CALCULATING STRENGTH OF ASSOCIATION

There are two main strength of association measures used in ANOVA contexts, omega squared ($\omega^2$), and eta squared ($\eta^2$). Omega squared is generally a more accurate estimate of the true population value of strength of association. Eta squared, however, is simpler to compute. In either case, a strength of association measure provides an estimate of the amount of variance in the dependent measure that can be explained or accounted for by the independent measure.

**Omega Squared for an independent $t$-test:**

$$\omega^2 = \frac{(t^2 - 1)}{(t^2 + N_1 + N_2 - 1)}$$

**Example:**

<table>
<thead>
<tr>
<th></th>
<th>Group 1</th>
<th>Group 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>65.5</td>
<td>69.0</td>
</tr>
<tr>
<td>Variance</td>
<td>20.69</td>
<td>28.96</td>
</tr>
<tr>
<td>$N$</td>
<td>30</td>
<td>30</td>
</tr>
</tbody>
</table>

$$t = \frac{65.5 - 69}{1.29} = -2.71$$

$$\omega^2 = \frac{(2.71)^2 - 1}{[(2.71)^2 + 30 + 30 - 1]} = 0.096$$

Strength of association measures can be multiplied by 100 and interpreted as the percent of variation explained (PVE). In this example, one could conclude that approximately 10% of the variation in the DV can be attributed to the difference between groups.

**Omega Squared for a one-factor ANOVA:**

$$\omega^2 = \frac{SS_{Between} - (a-1)(MS_{Within})}{SS_{Total} + MS_{Within}}$$

**Omega Squared for a two-factor ANOVA:**

$$\omega^2 = \frac{SS_A - (a-1)(MS_{S(AB)})}{SS_{Total} + MS_{S(AB)}}$$

$$\omega^2 = \frac{SS_B - (b-1)(MS_{S(AB)})}{SS_{Total} + MS_{S(AB)}}$$

$$\omega^2 = \frac{SS_{AB} - (a-1)(b-1)(MS_{S(AB)})}{SS_{Total} + MS_{S(AB)}}$$
Example:

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.78</td>
<td>2</td>
<td>0.39</td>
<td>0.05</td>
<td>NS</td>
</tr>
<tr>
<td>B</td>
<td>1.39</td>
<td>1</td>
<td>1.39</td>
<td>0.19</td>
<td>NS</td>
</tr>
<tr>
<td>AB</td>
<td>53.44</td>
<td>2</td>
<td>26.72</td>
<td>3.62</td>
<td>NS</td>
</tr>
<tr>
<td>S(AB)</td>
<td>88.67</td>
<td>12</td>
<td>7.39</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>144.28</td>
<td>17</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For Factor A:

\[ \omega^2 = SS_A - (a-1)(MS_{S(AB)}) / SS_{Total} + MS_{S(AB)} \]
\[ = 0.78 - (2)(7.39) / 144.28 + 7.39 = -0.10 = 0 \]

For Factor B:

\[ \omega^2 = SS_B - (b-1)(MS_{S(AB)}) / SS_{Total} + MS_{S(AB)} \]
\[ = 1.39 - (1)(7.39) / 144.28 + 7.39 = -0.04 = 0 \]

For the AB interaction:

\[ \omega^2 = SS_{AB} - (a-1)(b-1)(MS_{S(AB)}) / SS_{Total} + MS_{S(AB)} \]
\[ = 53.44 - (2)(7.39) / 144.28 + 7.39 = 0.25 \]

Note that it is possible to get an estimate for omega which is negative. Since a negative variance has no meaning, negative values are always set to zero. We conclude that the main effects of A and B are not associated with the DV. The AB interaction, however, accounts for approximately 25% of the variation in the DV.

Eta Squared, an alternative to Omega:

\[ \eta^2 = SS_{Effect} / SS_{Total} \]

Using the example above:

For Factor A: \[ \eta^2 = 0.78 / 144.28 = 0.005 \]

For Factor B: \[ \eta^2 = 1.39 / 144.28 = 0.010 \]

For the AB interaction: \[ \eta^2 = 53.44 / 144.28 = 0.370 \]

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