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DESY

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In hadron-hadron collision the picture is more complicated.

Decreasing the resolution scale more and more partons are visible and less absorbed by the incoming hadrons and the final state jets.

**Important observation:** The total cross section is independent of the resolution of the measurement (or detector).

We have to also consider the evolution of the final state jets.

*Does perturbative QCD support this nice intuitive picture?*
Hadron-Hadron Collision

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Resolution scale: 100 GeV

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*Does perturbative QCD support this nice intuitive picture?*
Cross section

The cross section is a phase space integral of all the possible matrix elements and the a convolution to the parton distribution functions.

\[ \sigma[F] = \sum_m \int [d\{p, f\}_m] \left( f_{a/A}(\eta_a, \mu_F^2) f_{b/B}(\eta_b, \mu_F^2) \right) \frac{1}{2\eta_a \eta_b p_A \cdot p_B} \]

\[ \times \langle M(\{p, f\}_m) | F(\{p, f\}_m) | M(\{p, f\}_m) \rangle \]

- This is formally an all order expression and it is impossible to calculate out.
- We can do it at LO, NLO and in some cases NNLO level.
- Lots of complication with IR singularities.
- Lots of complication with spin and colors.
- The idea is to approximate the matrix elements using factorization properties of the QCD matrix element.
  
  \[ \Rightarrow \text{ We need a general formalism to describe parton shower evolution.} \]
Cross section

The cross section is a phase space integral of all the possible matrix elements and the a convolution to the parton distribution functions.

\[
\sigma[F] = \sum_m \int [d\{p, f\}_m] \text{Tr}\left\{\rho(\{p, f\}_m)F(\{p, f\}_m)\right\}
\]

density operator in color \(\otimes\) spin space

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✗ We can do it at LO, NLO and in some cases NNLO level.
✗ Lots of complication with IR singularities.
✗ Lots of complication with spin and colors.
✓ The idea is to approximate the matrix elements using factorization properties of the QCD matrix element.
⇒ We need a general formalism to describe parton shower evolution.
The \( m+1 \) parton physical state is represented by density operator in the quantum space and by the statistical state in the statistical space.

\[
\rho(\{p, f\}_{m+1}) \iff |\rho(\{p, f\}_{m+1})| 
\]

This is based on the \( m+1 \) parton matrix elements. They are very complicated (especially the loop matrix elements). We try to approximate them by using their soft collinear factorization properties. For this we introduce operators in the statistical space:

\[
|\rho(\{\hat{p}, \hat{f}\}_{m+1})| \approx \int_{t_m}^{\infty} dt \left[ H_C(t) + H_S(t) \right] |\rho(\{p, f\}_m)| 
\]

This parameter represents the hardness of the splitting. We will call it shower time.

The total splitting operator is

\[
H_I(t) = H_C(t) + H_S(t) 
\]
The QCD matrix elements have universal factorization property when two external partons become collinear:

\[ \mathcal{H}_C \sim \sum_l t_l \otimes t_l^\dagger \mathcal{V}_{ij}(s_i, s_j) \otimes \mathcal{V}_{ij}^\dagger(s_i', s_j') \Leftrightarrow \frac{\alpha_s}{2\pi} \sum_l \frac{1}{p_i \cdot p_j} P_{f_l, f_i}(z) + \ldots \]

*Altarelli-Parisi splitting kernels*
Soft Singularities

The QCD matrix elements have universal factorization property when an external gluon becomes soft.

\[ \mathcal{H}_S \sim - \sum_{l,k} \frac{\hat{p}_l \cdot \varepsilon(s) \hat{p}_k \cdot \varepsilon(s')} {\hat{p}_l \cdot \hat{p}_{m+1} \hat{p}_k \cdot \hat{p}_{m+1}} t_l \otimes t_k^\dagger \]

Soft gluon connects everywhere and the color structure is not diagonal; quantum interferences in the color space.
Resolvable Splittings

Let us consider a physical state at shower time $t$, $|\rho(t)\rangle$. This means every parton is resolvable at this time (this scale). Now, we apply the splitting operator:

$${\mathcal H}_I(t) \text{ operator changes}$$
- the number of the partons, $m \to m+1$
- the color and spin structure
- flavors and momenta

$$|\rho_\infty^R\rangle = \int_t^\infty d\tau \, {\mathcal H}_I(\tau) \, |\rho(t)\rangle$$

This is good approximation if we allow only softer radiations than $t$, $\tau > t$

Now, let us consider a measurement with a resolution scale which correspond to shower time $t'$

$$|\rho_\infty^R\rangle \approx \int_t^{t'} d\tau \, {\mathcal H}_I(\tau) \, |\rho(t)\rangle + \int_{t'}^\infty d\tau \, {\mathcal V}_I^{(\epsilon)}(\tau) \, |\rho(t)\rangle$$

$${\mathcal V}_I(t) \text{ operator}$$
- changes only the color structure
- $(1|{\mathcal V}_I(t) = (1|{\mathcal H}_I(t)$

$\text{Resolved radiations}$

$\text{Unresolved radiations}$

This is a singular contribution

What can we do about it?
Virtual Contributions

There is another type of the unresolvable radiation, the virtual (loop graph) contributions. We have universal factorization properties for the loop graphs. E.g.: in the soft limit, when the loop momenta become soft we have

\[ m \rightarrow 0 \]

This is again a singular operator only in the color space.

We can use this factorization to dress up partonic states with virtual radiation. After careful analysis one can found that the virtual contribution can be approximated by

\[
|\rho^V_\infty\rangle \approx - \int_t^\infty d\tau \mathcal{V}_{I}^{(e)}(\tau) |\rho(t)\rangle
\]

Same structure like in the real unresolved case but here with opposite sign.
Combining the real and virtual contribution we have got

\[ |\rho_R^\infty \rangle + |\rho_V^\infty \rangle = \int_t^{t'} d\tau \left[ \mathcal{H}_I(\tau) - \mathcal{V}_I(\tau) \right]|\rho(t)\rangle \]

This operator dresses up the physical state with one real and virtual radiations that is softer or more collinear than the hard state. Thus the emissions are ordered. Now we can use this to build up physical states by considering all the possible way to go from \( t \) to \( t' \).

\[ |\rho(t')\rangle = |\rho(t)\rangle \]

\[ + \int_t^{t'} d\tau \left[ \mathcal{H}_I(\tau) - \mathcal{V}_I(\tau) \right]|\rho(t)\rangle \]

\[ + \int_t^{t'} d\tau_2 \left[ \mathcal{H}_I(\tau_2) - \mathcal{V}_I(\tau_2) \right] \int_{\tau_1}^{\tau_2} d\tau_1 \left[ \mathcal{H}_I(\tau_1) - \mathcal{V}_I(\tau_1) \right]|\rho(t)\rangle + \cdots \]

\[ = \mathcal{T}\exp \left\{ \int_t^{t'} d\tau \left[ \mathcal{H}_I(\tau) - \mathcal{V}_I(\tau) \right] \right\} \left[ \rho(t) \right] \]

\[ \mathcal{U}(t', t) \text{ shower evolution operator} \]

\[ |\rho(t')\rangle = \mathcal{U}(t', t)|\rho(t)\rangle \]
**Full Splitting Operator**

Very general splitting operator (no spin correlation) is

\[
\langle \{ \hat{p}, \hat{f}, \hat{c}', \hat{c} \}_{m+1} | \mathcal{H}(t) | \{ p, f, c', c \}_m \rangle = \sum_{l=a,b,1,...,m} \delta \left( t - T_l(\{ \hat{p}, \hat{f} \}_{m+1}) \right) \langle \{ \hat{p}, \hat{f} \}_{m+1} | \mathcal{P}_l | \{ p, f \}_m \rangle {\frac{m+1}{2}} \\
\times \frac{n_c(a)n_c(b) \eta_a \eta_b}{n_c(\hat{a})n_c(\hat{b}) \hat{\eta}_a \hat{\eta}_b} \frac{f_{\hat{a}/A}(\hat{\eta}_a, \mu_F^2) f_{\hat{b}/B}(\hat{\eta}_b, \mu_F^2)}{f_{a/A}(\eta_a, \mu_F^2) f_{b/B}(\eta_b, \mu_F^2)} \sum_k \Psi_{lk}(\{ \hat{f}, \hat{p} \}_{m+1}) \\
\times \sum_{\beta=L,R} (-1)^{1+\delta_{lk}} \langle \{ \hat{c}', \hat{c} \}_{m+1} | \mathcal{G}_\beta(l, k) | \{ c', c \}_m \rangle
\]

Splitting kernel is

\[
\Psi_{lk} = \frac{\alpha_s}{2\pi} \frac{1}{\hat{p}_l \cdot \hat{p}_{m+1}} \left[ A_{lk} \frac{2\hat{p}_l \cdot \hat{p}_k}{\hat{p}_k \cdot \hat{p}_{m+1}} + H_{ll}^{\text{coll}}(\{ \hat{f}, \hat{p} \}_{m+1}) \right] \quad \text{Important:} \quad A_{lk} + A_{kl} = 1
\]

Color operator for gluon emission is

\[
\langle \{ \hat{c}', \hat{c} \}_{m+1} | \mathcal{G}_R(l, k) | \{ c', c \}_m \rangle = D \langle \{ \hat{c} \}_{m+1} | t_l^\dagger | \{ c \}_m \rangle \langle \{ c' \}_m | t_k | \{ \hat{c}' \}_{m+1} \rangle_D .
\]

Important: \( A_{lk} + A_{kl} = 1 \)
Full Splitting Operator

Very general splitting operator (no spin correlation) is

\[
\left\{ \hat{p}, \hat{c}_a, \hat{c}_b \right\}_{m+1} = D \left\{ \hat{c}_{m+1} | \mathcal{G}_R(l, k) | \{c', c\}_m \right\}
\]

\[
= \left\langle \{\hat{c}\}_{m+1} | t^\dagger_l \{c\}_m \right\rangle \left\langle \{c'\}_m | t_k \{\hat{c}'\}_{m+1} \right\rangle_D.
\]
Let us see how it looks at hadron collider

\[ \mu = 100 \text{ GeV} \]

In hadron-hadron collision the parton distribution function also absorbs the contribution of the multiple interactions and rescattering.

**Our strategy:**
- Identify *factorizable* singular contributions.
- Sum up the *strongly ordered* radiations.
- Minimize the number of the *ad-hoc* assumptions and tuning parameters.
Let us see how it looks at hadron collider

\[ \mu = 125 \text{ GeV} \]

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Let us see how it looks at hadron collider

\[ \mu = 50 \text{ GeV} \]

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Let us see how it looks at hadron collider

\[ \mu = 25 \text{ GeV} \]

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Multi Parton Interaction

Let us see how it looks at hadron collider

\[ \mu = 15 \text{ GeV} \]

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Now, one can try to define the evolution operator in the following form

\[ \mathcal{U}(t, t') = T \exp \left\{ \int_t^{t'} d\tau \left[ \mathcal{H}_I(\tau) - \mathcal{V}_I(\tau) + \sum_{\beta = \text{MI, RS}} \left\{ \mathcal{H}_\beta(\tau) - \mathcal{V}_\beta(\tau) \right\} \right] \right\} \]

Everything else
This is important in the very small pT regions and negligible in the large pT regions but it is hard to tell how important in the intermediate region. The cumulative effect could be sizable.

Important to note that this is an NLO contributions. Thus, compared to the standard shower this is also suppressed by an extra power of $\alpha_s$.

Requires multi parton PDF (mPDF).

Implemented in HERWIG & PYTHIA. (No “proper” mPDF implemented.)
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Requires multi parton PDF (mPDF).

Implemented in HERWIG & PYTHIA. (No “proper” mPDF implemented.)
This is the standard shower evolution. Adds LL and NLL contributions. Not power suppressed.

Since the MPI kernel is NLO contribution we should consider the standard shower at NLO level as well. (Just to be systematic.)

If we consider NLO terms then we need subleading color contributions, too.

Adds correction to the primary interaction as well as to the MPI contributions.

It is implemented only at LO level in HERWIG & PYTHIA.
This operator can be applied on states with at least two chains. (They are already power suppressed.)

No corresponding factorizable virtual contribution. No associated Sudakov factor.

Actually this is not a singular contribution. It looks singular but that is spurious singularity.

Some level it is implemented in PYTHIA.

\[ \mathcal{H}_{RS}(t) = \frac{\alpha_s}{2\pi} \mathcal{O}(t) + \left(\frac{\alpha_s}{2\pi}\right)^2 \mathcal{O}(t^2) \]

This is the most problematic contribution

\[ \mathcal{V}_{RS}(t) = 0 \]
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\[ \mathcal{V}_{RS}(t) = 0 \]
The PDF has an operator product definition:

\[ \mu \frac{d}{d\mu} f_{a/H}(x, \mu) = \sum_b [P_{a,b} \otimes f_{b/H}](x, \mu) \]

This expression is UV divergent and needed to be renormalized.

The UV singularity in the PDF corresponds to the IR singularity in hard part of the cross section. Everything is consistent.

*How does it work in the mPDF case?*
This operator is also UV divergent and needed to be renormalized. RGE provides the generalized DGLAP equation.

For \( y \neq 0 \) we have a homogeneous DGLAP equation, there is no contribution from \( 2 \rightarrow 4 \) transitions

\[
\frac{d}{dt} F(x_i, y) = P \otimes_{x_1} F + P \otimes_{x_2} F
\]

For \( \int dy \ F(x,y) \) we have contribution from \( 2 \rightarrow 4 \) transitions

Marcus Diehl talk in DESY
Let us study the $2\rightarrow 4$ transitions in the hard matrix elements. In this example we have double $Z$ boson production.

There is a 1-loop graph in this process. This loop integral is perfectly finite, there is **NO IR singularities**.

This tells we should **NOT** consider the $2\rightarrow 4$ transitions.

*Look like there is some inconsistency between the two approaches...*
From this approach one can find that the full evolution operator is

\[ \mathcal{U}(t, t') = \mathcal{T} \exp \left\{ \int_t^{t'} d\tau \left[ \mathcal{H}_I(\tau) - \mathcal{V}_I(\tau) + \mathcal{H}_{MI}(\tau) - \mathcal{V}_{MI}(\tau) \right] \right\} \]

\[
\frac{f\{a_1, a_2\} / A(\hat{\eta}_{a_1}, \hat{\eta}_{a_2}, \mu_F^2)}{f\{a_1, a_2\} / A(\eta_{a_1}, \eta_{a_2}, \mu_F^2)} \]

\[
\frac{f\{a_1, a_2\} / A(\hat{\eta}_{a_1}, \hat{\eta}_{a_2}, \mu_F^2)}{f\{a_1\} / A(\eta_{a_1}, \mu_F^2)}
\]
MPI Evolution Operator

From this approach one can find that the full evolution operator is

\[ U(t, t') = T \exp \left\{ \int_t^{t'} d\tau \left[ \mathcal{H}_I(\tau) - \mathcal{V}_I(\tau) + \mathcal{H}_{MI}(\tau) - \mathcal{V}_{MI}(\tau) \right] \right\} \]

\[ \frac{f\{a_1, a_2\}/A(\hat{\eta}_{a_1}, \hat{\eta}_{a_2}, \mu_F^2)}{f\{a_1, a_2\}/A(\eta_{a_1}, \eta_{a_2}, \mu_F^2)} \]

\[ \frac{f\{a_1, a_2\}/A(\hat{\eta}_{a_1}, \hat{\eta}_{a_2}, \mu_F^2)}{f\{a_1\}/A(\eta_{a_1}, \mu_F^2)} \]
In Pythia or Herwig the effective mPDF is always color singlet state. But it can be a color octet state.

\[ a_1 \rightarrow a_2 \rightarrow a_2 \rightarrow a_1 = \]

Two independent interactions

\[ \Rightarrow \text{Color reconnection effect in the perturbative part.} \]

These two gluons became color connected.
Virtual Contributions

In standard parton shower this operator is obtained from the \textit{unitarity} condition

\[
(1|\mathcal{V}_I(t)) = (1|\mathcal{H}_I(t)) \quad \text{Always real}
\]

But it turns out that we have imaginary contribution from the virtual graphs

\[
\int \frac{d^dl}{(2\pi)^d} \propto \mathcal{V}_I(t) + i\pi \tilde{\mathcal{V}}(t) \quad \text{and} \quad (1|\tilde{\mathcal{V}}(t) = 0
\]

\textit{Coulomb gluon}

What can Coulomb gluon do?
1. Coulomb gluon changes the color configuration and the color flow. It is pure virtual contribution, thus it is unresolvable. *It does the same thing what color reconnection does.*

2. It always make color correlation between the two incoming partons. Let’s consider a color octet hard state:

3. Leads to “*Super Leading Logs*” in the case of some non-global observables.

*Do we have Coulomb like contribution in the MPI virtual graphs?*
**MPI: Coulomb Gluon**

In the MPI part the “resolvable” radiation comes from extra $2 \rightarrow 2$ scattering. This is very singular in the low $p_T$ region. This singularity must be cancelled by the corresponding virtual graphs.

![Diagram](image)

*Real $2 \rightarrow 2$ scattering adds two extra jets*

*Pythia and Herwig put this graph into a simple probabilistic framework and exponentiate the extra $2 \rightarrow 2$ process.*
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Real $2 \rightarrow 2$ scattering adds two extra jets

Corresponding virtual graph.
This is a forward elastic scattering contribution.
It can produce Coulomb gluon term => Color reconnection effect

Pythia and Herwig put this graph into a simple probabilistic framework and exponentiate the extra $2 \rightarrow 2$ process.
Conclusion

- Multiple Interaction is very complicated from theory point of view.
- There are MC tool available mostly based on some tunable models (Color reconnection, simple mPDF assumption,...)
- Running of the mPDF, modeling mPDF
- Some perturbative effects are not included in our MC (Coulomb gluon,...)
- Lack of theorems (factorization,...)
Using the factorization properties of the QCD the approximated order by order calculation can be organized according to

\[ U(t, t') = 1 + \int_{t'}^t d\tau U(t, \tau) [\mathcal{H}_I(\tau) - \mathcal{V}(\tau)] \]

From the unitary condition:

\[ (1|\mathcal{V}(t)) = (1|\mathcal{H}_I(t)) \]

The shower form of the solution is

\[ U(t, t') = \mathcal{N}(t, t') + \int_{t'}^t d\tau U(t, \tau) \mathcal{H}_I(\tau) \mathcal{N}(\tau, t') \]

and the Sudakov operator is

\[ \mathcal{N}(t, t') = \mathbb{T} \exp\left( -\int_{t'}^t d\tau \mathcal{V}(\tau) \right) \]