

Wild representation theories

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Representation theory studies symmetries or actions of groups on vector spaces. In a nicely structured world one could hope to classify such actions given some reasonable constraints: a classical example is the classification of regular tilings of a 2-dimensional plane beautifully illustrated in Escher's collection of "symmetry" prints. In the world of groups acting on complex vector spaces the classical result of Maschke from 1898 points towards such general classifications.

I'll review the classical theory and move to the settings where such classifications are utterly impossible giving the theories a technical but eloquent name of being "wild". For example, the actions of a non-cyclic group of order 9 on vector spaces over a field of positive characteristic 3 already constitute a "wild" representation theory. Associating geometric invariants to wildly behaving representations allows one to give some structure to this unruly territory and even parameterize naturally occurring classes of representations. For finite groups these geometric invariants are governed by purely combinatorial structures which allows one to draw diagrams and turn them into pretty pictures.