Three problems in special relativity\textsuperscript{1}
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1 Moving clock

A clock moves with velocity $v$. What is the ratio $dt/d\tau$ between coordinate time $t$ as seen by clocks at rest in the coordinate system and proper time $\tau$ as measured by the moving clock?

2 Moving meter sticks

We can describe a meter stick in a covariant fashion by saying that coordinate markers on the meter stick are given by a parameter $\sigma$ with $0 < \sigma < L$ and that a clock attached to each coordinate marker measures proper time $\tau$ with $-\infty < \tau < \infty$. Then the space-time coordinates traced out by the points on the meter stick are

$$x^\mu = u^\mu \tau + n^\mu \sigma.$$  \hspace{1cm} (1)

In the rest frame of the meter stick, $u = (1, 0, 0, 0)$ and $n = (0, 0, 0, 1)$. In any other frame, $u$ and $n$ are different. However we have the invariant relations $u^2 = 1$, $n^2 = -1$ and $n \cdot u = 0$.

Consider a frame in which the meter stick is aligned along the $z$ axis and is moving in the $z$ direction. Then $u = (u^0, 0, 0, u^3)$ and $n = (n^0, 0, 0, n^3)$. How are $n^0$ and $n^3$ related to $u^0$ and $u^3$? How is $u^3$ related to the velocity $v$ of the meter stick? Use these relations to find the length $L'$ of the meter stick as measured in this reference frame. That is, how is $L'$ related to $L$ and $v$.

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3 Motion in a magnetic field

A particle of mass $m$ and charge $q$ moves in a uniform magnetic field $B$ directed in the $+z$ direction. The particle moves in the $x$-$y$ plane. Suppose that the speed of the particle is $v$. Show that the particle moves in a circular orbit. Find the radius $R$ and the angular velocity $\omega$ of the orbit as functions of $v$ and $B$.

4 Motion in an electric field

A particle of mass $m$ and charge $q$ moves in a uniform electric field $E$ directed in the $+z$ direction. At time $t = 0$, the particle is at rest. Find the subsequent motion of the particle.