INDEPENDENCE AND METHODS OF PARTITIONING VARIANCE (SS Types)

An important issue in estimating and interpreting main effects and interactions in ANOVA is how to partition variances associated with the component parts of the design. In the experimental agriculture context in which Fisher developed ANOVA, estimation was relatively straightforward. In between-subjects designs, when the factors are true independent variables (manipulated and controlled by the researcher) and there is no missing data and equal sample sizes in each group, the sources of variance in the ANOVA are statistically independent of each other. This means that the variance estimated for one factor or effect is unrelated to the estimation of the variance for every other factor or effect in the design. Another statistical name for this situation is a “balanced, orthogonal” design. The term “orthogonal” means the effects are independent of each other. The term “balanced” refers to whether the subjects are equally distributed in each group or cell in the design.

However, when quasi-IVs are used, it is likely that they are correlated or related to each other to some degree (i.e., they are nonorthogonal). Another way in which a relationship can arise between effects is due to correlated patterns of sample sizes. When researchers study existing variables and intact, existing groups, it is likely that cell sample sizes in an ANOVA design will be unequal. Another common reason cell sizes are unequal is as a result of missing data or attrition of subjects. Attrition rarely occurs equally across all groups. ANOVA designs with unequal sample sizes are referred to as “unbalanced” designs. In these situations, estimation and interpretation of variance estimates is more complex. The essential issue is that the same portion of variance in the dependent measure explained by one effect may also be explained by other effects in the design. The question is where should this common or shared variance be attributed? Several ways of answering this question and estimating variances have been developed and used in the social sciences by calculating the sums of squares (SS) using different methods.

**Type I, Hierarchical Method.** In this method, the researcher decides, based on theory or prior research, that a particular hierarchy or ordering of effects should be established. Then in
analysis, each effect is adjusted for any effects that precede it in the hierarchy. The figure below shows an example of a two-factor, unbalanced, nonorthogonal design with main effects A and B and the interaction effect AB. Note in the figure the way the effects overlap with each other. The background in the figure represents the total variance and so whatever variance is not contained in the overlapping circles is residual variance or MSE. As an example of the application of the Hierarchical Method of computing sums of squares, assume that the researcher considers the AB effect as most important theoretically, then the B effect, and last the A effect. In this example, all common variance of AB with A or AB with B would be assigned to AB. That is, the sums of squares AB would be composed of the areas labeled AB + 1 + 3 + 2. The B effect would be calculated as the sum of areas B + 4, and the A effect would contain only area A. Obviously, the size of an effect depends a lot on its precedence in the hierarchy, so this method should be applied carefully and only when there is strong theoretical rationale for the hierarchy.

**Type II. Experimental.** In this approach the main effects are estimated ignoring the interaction, but the interaction is estimated after adjusting for the main effects. In the figure, this means that the SS for A is composed of A + 1; the SS for B is composed of B + 2; and the SS for AB is composed of AB only. While historically popular, this method is not recommended.

**Type III Unique Sums of Squares (also known as the regression method).** This method is the most common and accepted solution to unbalanced designs and is the default method in SPSS. In this approach, each effect is adjusted for all other effects in the design to obtain the unique contribution of that effect. In the figure, this means that the SS for each effect will only contain the area labeled with that letter. The areas labeled 1 - 4 are not included in the estimation of any of the effects. It is important in this approach to recognize that when there is a great deal of overlap among the effects, there may be little unique variance even though the effects as a group are strongly related to the dependent variable.

[Note: SPSS also provides estimation of sums of squares for a Type IV design. This method is specifically for situations where there are completely empty cells in the design which is beyond the research designs considered in 603].
Also note that when a design is balanced (equal cell sizes) all three methods above yield the same F tests. When cell sizes are unequal, results from one method to another can differ substantially. Another complication that arises in unbalanced designs is that contrast weights used in comparison procedures are affected by the n-sizes for the groups. In order to correct for this in applications like planned comparisons, weights must be created that take group sample sizes into account.

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