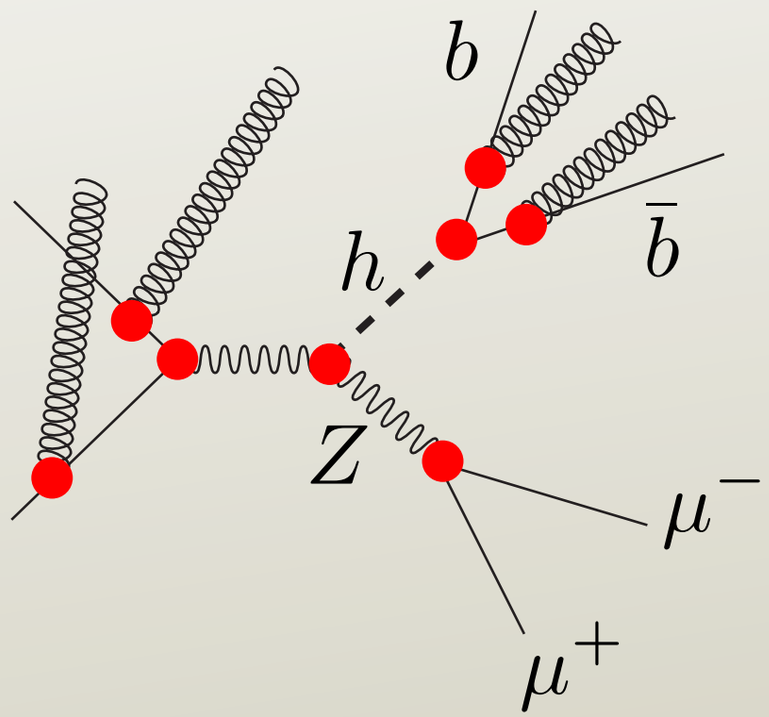


What gauge field theory has to say

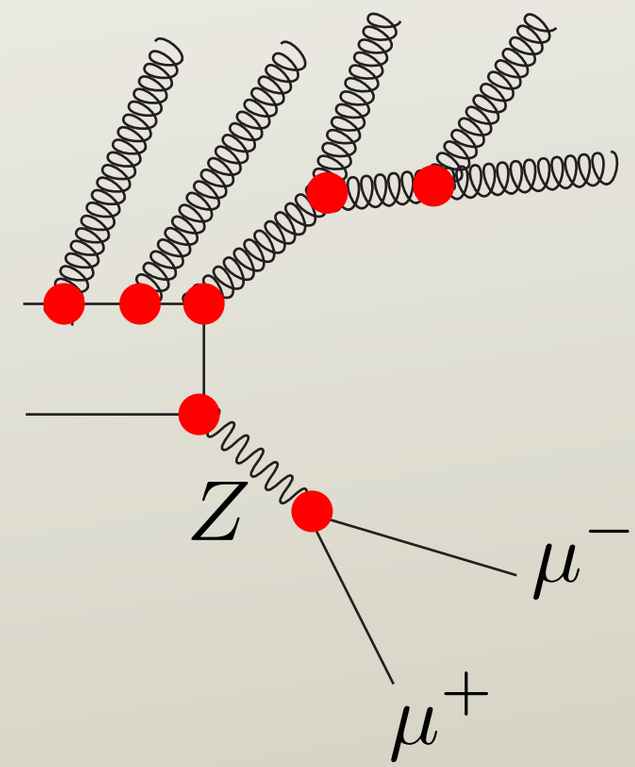
- One can use field theory to understand the characteristics of background events and the characteristics of signal events (with heavy particle decays).
- Schemes for selecting signal events depend on using the similarities and differences in signal and background events.
- I will try to outline the characteristics that can be used.

Initial state radiation

- One immediate similarity is that both signal and background events have initial state radiation.



A Higgs signal event



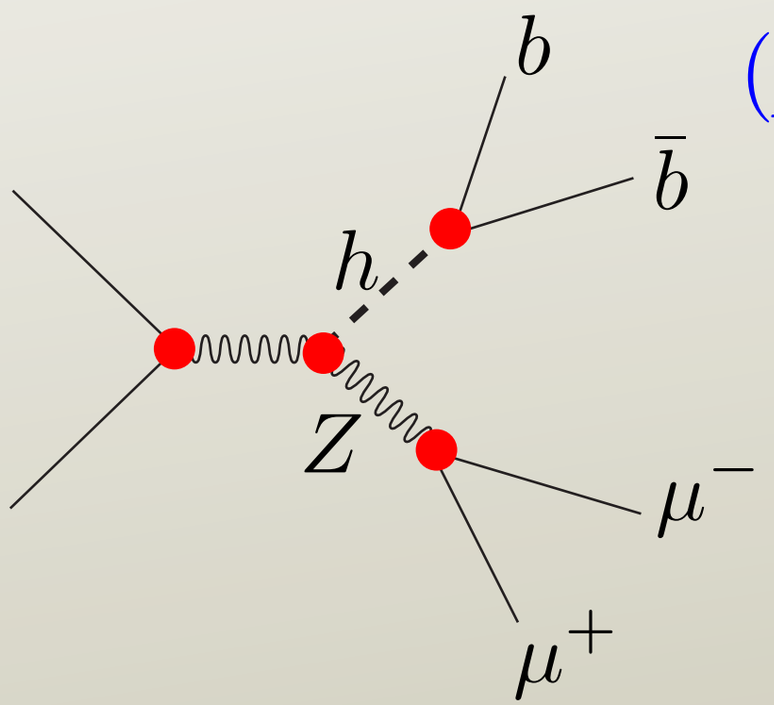
A similar background event

- This makes the signal/background separation difficult.

- More on this later.

Mass bumps

- One key feature of signal events is that the decay products of a heavy particle have a fixed mass.

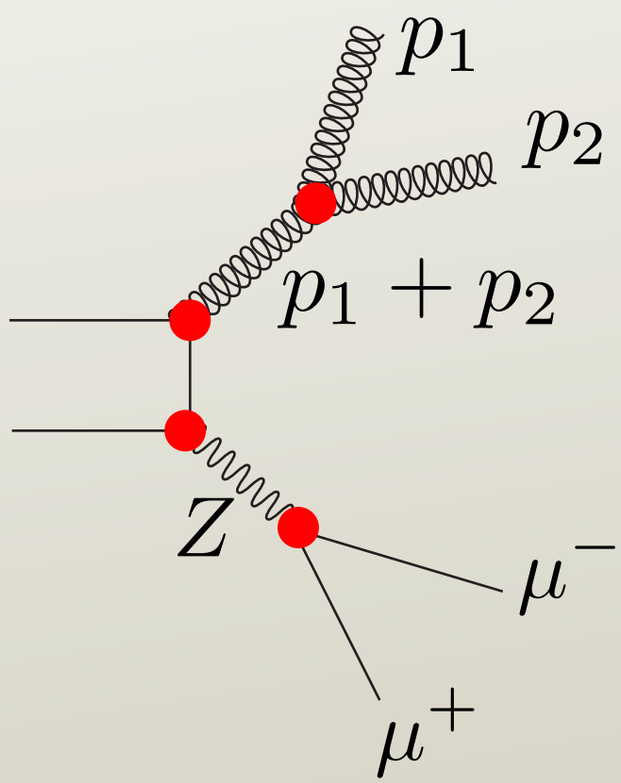


$$(p_b + p_{\bar{b}})^2 = M_h^2$$

- The theoretical uncertainty of this is $M_h \Gamma_h$.
- The practical uncertainty is much larger.

Mass in QCD splittings

- QCD splittings have a very different distribution as a function mass.



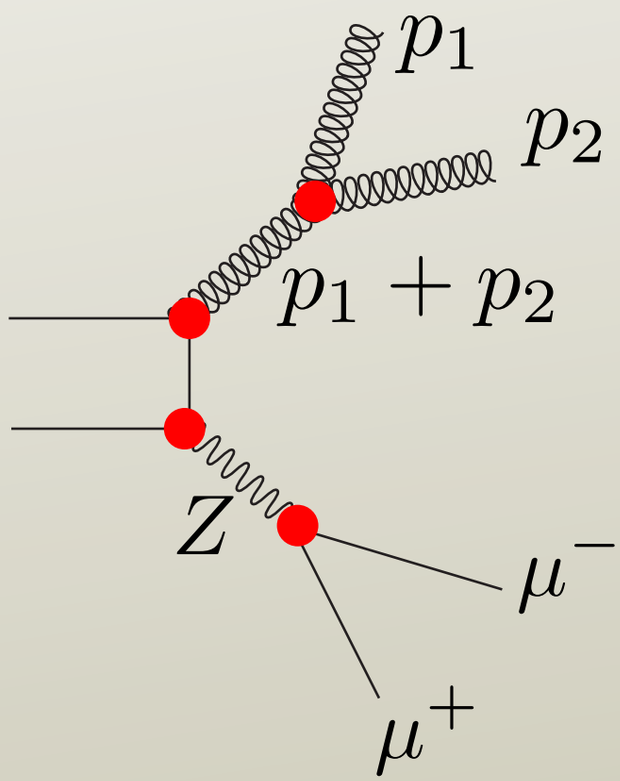
- I will use a scaled virtuality as a "hardness variable":

$$v = \frac{(p_1 + p_2)^2}{(p_1 + p_2) \cdot Q_0}$$

- Q_0 is the momentum of the outgoing partons in the hard process.

QCD tree graph results

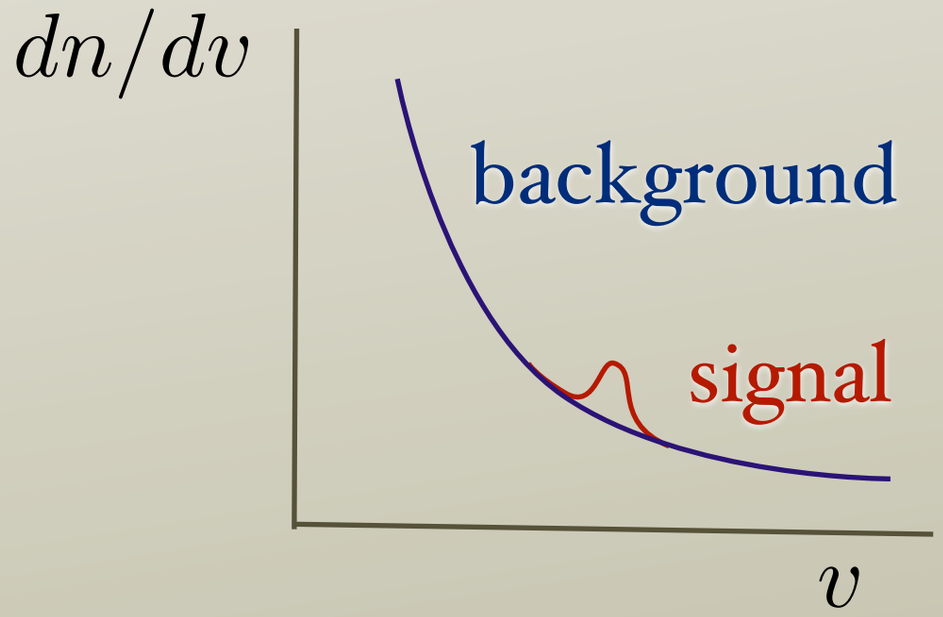
$$v = \frac{(p_1 + p_2)^2}{(p_1 + p_2) \cdot Q_0}$$



- Differential probability is

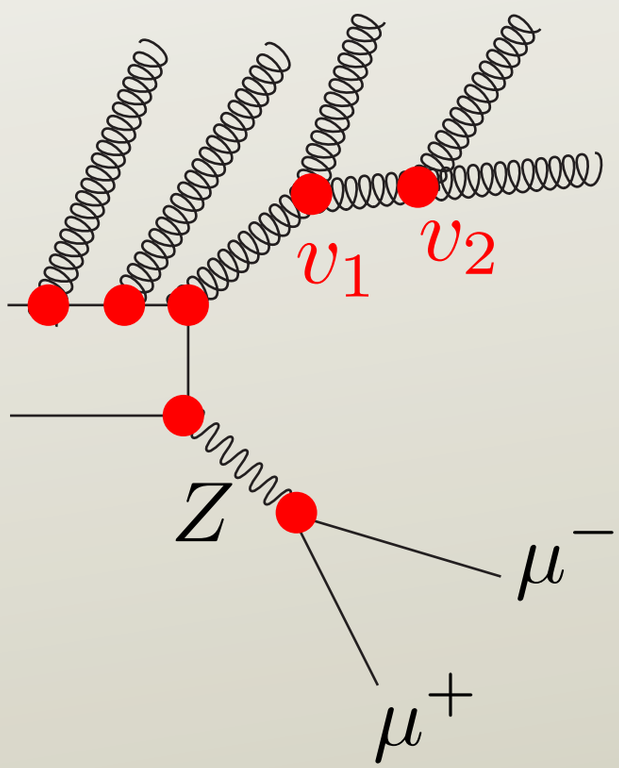
$$dn \propto \frac{dv}{v}\ \alpha_s \log(v/\tilde{v})$$

- The log (and $\tilde{v}$) comes from $\int dz$.



Nested splittings

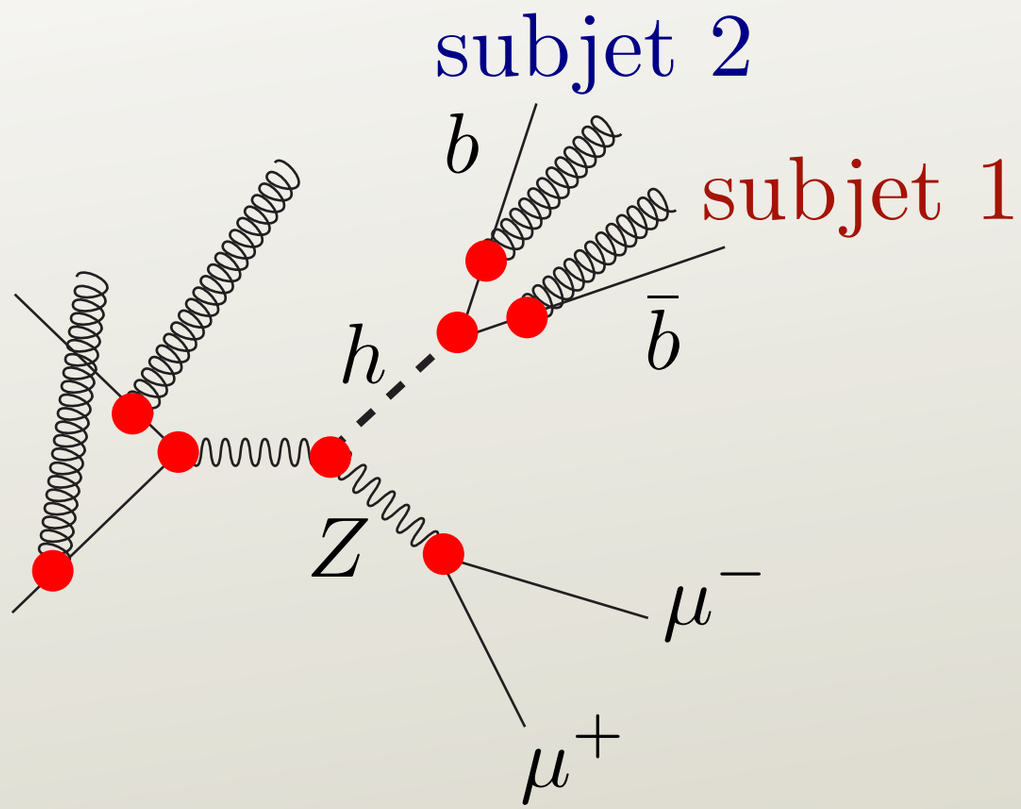
- The same structure can be (approximately) iterated.

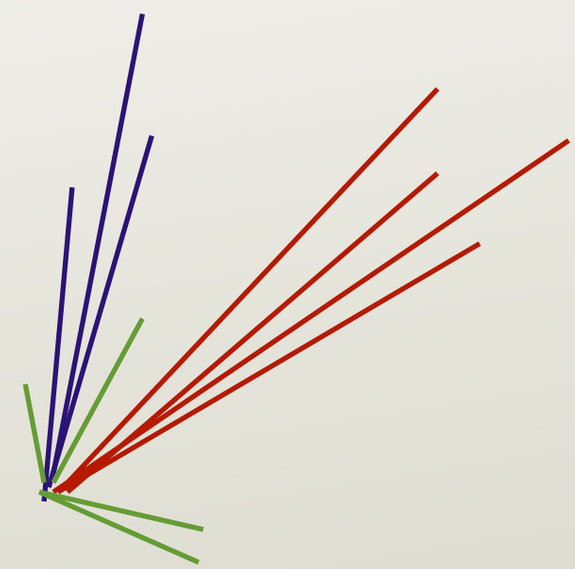


- $v_2 < v_1$.
- Set $v_2 = 0$ in splitting 1.

$$dn = \frac{dv_1}{v_1}\ \alpha_s \log(v_1/\tilde{v}_1)$$
$$\times\ \frac{dv_2}{v_2}\ \alpha_s \log(v_1/\tilde{v}_2)\theta(v_2 < v_1)$$

An application of this

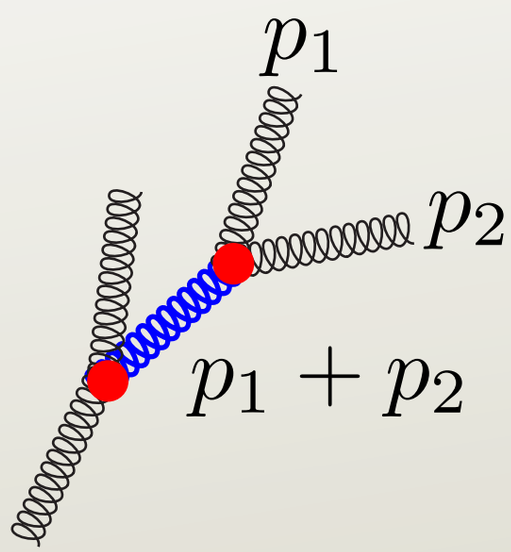




- A Higgs boson will likely decay to two low mass subjets, plus soft gluons.
- Try to find the low mass subjets.
- A "mass drop" condition can be part of this.
  (Butterworth, Davison, Rubin & Salam.)
- Measure the mass of subjet pair.

Sudakov factor

- A better approximation for dn includes a Sudakov factor.



$$dn \propto \frac{dv}{v}\ \alpha_s \log(v/\tilde{v})$$

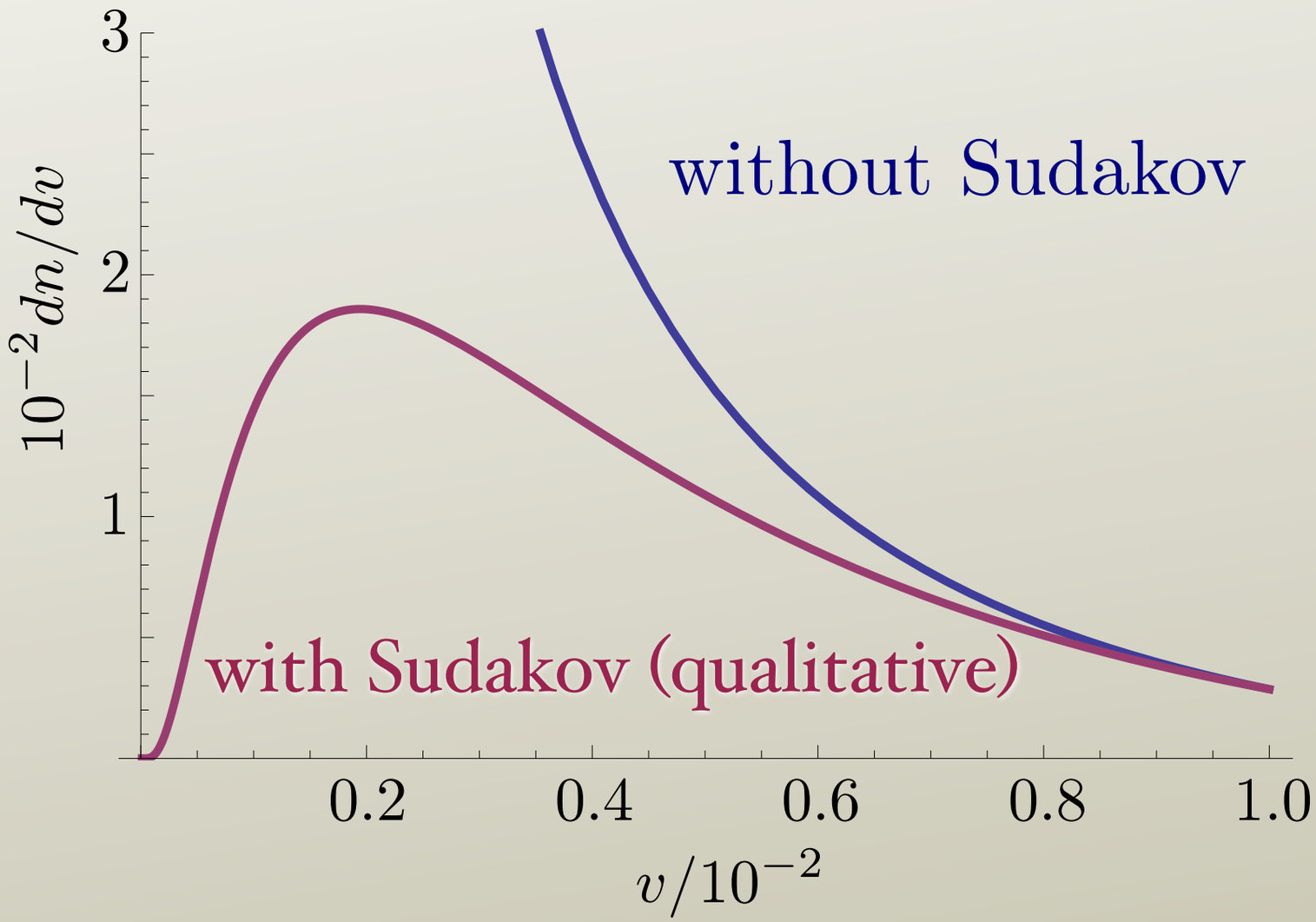
$$\exp\left(-\int_v^{v_0} \frac{d\bar{v}}{\bar{v}}\ \alpha_s \log(\bar{v}/\tilde{v})\right)$$

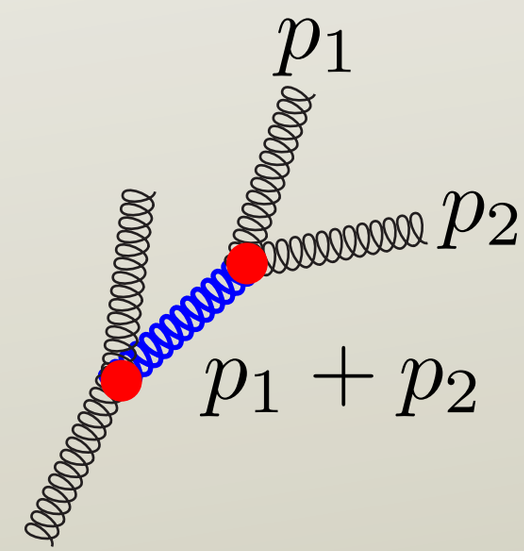
- We think of a renormalization group approach with a running resolution scale.

- The Sudakov factor accounts for virtual graphs and unresolved real parton emissions.

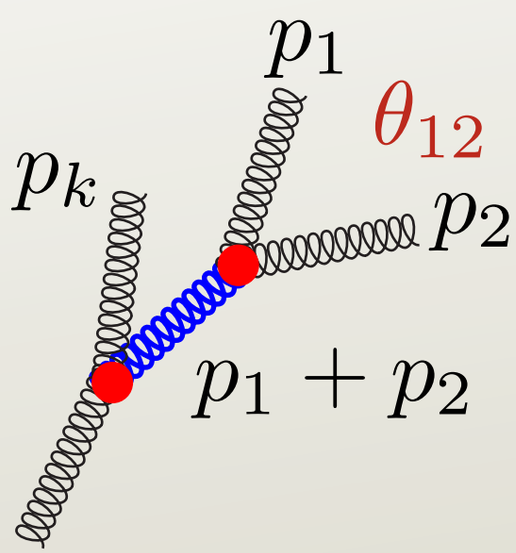
Example of effect of Sudakov

$$dn \propto \frac{dv}{v}\ \alpha_s \log(v/\tilde{v}) \exp\left(-\int_v^{v_0} \frac{d\bar{v}}{\bar{v}}\ \alpha_s \log(\bar{v}/\tilde{v})\right)$$





Distribution in angles



- For a given v, what is the distribution in the angle θ_{12} between $\vec{p}_1$ and $\vec{p}_2$?

- Kinematically,

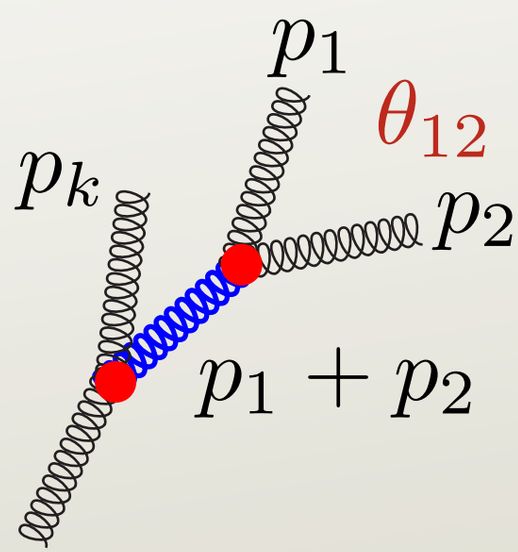
$$\theta_{12}^2 > v \, \frac{4Q_0^2}{(p_1 + p_2) \cdot Q_0}$$

- For larger, but not too large, θ_{12},

$$v = \frac{(p_1 + p_2)^2}{(p_1 + p_2) \cdot Q_0}$$

$$dn \propto \frac{d\theta_{12}^2}{\theta_{12}^2}$$

Large angles



- Large θ_{12} corresponds to $|\vec{p}_1| \ll |\vec{p}_2|$.
- (Or gluon 2 could be the soft one.)
- For emission of soft gluon 1, interference with emission from parton k is important.
- Parton k is the color connected partner.

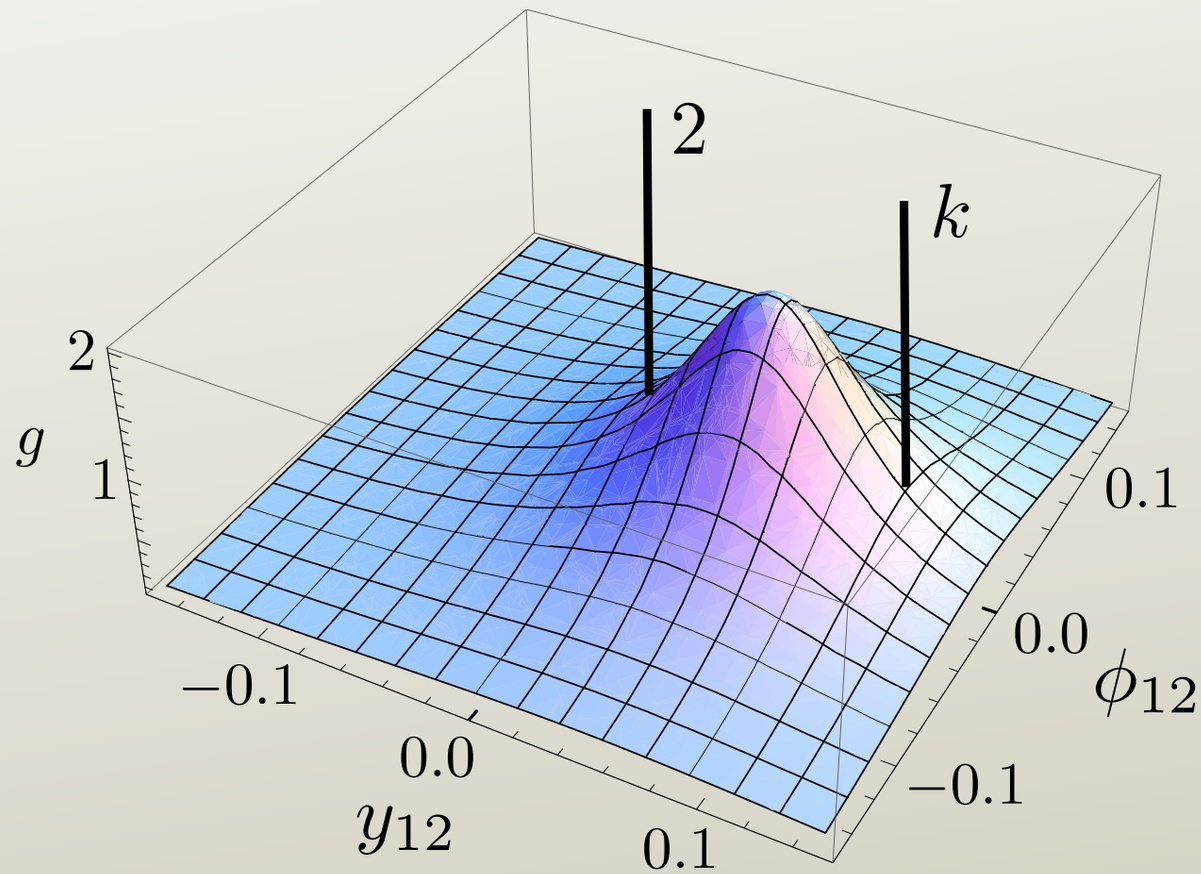
- With interference,

$$dn \propto \frac{d\theta_{12}^2}{\theta_{12}^2} g(\theta_{12}^2) \qquad g(\theta_{12}^2) = \frac{\theta_{2k}^2}{\theta_{12}^2 + \theta_{1k}^2}$$

Effect of the angle factor

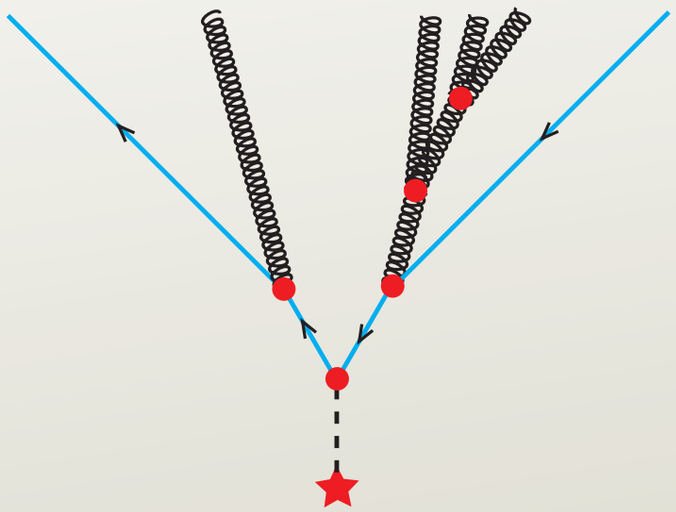
$$dn \propto \frac{d\theta_{12}^2}{\theta_{12}^2} g(\theta_{12}^2)$$

$$g(\theta_{12}^2) = \frac{\theta_{2k}^2}{\theta_{12}^2 + \theta_{1k}^2}$$

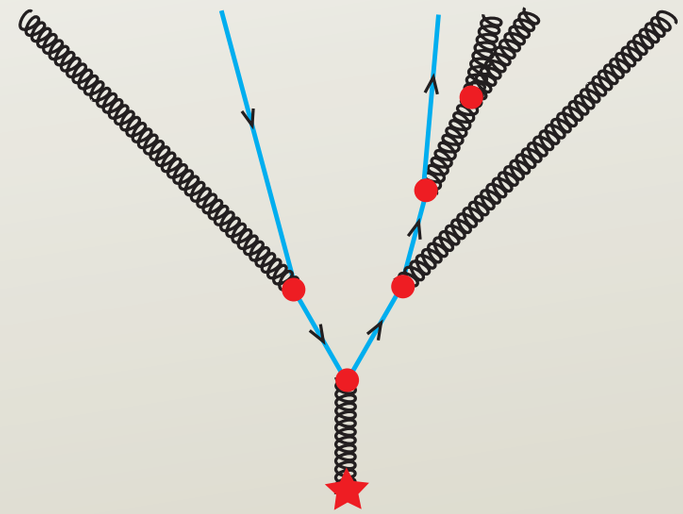


- Emission with $\theta_{12} \gg \theta_{2k}$ is suppressed.
- Emission between partons 2 and k is enhanced.

An application of this



After $H \to b + \bar{b}$
Soft gluons between b and $\bar{b}$



After $g \to b + \bar{b}$
Soft gluons away from b and $\bar{b}$

- *Cf.* "pull" (Gallicchio & Schwartz).

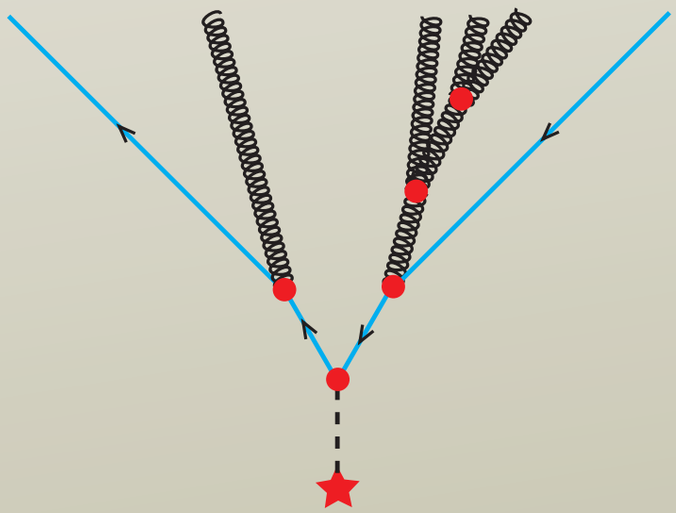
Gluons want to be soft

- Large angles $\implies$ small z:

$$z(1-z) = \frac{v}{\theta^2}\,\frac{Q_0^2}{(p_1+p_2)\cdot Q_0}$$

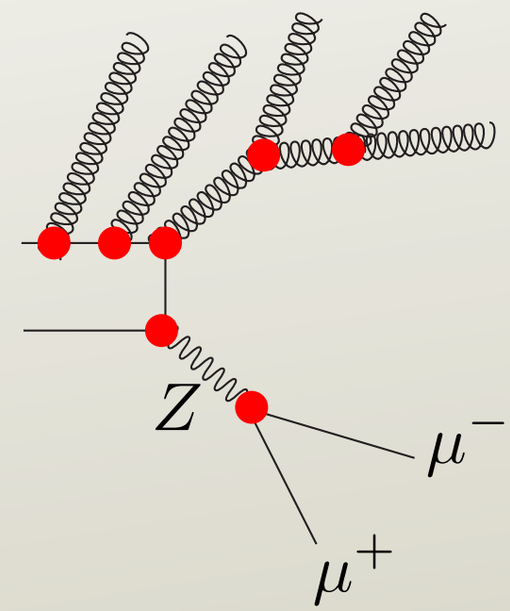
- So we expect lots of soft gluon radiation.

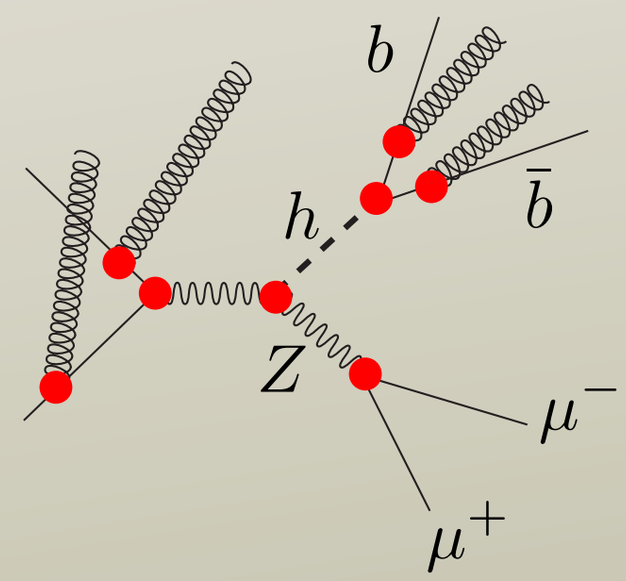
- However, soft gluon radiation is limited by "angular ordering" from $g(\theta)$.



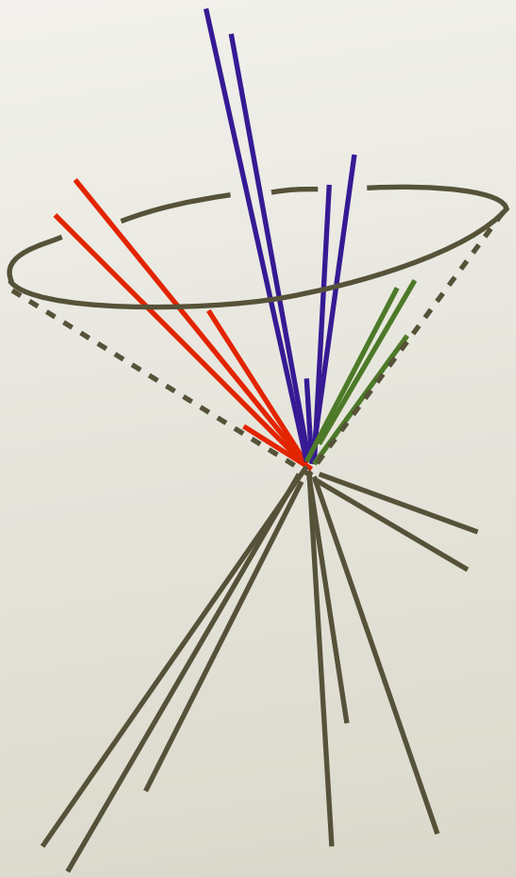
More soft gluons come from initial state radiation

- There is initial state radiation in both signal and background events.
- It can come at central rapidities.
- It is largely rather low transverse momentum.

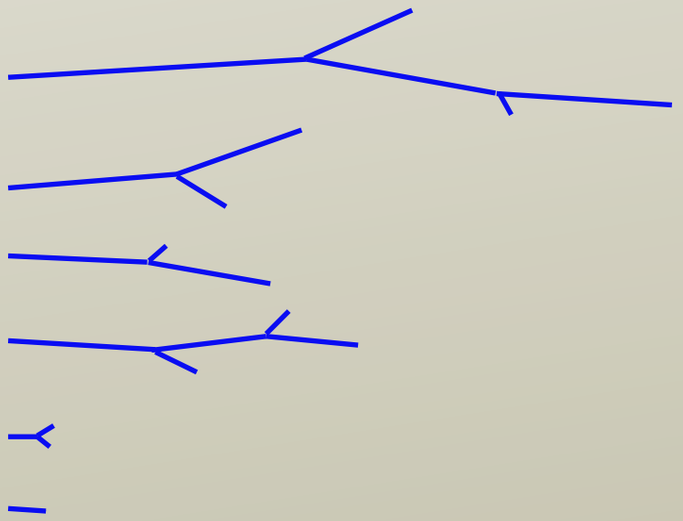




An application



- Look at all of the hadrons or calorimeter clusters in an angular region that might contain a boosted heavy particle.
- The k_T jet algorithm can put the starting clusters together into jets in something like a shower history.
- Use a fairly small angular size parameter.



- The smallest P_T jets are likely from initial state radiation.
- Throw them out. (Krohn, Thaler & Wang).

Another application

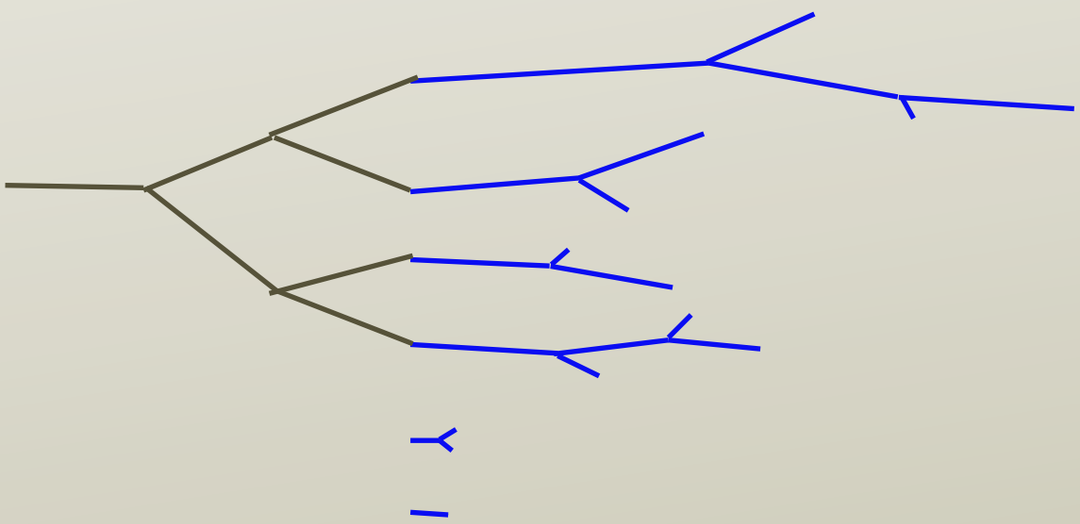
- It's hard to tell where soft gluons belong, so try to get rid of them.

- For instance, "pruning" joins protojets starting at small angular separations. But if

$$z = \frac{\min(p_{T,i}, p_{T,j})}{|\vec{p}_{T,i} + \vec{p}_{T,j}|}$$

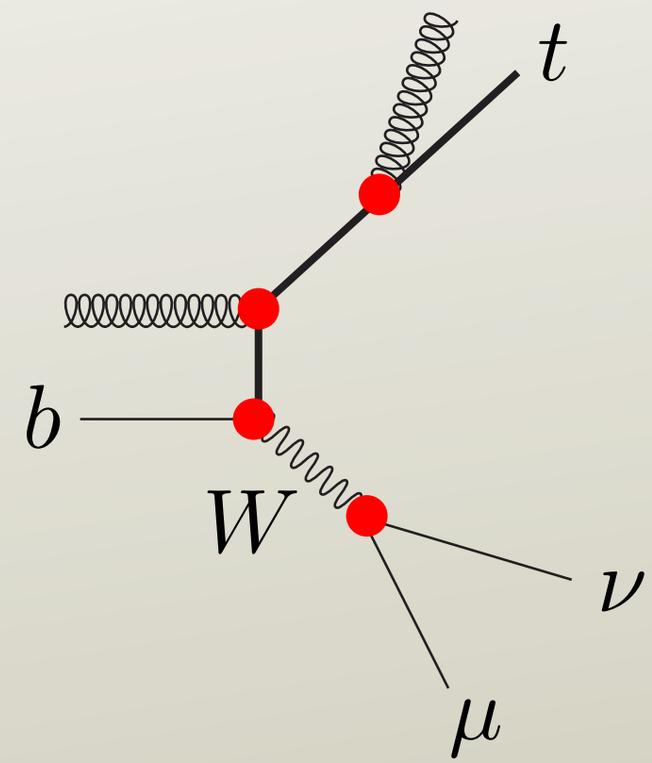
is too small, one throws the softer protojet away.

(Ellis, Vermilion & Walsh)



Heavy particles with color

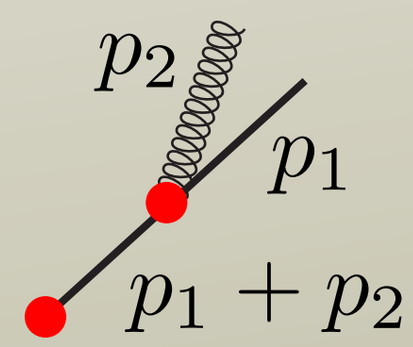
- A heavy particle with color will radiate gluons.
- There is singularity in $|\mathcal{M}|^2$.



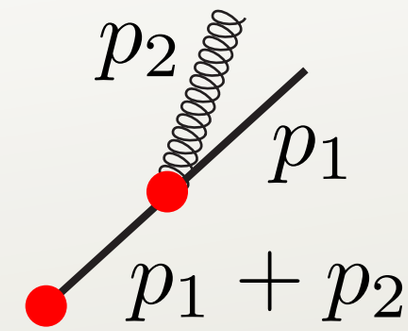
$$dn \propto \frac{dv}{v}\ \alpha_s \log(\theta_{\rm min}/\theta_{\rm max})$$

where the hardness v is now

$$v = \frac{(p_1+p_2)^2 - M^2}{(p_1+p_2)\cdot Q_0}$$



- The collinear singularity is cut off:

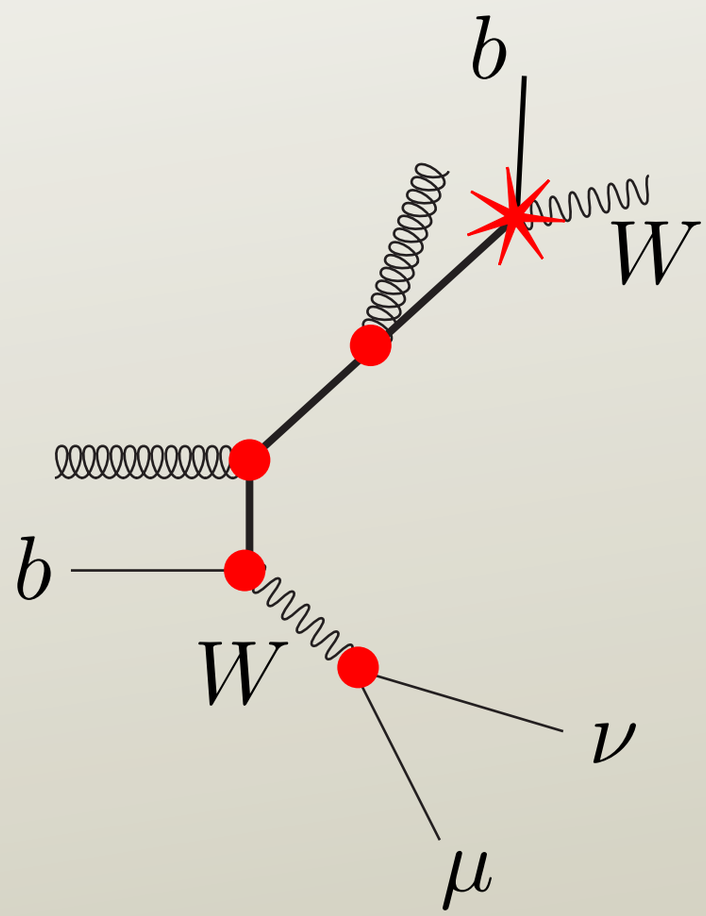


$$\theta^2_{\min} \sim \min\left(\frac{M^2 Q_0^2}{((p_1+p_2)\cdot Q_0)^2}, v\,\frac{4Q_0^2}{(p_1+p_2)\cdot Q_0}\right)$$

- There are no emissions that take longer than the particle lifetime:

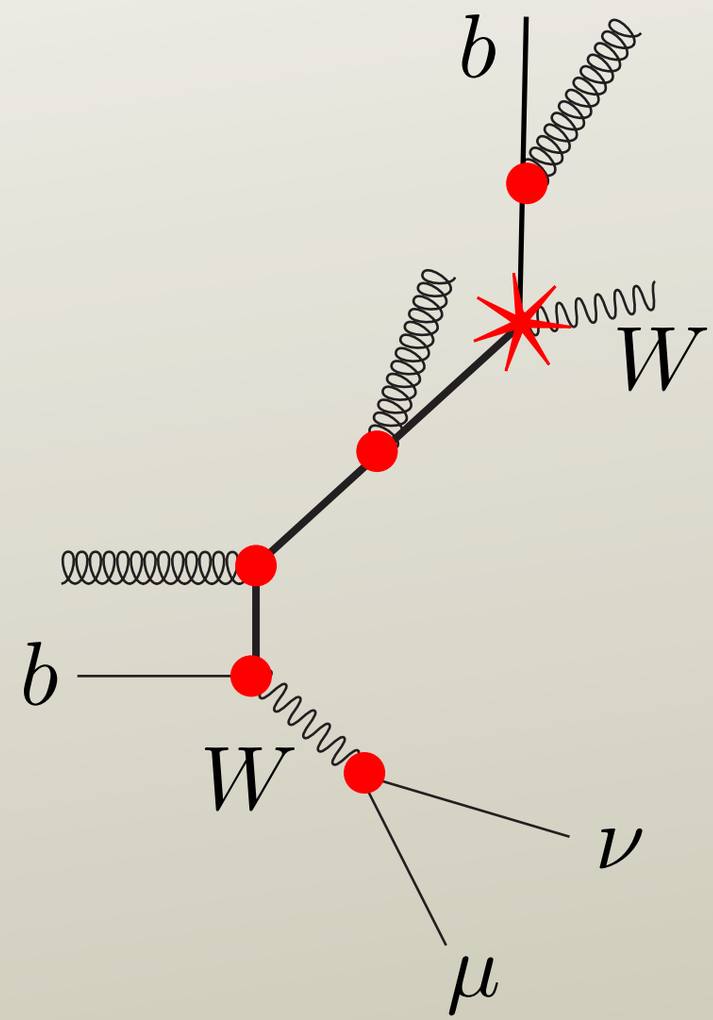
$$v > \frac{M\Gamma}{(p_1+p_2)\cdot Q_0}$$

Heavy particles decay



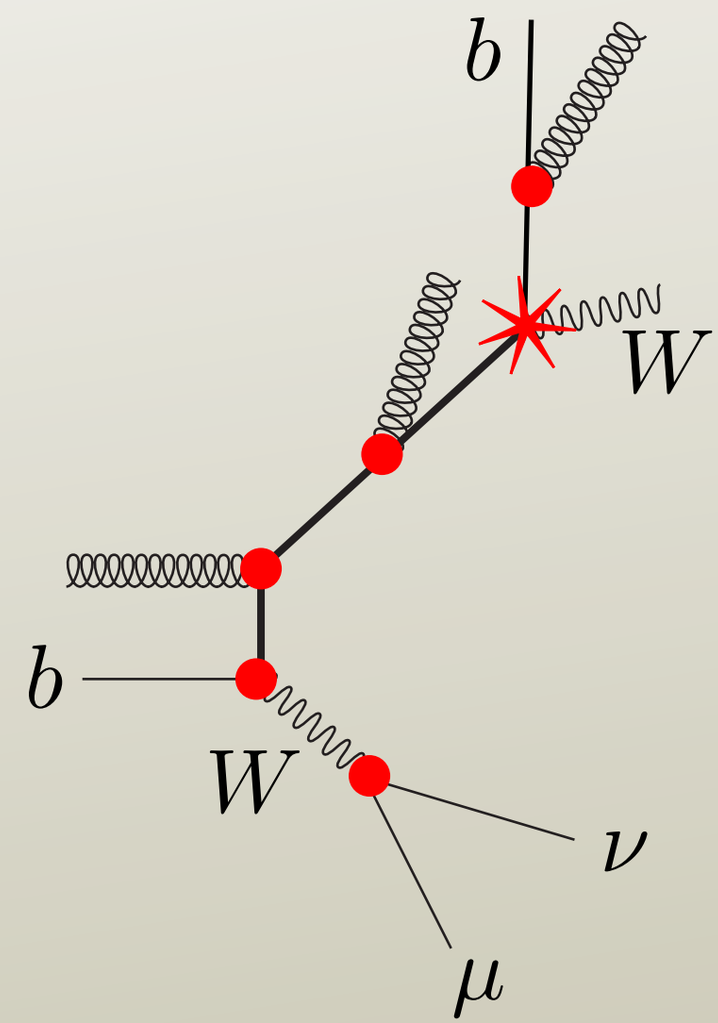
Daughters radiate

- In $t \to b + W$, a lot of energy is released.
- Color charge is accelerated.
- Gluons are radiated.
- The first radiation is from a color dipole of
  - The final state b-quark;
  - The "initial state" t-quark;

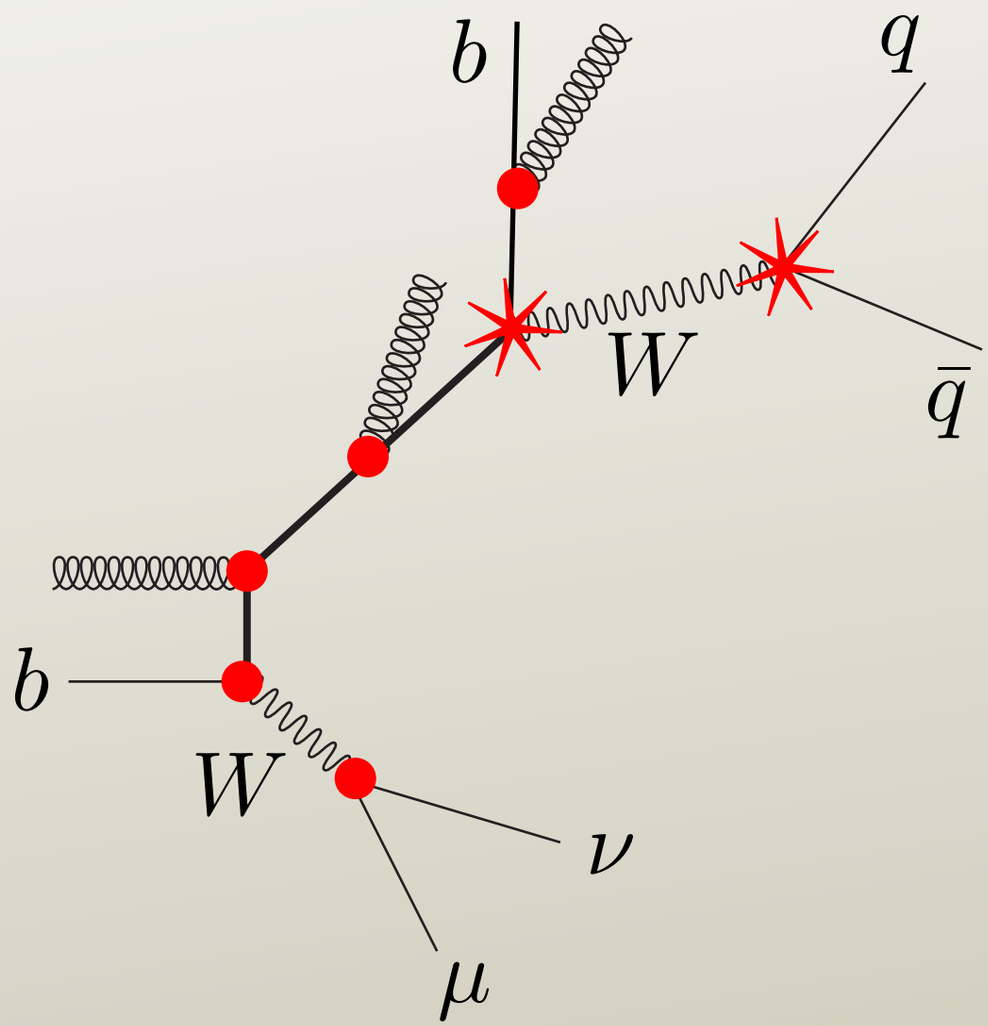


Where does the radiation go?

- The first radiation is from a color dipole of
  - The final state b-quark;
  - The "initial state" t-quark;
- Direction of g is likely collinear with the b.
- Direction of g is likely within angle $M/|\vec{P}_t|$ of the top.
- If the top is highly boosted, this is a narrow cone.

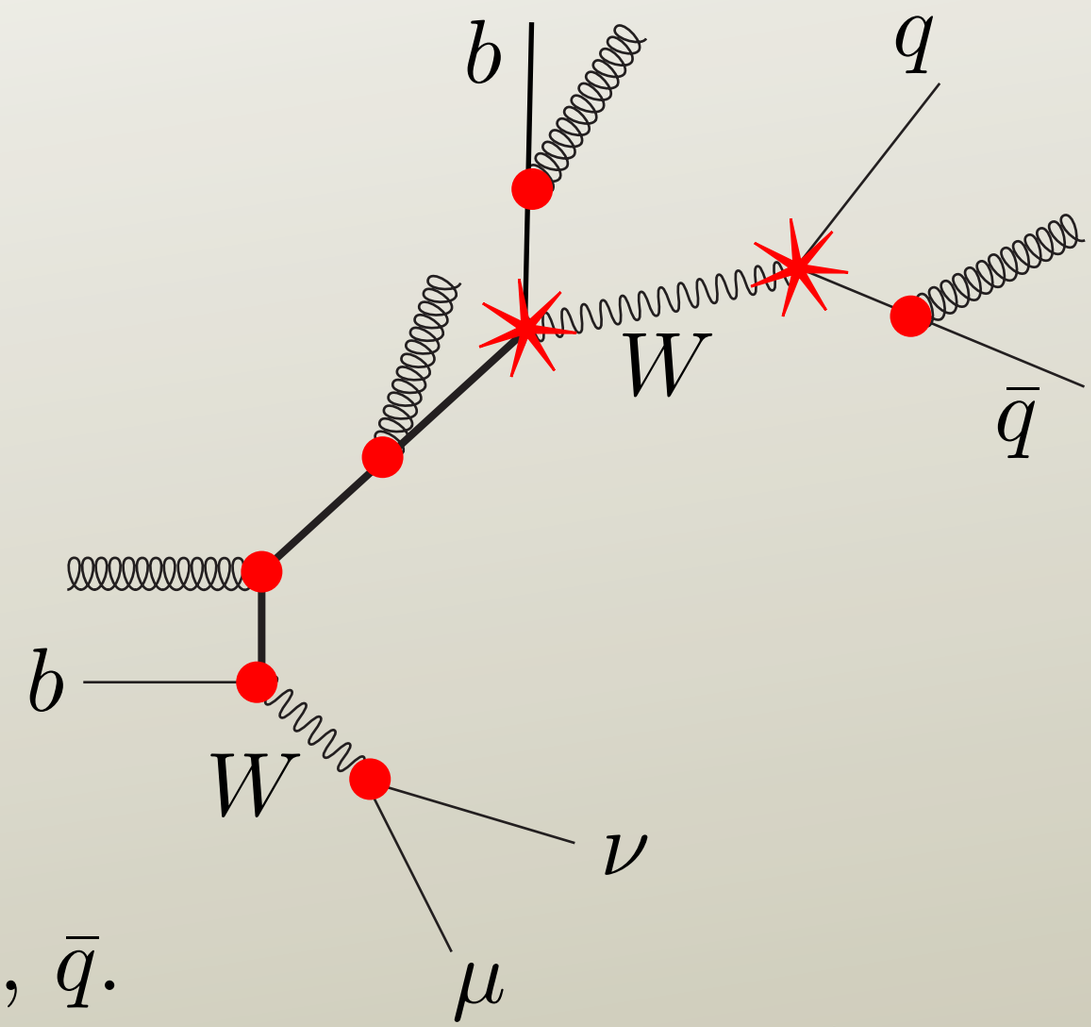


Heavy particles decay



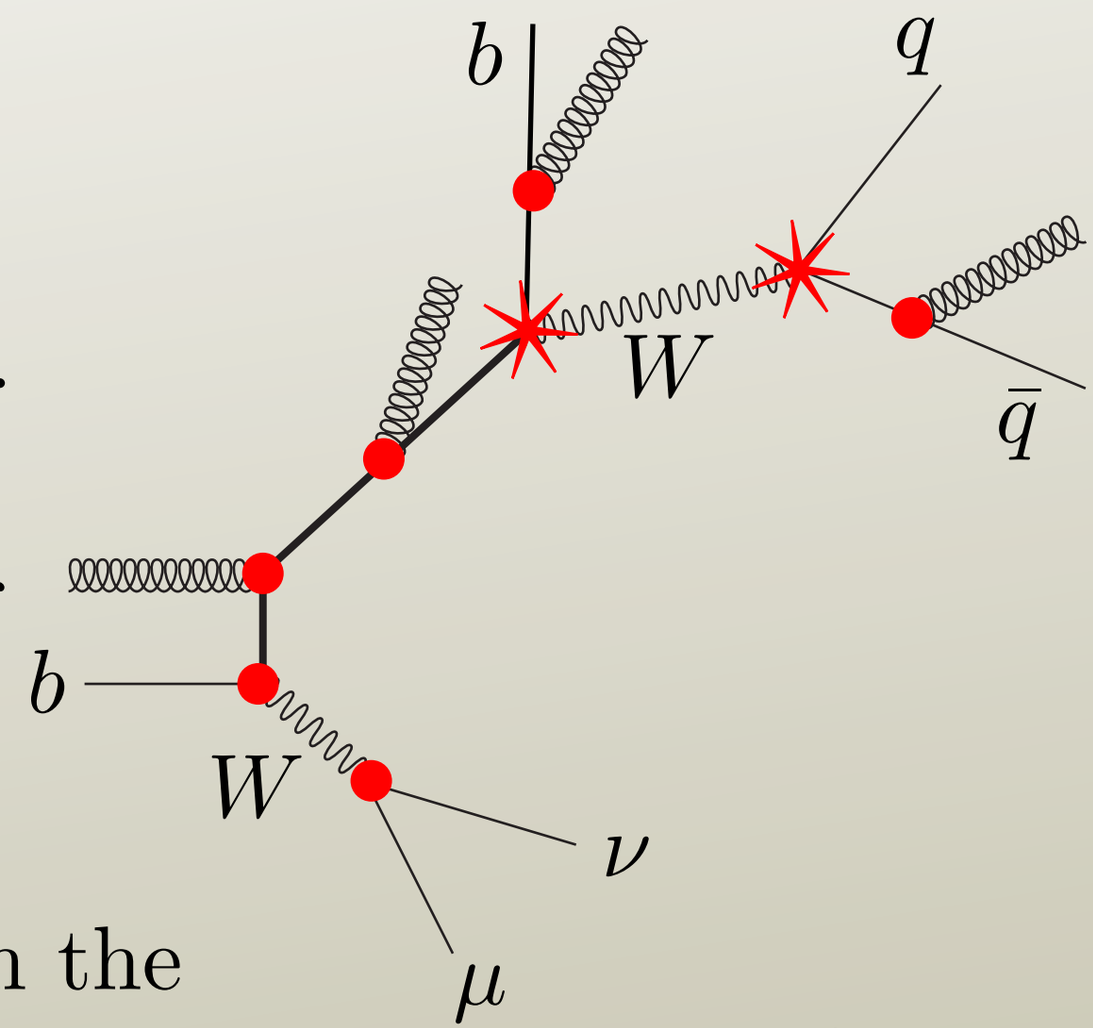
Daughters radiate

- In $W \to q + \bar{q}$, a lot of energy is released.
- Color charge is accelerated.
- Gluons are radiated.
- The first radiation is from a color dipole of
  - The final state quark, q;
  - The final state anti-quark, $\bar{q}$.



Where does the radiation go?

- The first radiation is from a color dipole of
  - The final state quark, q;
  - The final state anti-quark, $\bar{q}$.
- Direction of g is likely collinear with the q or $\bar{q}$.
- In any case, it is likely between the q and $\bar{q}$.

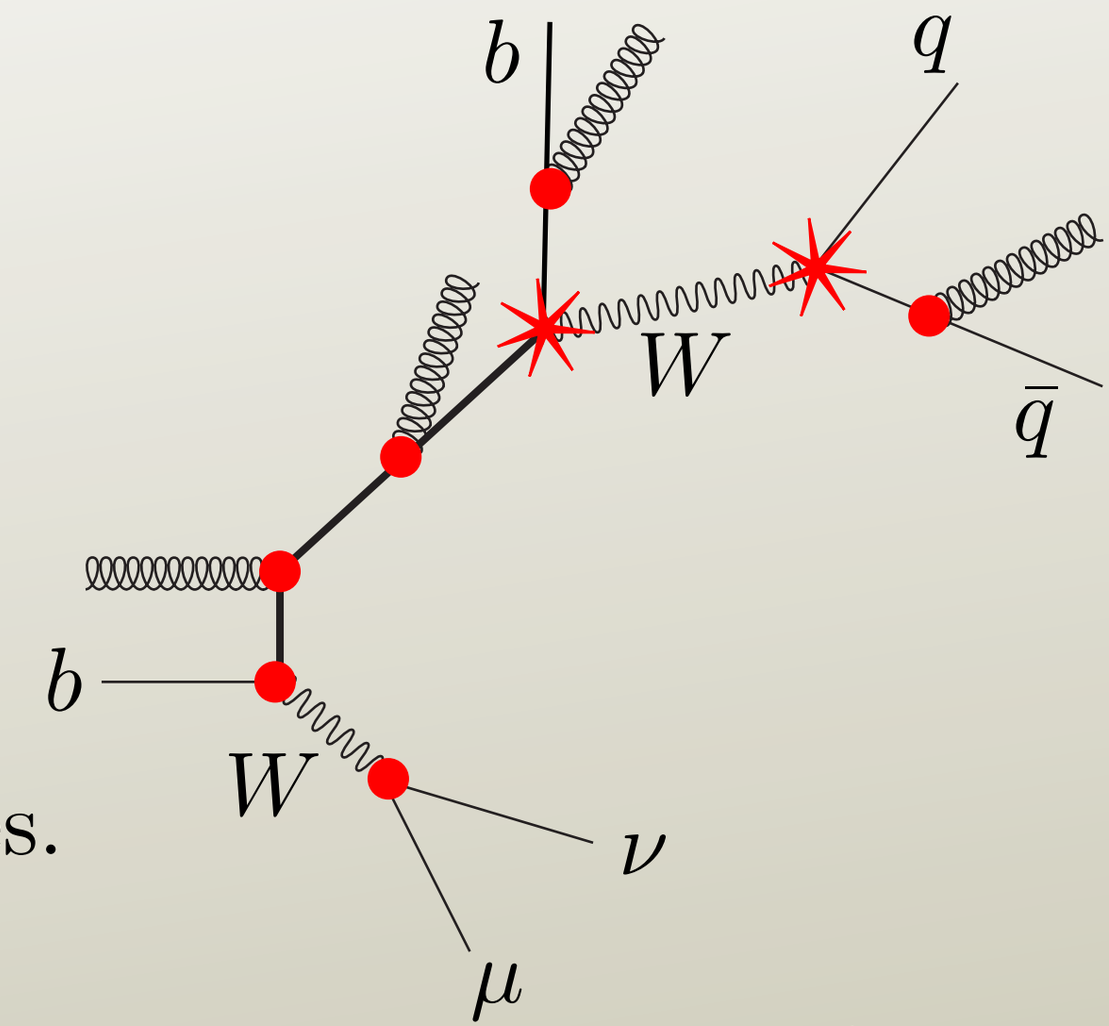


Very soft radiation

- For very soft emissions,

$$v < \frac{M_t \Gamma_t}{p_t \cdot Q_0} \sim \frac{M_W \Gamma_W}{p_W \cdot Q_0}$$

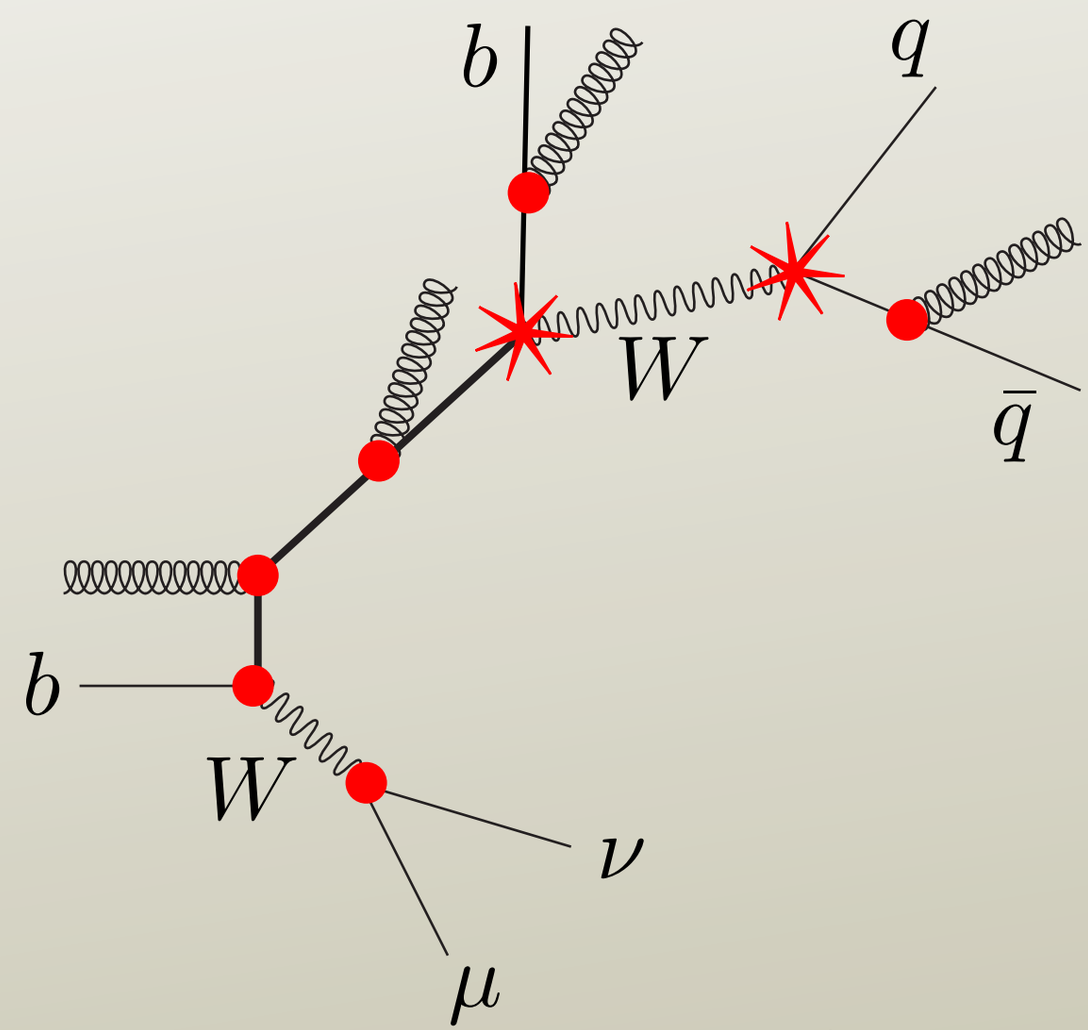
there is quantum interference between emissions from any two (color connected) particles.



- Probably the structure of these emissions is too complicated to be useful for separating signal from background.

Comments

- Since we know something abut where the collinear or soft radiation goes in a heavy particle decay, sequential heavy particle decays should have good signatures.
- We do have the problem of contamination from initial state radiation.
- Having highly boosted heavy objects helps with this.



Summary

- With a model for sequential heavy particle decays and knowledge of the structure of radiation in gauge theories, one can build filters for separating background events from signal events.

Shower deconstruction

with Michael Spannowsky

Our example of a signal

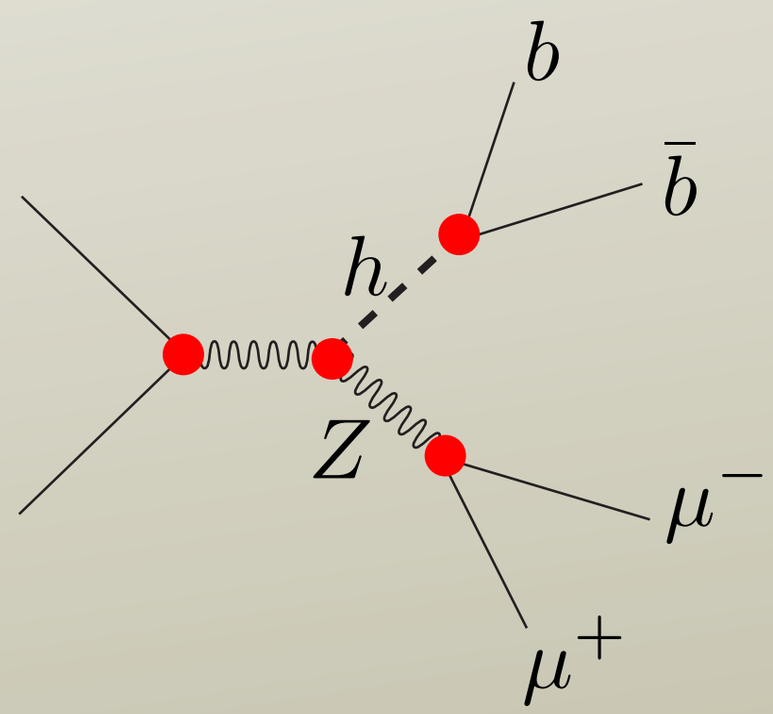
- $p + p \rightarrow h + Z$

  $Z \rightarrow \mu^+ + \mu^-$

  $h \rightarrow b + \bar{b}$

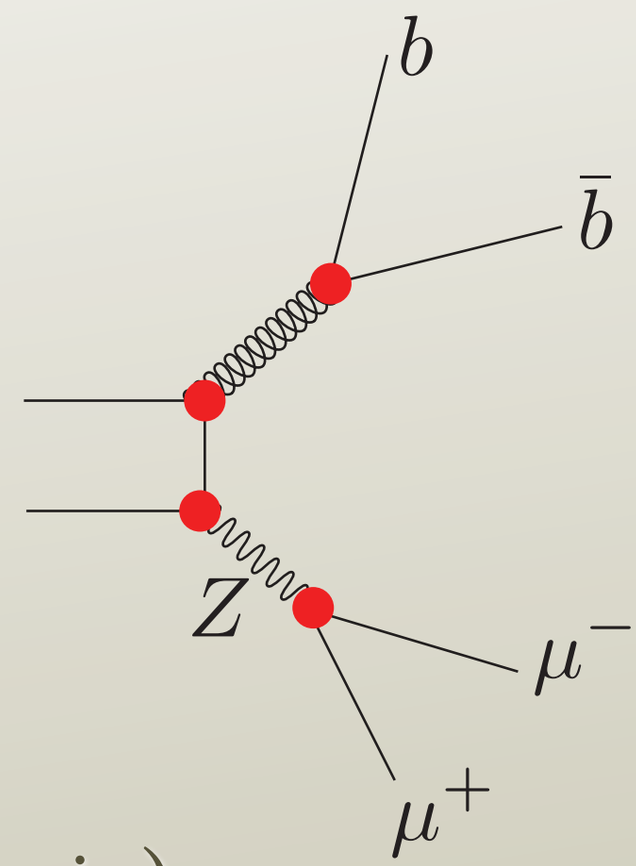
- The Z has lots of p_T so the h is highly boosted.

A Higgs boson signal event



Background events

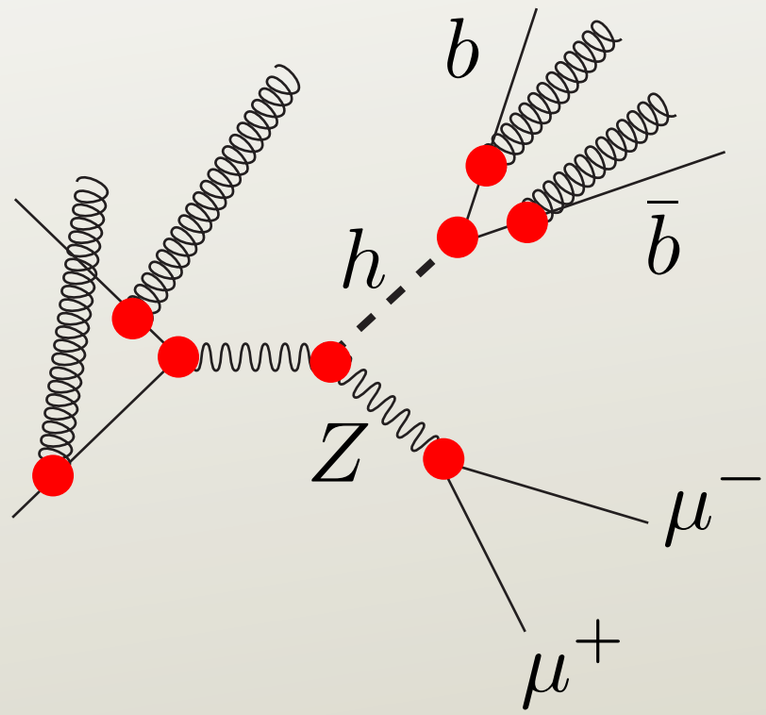
- We need to be able to tell the signal events from QCD background events.



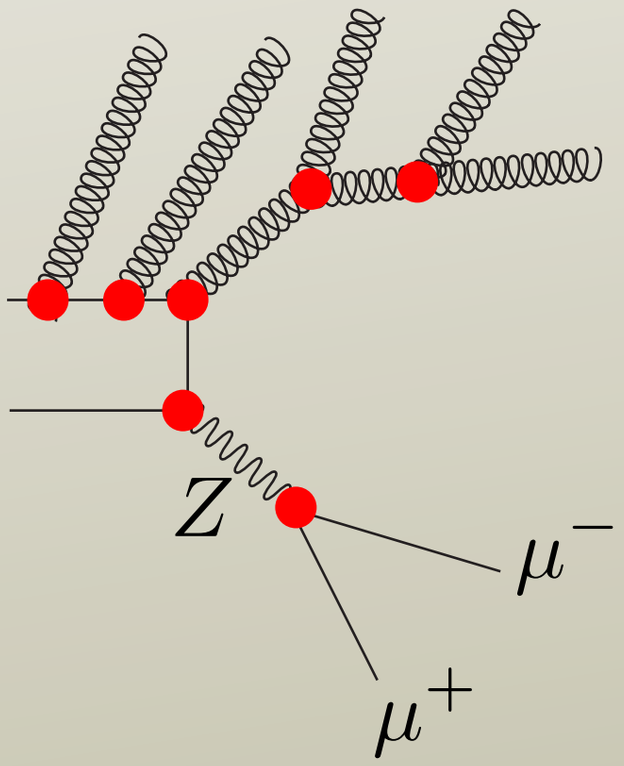
...(at least on a statistical basis.)

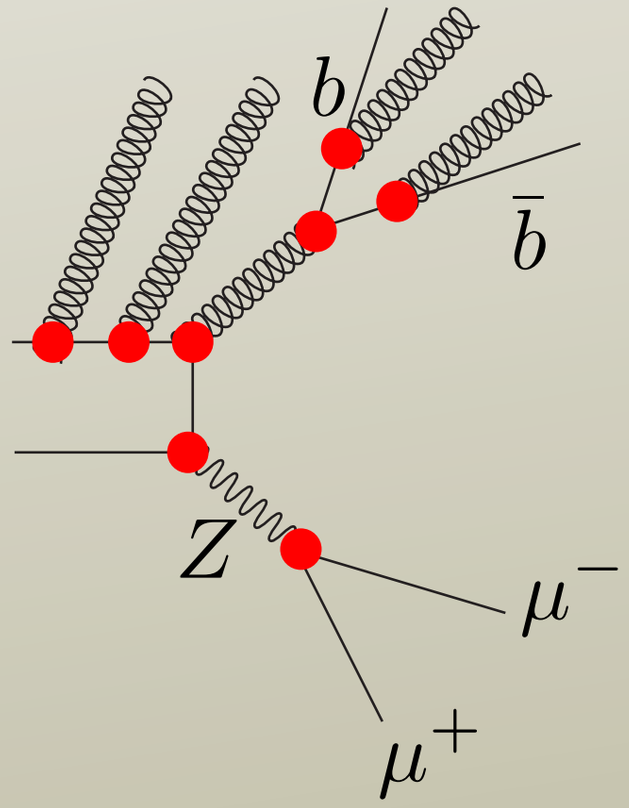
Reality is a bit worse...

We want to find this.

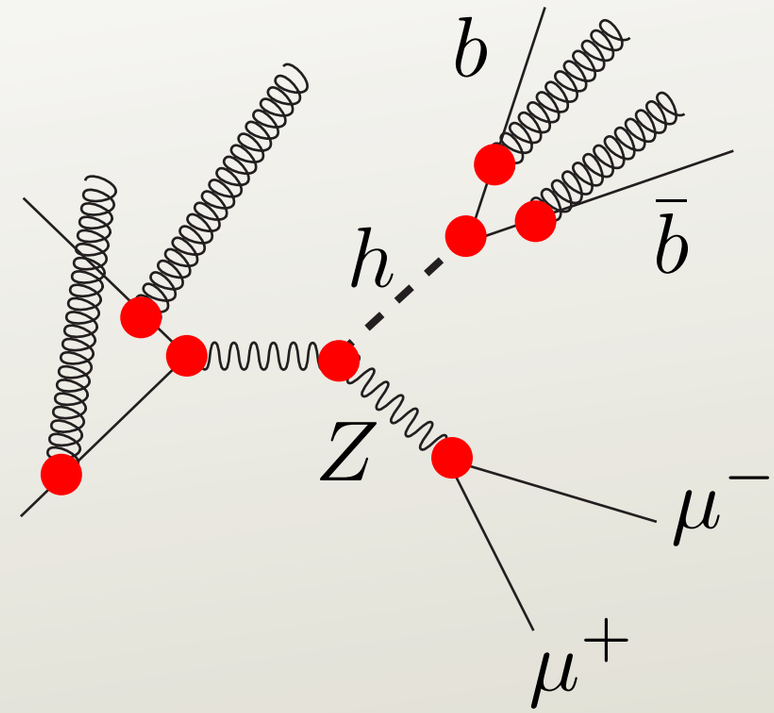


In a background of this.





Event selection



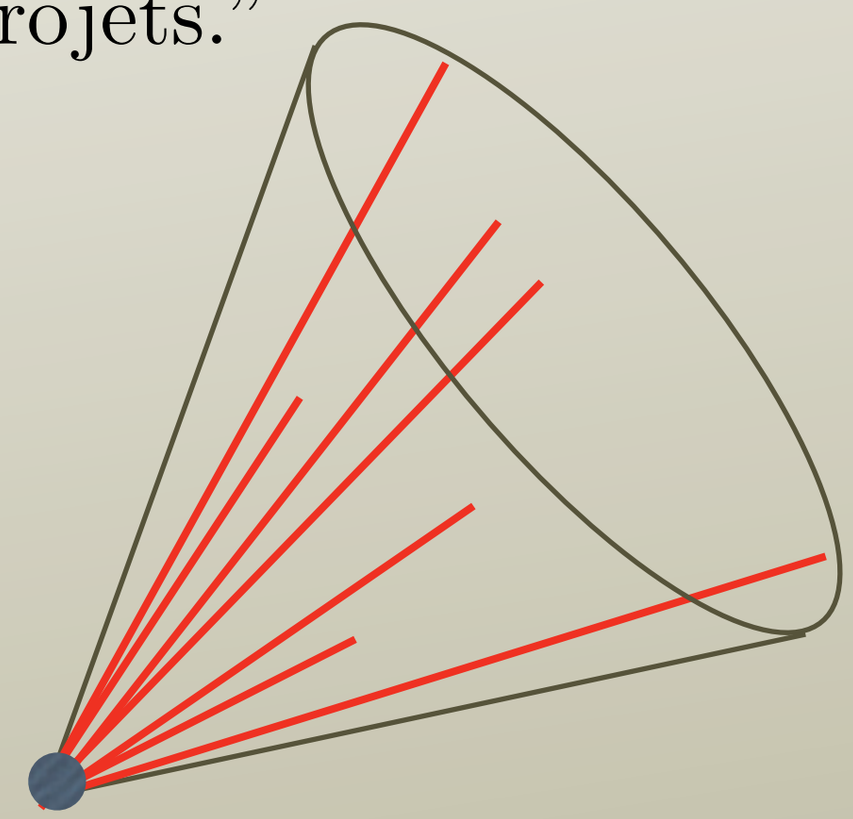
- Demand $\mu^+\mu^-$ or e^+e^- with

$$|m_{l^+l^-} - m_Z| < 10 \text{ GeV}$$
$$p_{T,l^+l^-} > 200 \text{ GeV}$$

- Combine final state hadrons in cells of size 0.1×0.1.
- Adjust $|\vec{p}|$ to make each cell momentum massless.
- Remove cells with energy less than 0.5 GeV.
- Apply anti-k_T jet algorithm with $R = 1.2$.
- The jet with the highest p_T is the "fat jet."
- Demand $p_T^{\text{fat jet}} > 200$ GeV.

Define the microjet constituents

- Use the k_T algorithm to group the fat jet into subjets.
- Use $R = 0.15$.
- This is more or less like an Atlas "topocluster."
- If too many subjets (*e.g.* > 7) drop those with smallest p_T.
- We call the resulting subjets the "microjets."
- Add 0.1 GeV to the energy of each microjet.



The variables

- Microjets described by momenta $\{p\}_N = \{p_1, \ldots, p_N\}$.
- Also provide b-tags, t_j: T, F, or "none."
- T or F tags to three highest p_T microjets if $p_T > 15$ GeV.
- In Monte Carlo events,
  - If any hadron in microjet j contains a b or $\bar{b}$ quark,
    $t_j = \mathrm{T}$ with a probability 0.6
    $t_j = \mathrm{F}$ with a probability 0.4.
  - If no hadron in microjet j contains a b or $\bar{b}$ quark,
    $t_j = \mathrm{T}$ with a probability 0.02
    $t_j = \mathrm{F}$ with a probability 0.98
- So microjets described by momenta $\{p, t\}_N$.

What we would like

- Our data: momenta p and b-tags for N microjets, $\{p,t\}_N$.

- Define probabilities for signal and background events to have $\{p,t\}_N$ according to a trusted Monte Carlo:

$$P_{\mathrm{MC}}(\{p,t\}_N|\mathrm{S}) = \frac{1}{\sigma_{\mathrm{MC}}(\mathrm{S})}\frac{d\sigma_{\mathrm{MC}}(\mathrm{S})}{d\{p,t\}_N}$$

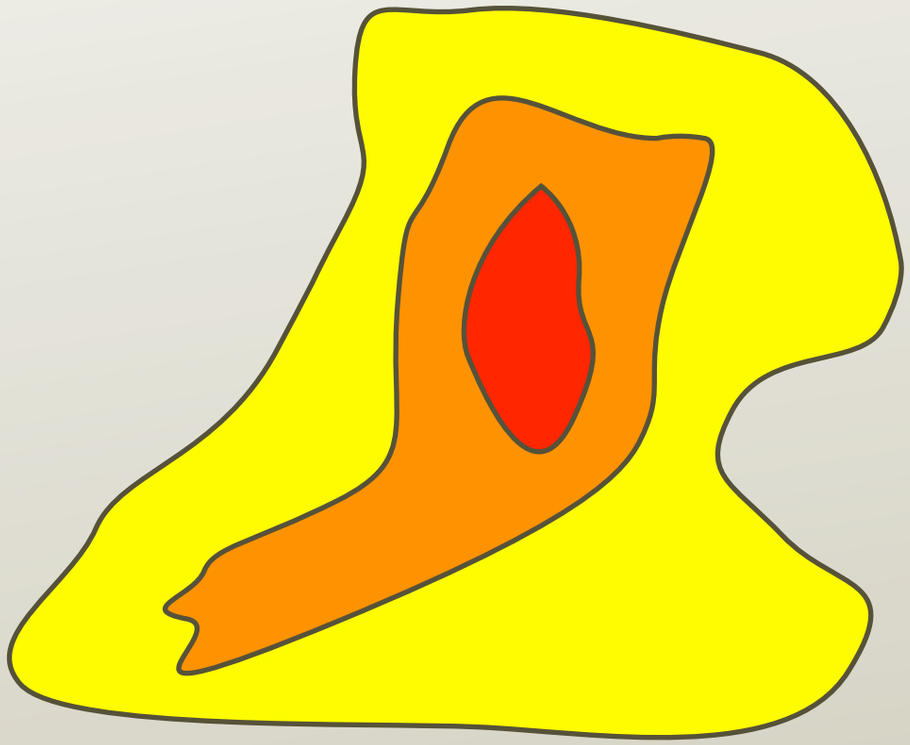
$$P_{\mathrm{MC}}(\{p,t\}_N|\mathrm{B}) = \frac{1}{\sigma_{\mathrm{MC}}(\mathrm{B})}\frac{d\sigma_{\mathrm{MC}}(\mathrm{B})}{d\{p,t\}_N}$$

- We would like to separate signal from background using

$$\chi_{\mathrm{MC}}(\{p,t\}_N) = \frac{P_{\mathrm{MC}}(\{p,t\}_N|\mathrm{S})}{P_{\mathrm{MC}}(\{p,t\}_N|\mathrm{B})}$$

Why?

- Assuming that you believe your Monte Carlo, to get the most signal cross section for a given background cross section by making a cut, your cut should be along a contour line of



$$\chi_{\mathrm{MC}}(\{p,t\}_N) = \frac{P_{\mathrm{MC}}(\{p,t\}_N|\mathrm{S})}{P_{\mathrm{MC}}(\{p,t\}_N|\mathrm{B})}$$

What we do

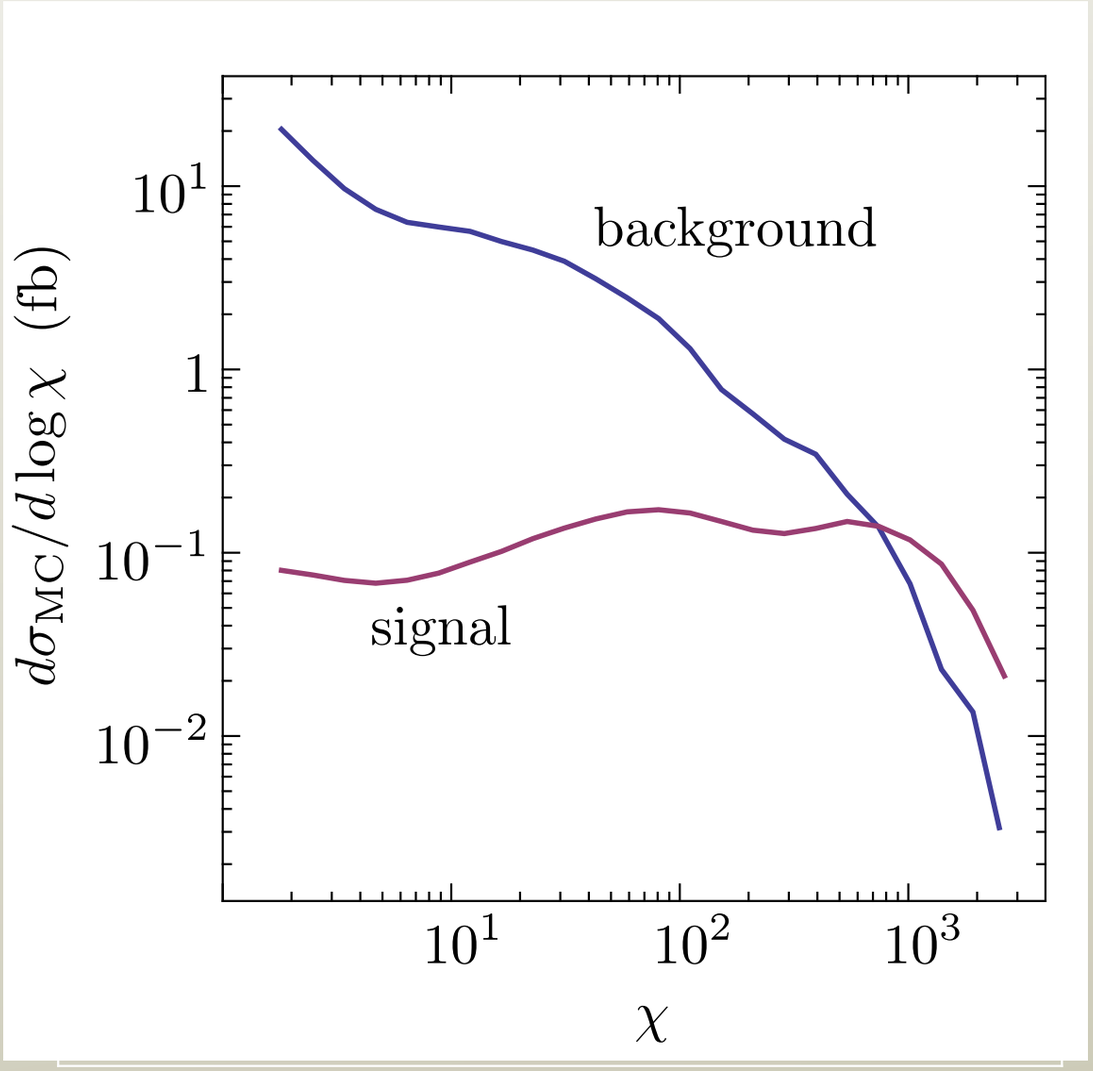
- Calculate

$$\chi(\{p,t\}_N) = \frac{P(\{p,t\}_N|\mathrm{S})}{P(\{p,t\}_N|\mathrm{B})}$$

according to a “simplified parton shower” algorithm.

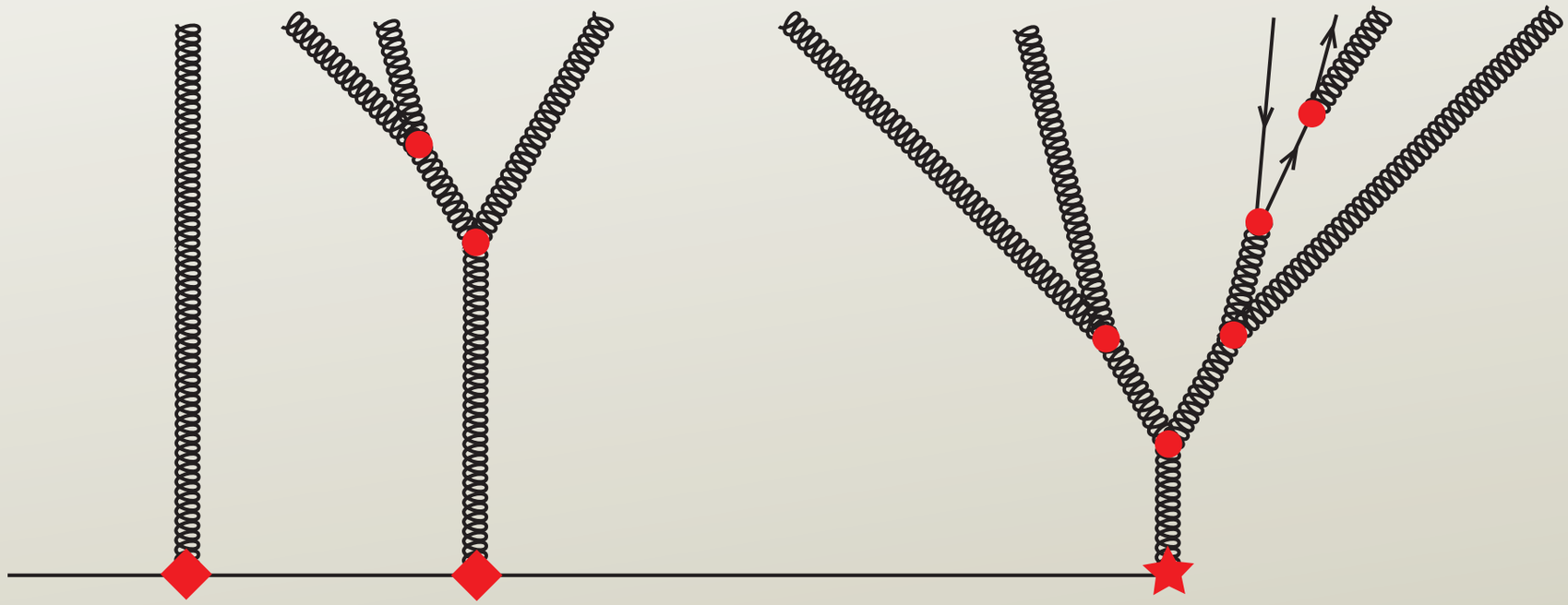
Result

• We calculate χ for samples of signal and background events generated with PYTHIA.



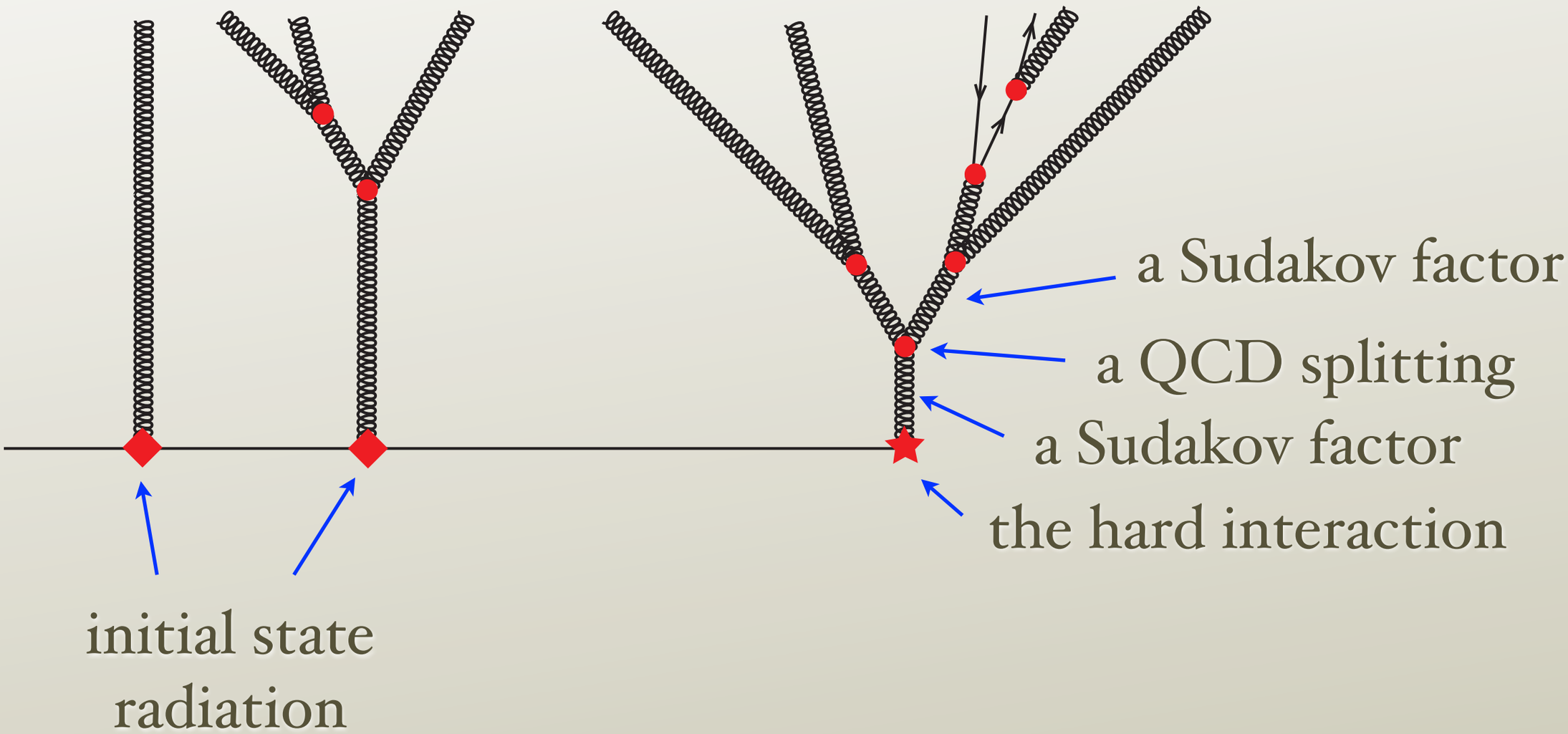
How does it work?

- We sum over event histories.



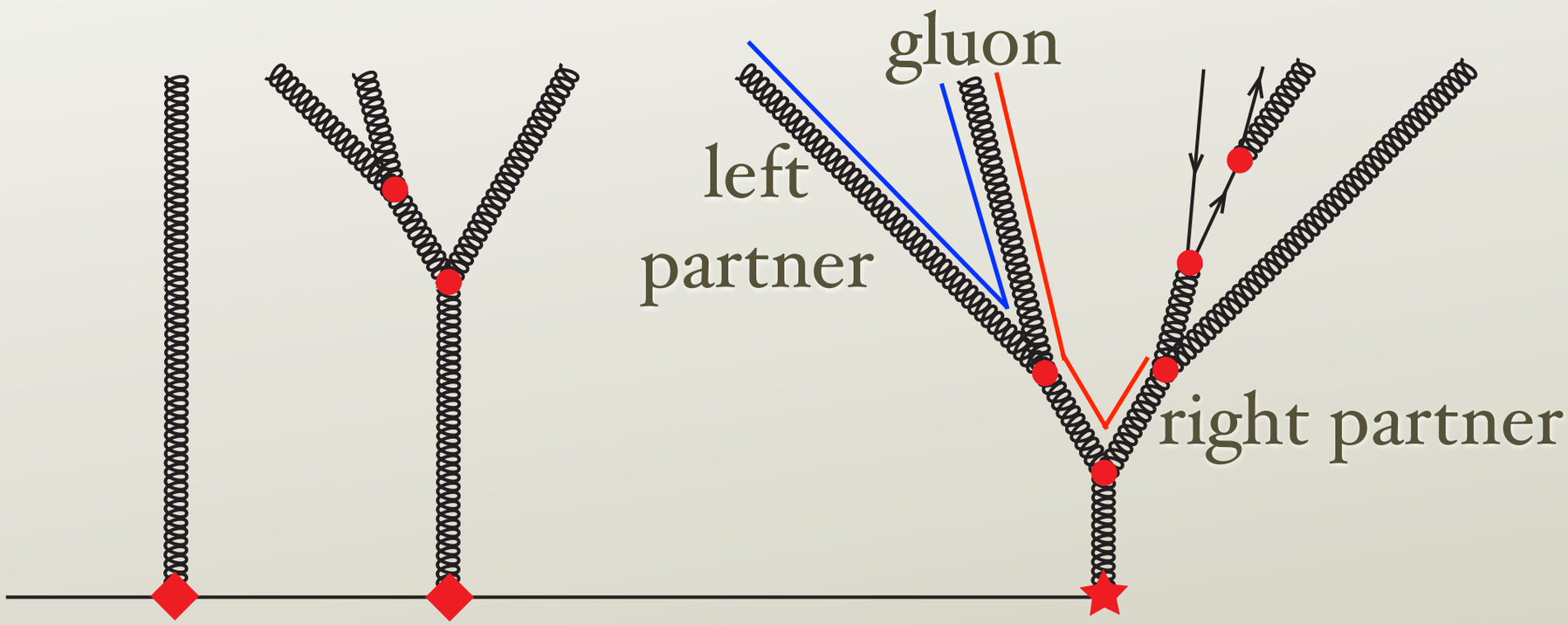
- Each vertex and propagator corresponds to a shower algorithm factor.

About histories



Color connections

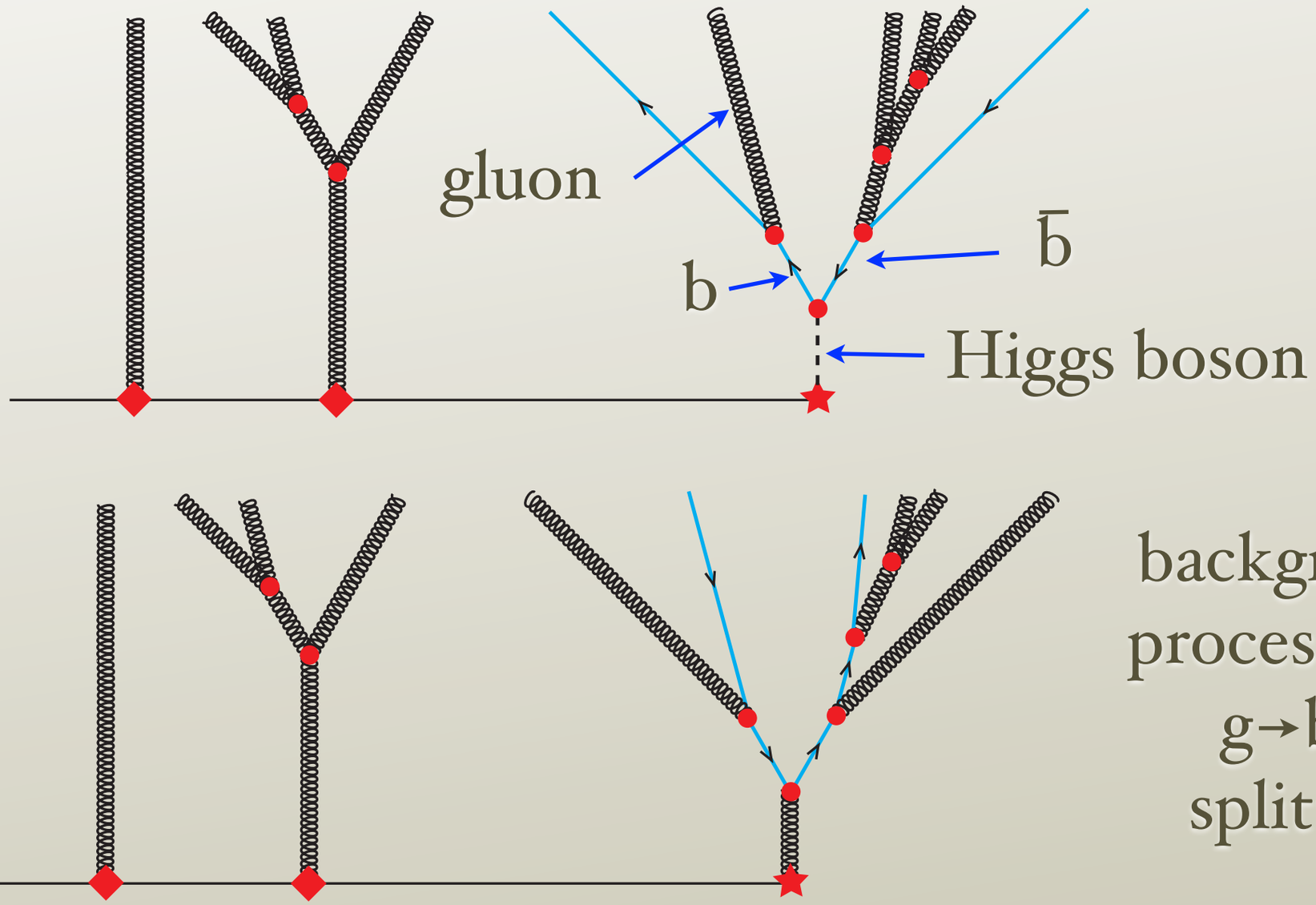
Each gluon has two color connected partners.



Each quark has one color connected partner.

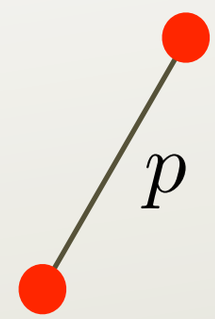
Some partners are unknown, likely outside the fat jet.

There are more kinds of histories

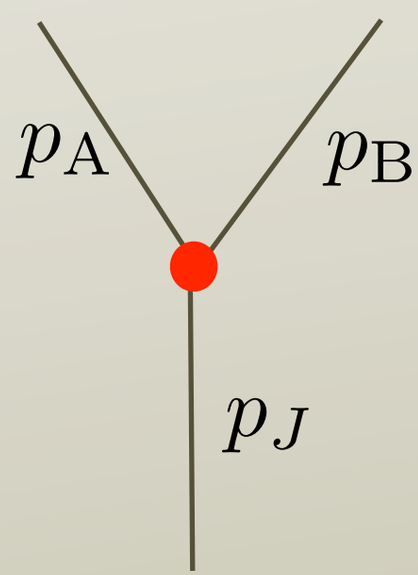


background process with $g \to b\,\bar{b}$ splitting

Kinematics



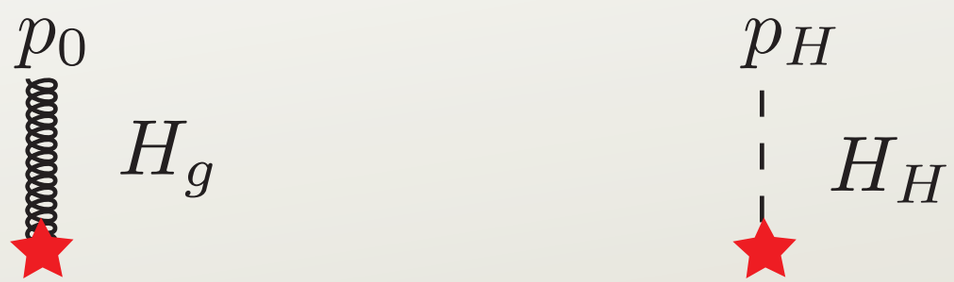
$$p = \left(\frac{1}{\sqrt{2}} \sqrt{k^2 + \mu^2}\, e^{y}, \frac{1}{\sqrt{2}} \sqrt{k^2 + \mu^2}\, e^{-y}, k \cos\phi, k \sin\phi \right)$$



$$p_J = p_{\mathrm{A}} + p_{\mathrm{B}}$$

The daughter partons are not on shell.

The hard interaction

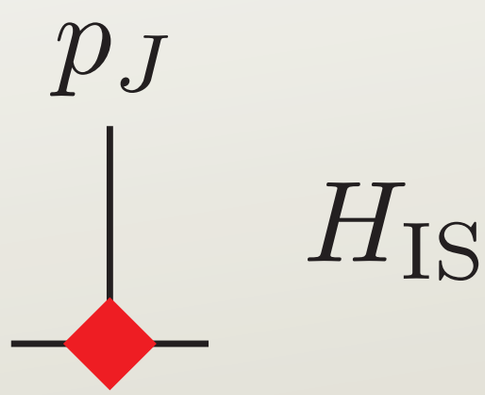


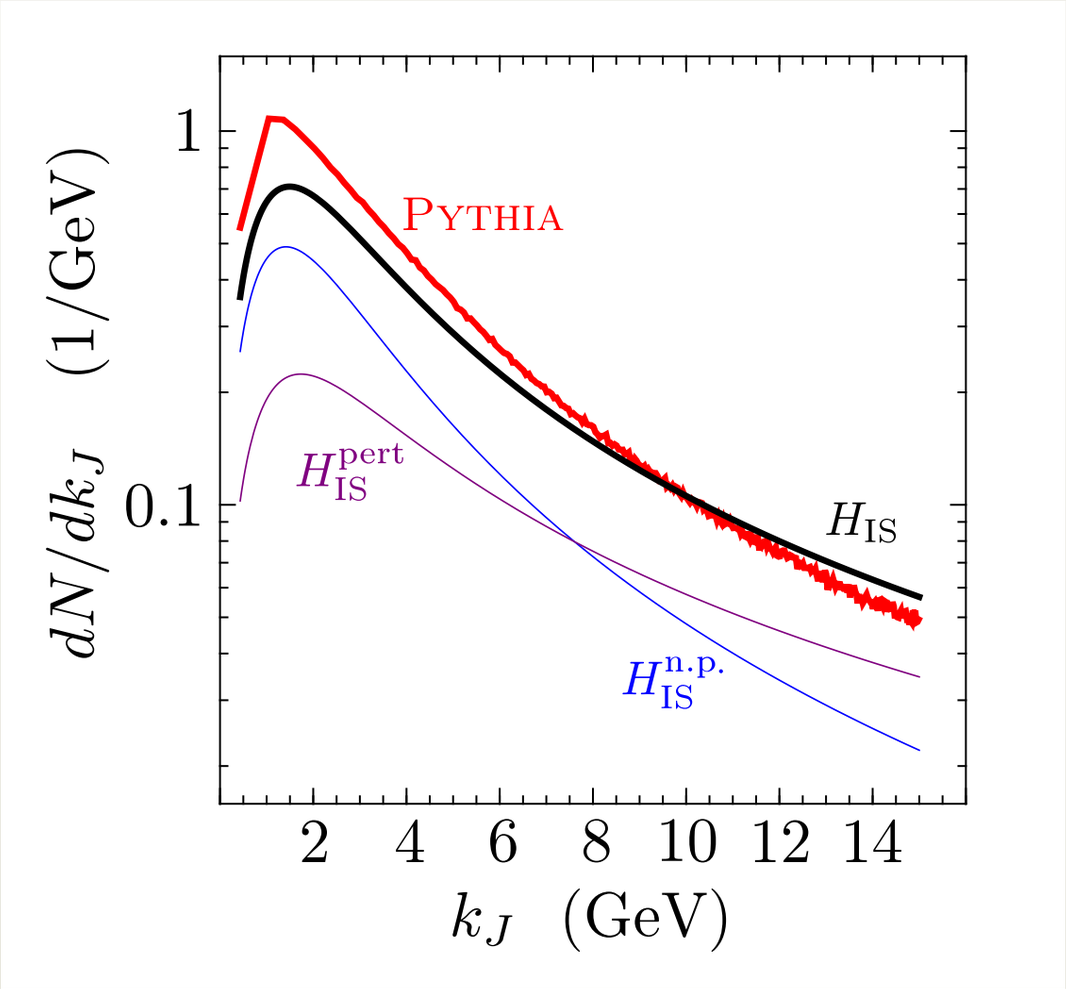
$$H_g = N_{\rm pdf} \left(\frac{p_{T,\rm min}^2}{k_0^2} \right)^{N_{\rm pdf}} \frac{1}{k_0^2} \; \Theta(k_{T,\rm I}^2 < Q^2/4)$$

$$H_H = N_{\rm pdf} \left(\frac{p_{T,\rm min}^2 + m_H^2}{k_H^2 + m_H^2} \right)^{N_{\rm pdf}} \frac{1}{k_H^2 + m_H^2} \; \Theta(k_{T,\rm I}^2 < Q^2/4)$$

$$Q^2 = \left(\sum_{i \in \text{fat jet}} \vec{p}_{T,i} \right)^2 + \left(\sum_{i \in \text{fat jet}} p_i \right)^2$$

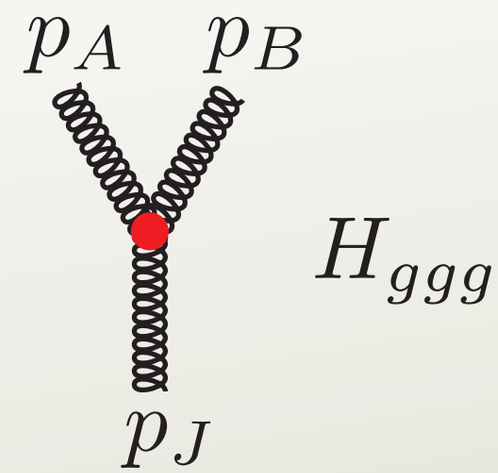
Initial state radiation





$$H_{\rm IS} = \frac{\alpha_s(k_J^2+\kappa_{\rm p}^2)}{k_J^2+\kappa_{\rm p}^2}\,\frac{8\pi\,C_{\rm A}}{(1+c_R\,k_J/Q)^{n_R}} + \frac{16\pi\,c_{\rm np}(\kappa_{\rm np}^2)^{n_{\rm np}-1}}{[k_J^2+\kappa_{\rm np}^2]^{n_{\rm np}}}$$

Gluon splitting



"s" = softer of A and B

"h" = harder of A and B

$$\frac{[1-z(1-z)]^2}{z(1-z)}$$

angle factor

$$H_{ggg} = 8\pi C_A \, \frac{\alpha_s(\mu_J^2)}{\mu_J^2} \, \frac{k_J^2}{k_s k_h} \left[1 - \frac{k_s k_h}{k_J^2}\right]^2 \frac{\theta_{hk}^2}{\theta_{sh}^2 + \theta_{sk}^2}$$

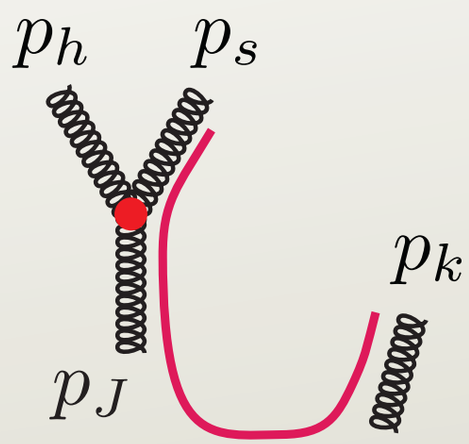
$$\mu_J^2 = p_J^2$$

$$\times \Theta\left(2\,\frac{\mu_J^2}{k_J} < \frac{\mu_K^2}{k_K}\right)$$

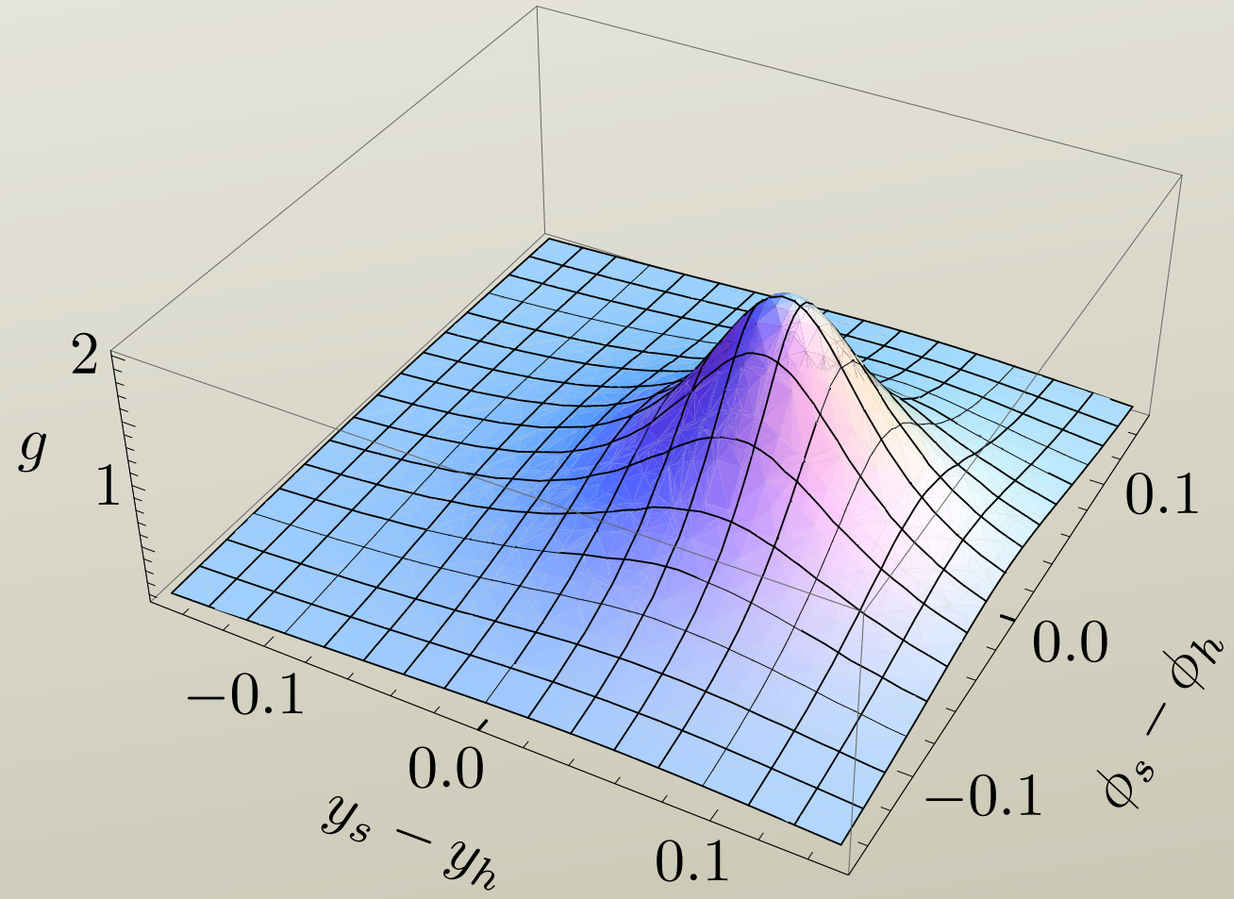
ordering of shower times

"K" = grandmother parton

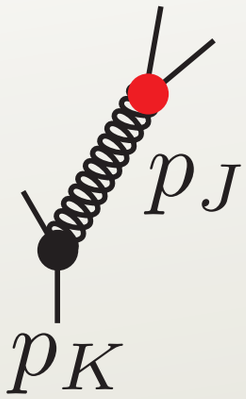
The angle factor



$$g = \frac{\theta_{hk}^2}{\theta_{sh}^2 + \theta_{sk}^2}$$



Sudakov factor



e^{-S_g}

$$S_g = S_{ggg}\,\Theta(S_{ggg} > 0) + n_{\mathrm{f}} S_{g\bar{q}q}$$

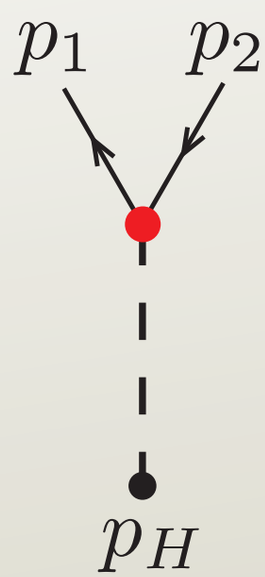
$$S_{ggg} \approx \int \frac{d\bar{\mu}_J^2}{\bar{\mu}_J^2}\,\Theta\left(\mu_J^2 < \bar{\mu}_J^2 < \frac{k_J}{2k_K}\,\mu_K^2\right)\,\frac{\alpha_{\mathrm{s}}(\bar{\mu}_J^2)}{2\pi}$$

$$\times \int dz\,\Theta\left(\frac{1}{\theta_{k(A)}^2}\,\frac{\bar{\mu}_J^2}{k_J^2} < z < 1 - \frac{1}{\theta_{k(B)}^2}\,\frac{\bar{\mu}_J^2}{k_J^2}\right)$$

$$\times C_{\mathrm{A}}\,\frac{[1 - z(1-z)]^2}{z(1-z)}$$

limits on z
from the angle factor
(approximately)

Higgs decay



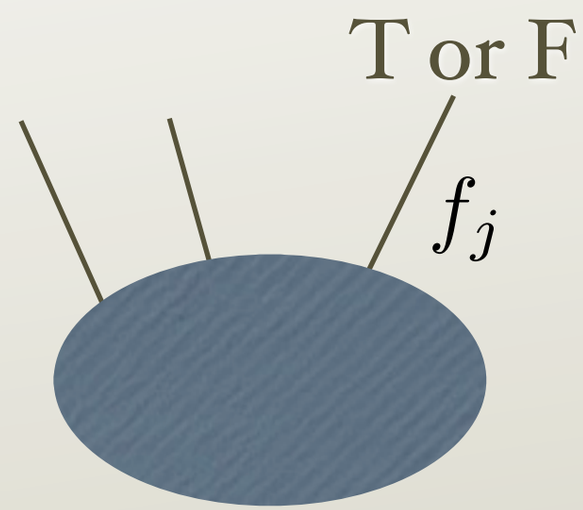
$$H_{Hb\bar{b}} \exp(-S_{Hb\bar{b}})$$

$$He^{-S} = 16\pi^2 \, \frac{\Theta(|m_{b\bar{b}} - m_H| < \Delta m_H)}{4 m_H \, \Delta m_H}$$

default: $\Delta m_H = 10$ GeV

Probabilities for b-tags

- b tags are assigned to the three highest p_T microjets (with $p_T > 15$ GeV).



- If microjet j is a b or $\bar{b}$, we say that $t_j = T$ with probability 0.6 and $T_j = F$ with probability 0.4.

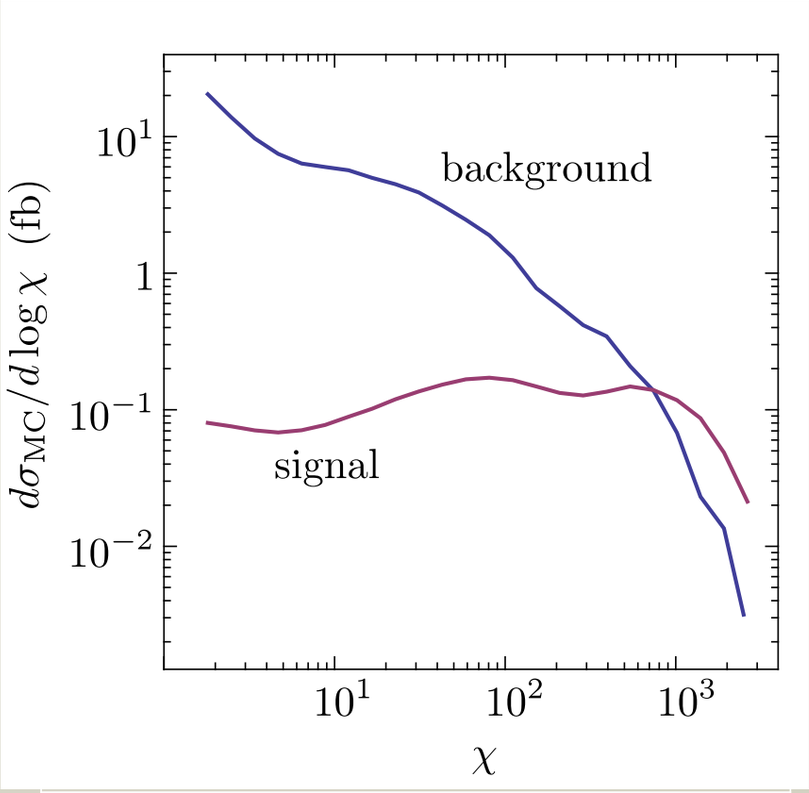
- If microjet j is not a b or $\bar{b}$, we say that $t_j = T$ with probability 0.02 and $T_j = F$ with probability 0.98.

Results

- Best to construct likelihood ratio, but let's use a simple cut.
- Define signal and background cross sections above a cut:

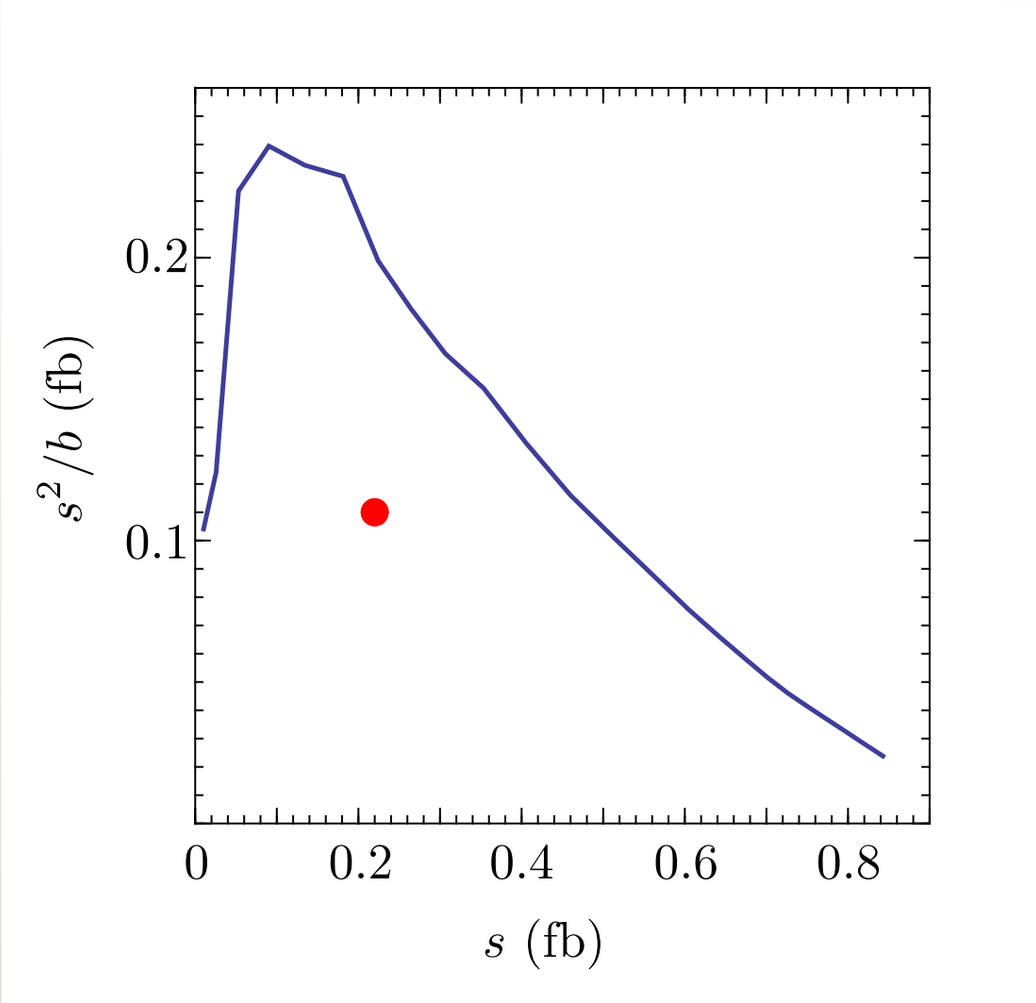
$$s(\chi) = \int_{\chi}^{\infty} d\bar{\chi}\ \frac{d\sigma_{\mathrm{MC}}(\mathrm{S})}{d\bar{\chi}}$$

$$b(\chi) = \int_{\chi}^{\infty} d\bar{\chi}\ \frac{d\sigma_{\mathrm{MC}}(\mathrm{B})}{d\bar{\chi}}$$



- We can choose the signal cross section s by adjusting χ.
- We try to make the statistical significance s^2/b large.

- We find



- *e.g.* with $\int dL = 100$ fb^{-1} we can choose $s = 0.1$ fb.
- Then $N(S) = 10$.
- $s^2/b = 0.25$ fb gives $N(S)^2/N(B) = 25$.
- That is $N(S)/\sqrt{N(B)} = 5$.
- The red point is what you get with the BDRS method.

Conclusions

- Shower deconstruction needs a lot of development.
- So far, it is a little better than existing methods.
- It is modular and the parts can be improved.
- We expect that it will be useful for complicated problems.