QCD calculations at NLO: doing integrals numerically

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Work with

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Points to take home

• For flexibility, do the virtual loop integrations numerically.

• Real/virtual cancellations can happen without subtractions.

• You may want subtractions, nevertheless.

• Numerical virtual loop integrals need contour deformation.
For flexibility, do the virtual loop integrations numerically.

- You don’t need to compute complicated integrals analytically.
- You do need a few simple integrals for renormalization.
- Feynman rules → integrand.
- Code exists for $e^+ + e^- \rightarrow 3$ jets.
- Example of flexibility: Coulomb gauge (with M. Krämer).
Real/virtual cancellations can happen without subtractions.

- Insert $\int d\sqrt{s} \, h(\sqrt{s})$.
- Real graph becomes $\int dl_1 \, dl_2 \, dl_3 \, f_R(l)$.

- In loop, do $\int dE$.
- Loop graph becomes $\int dl_1 \, dl_2 \, dl_3 \, f_V(l)$.

\[
\int dl_1 \, dl_2 \, dl_3 \, [f_R(l) + f_V(l)].
\]

- Leading singularities cancel between $f_R$ and $f_V$. 
You may want subtractions, nevertheless.

- Customarily, one subtracts for collinear and soft singularities separately in real and virtual graphs.

- Nothing prevents this.

- If you do subtract, you can have $\int d^4l$ for a loop.

- This may be simpler for $n$-point functions with $n \geq 5$.

- There is an advantage with respect to adding showers.
Numerical virtual loop integrals need contour deformation.

In

\[ \int d\vec{l} \frac{i}{E(\vec{l})^2 - \vec{l}^2 + i\epsilon} \ldots \]

the \( +i\epsilon \) instructs you to deform the integration contour in \( \vec{l} \),

\[ \vec{l} \rightarrow \vec{l} + i\kappa(\vec{l}). \]

- The result is independent of the contour choice (as long as you don’t cross the singularity).
- This works.
Singularities and deformations

Where the singularities are for a 4 point function.

- Deform past the scattering singularities.
- Don’t deform at the collinear or soft singularities.
Singularities and deformations for 4 dimensions

• In 4 dimensions, singularities to be deformed around are on cones.
• The soft singularities are at the tips of the cones.
• The collinear singularities are at points where two cones are tangent.

• Starting at five point functions there are some difficulties with the picture in 3 dimensions

• This 4-dimensional version may be more powerful.
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