1. (a) Consider the differential form “$d\theta''$" on $\mathbb{R}^2 - 0$ given by the formula

$$
``d\theta'' = \frac{xdy - ydx}{x^2 + y^2}.
$$

Prove that “$d\theta''$" is not exact. (Recall that a smooth 1-form $\eta$ on a manifold $M$ is said to be exact if there is a smooth function $f$ on $M$ such that $df = \eta$.) Also show that the restriction of “$d\theta''$" to the subset $U = \{(x, y) \in \mathbb{R}^2 | x \neq 0\}$ is exact.

(b) Show that every smooth 1-form $\eta$ on $\mathbb{R}^2 - 0$ can be written as

$$
\eta = f(x, y) ``d\theta'' + g(x, y) dr,
$$

where $r(x, y) = \sqrt{x^2 + y^2}$ and $f, g$ are smooth functions. In particular, write $dx$ and $dy$ in the above form.

(c) Show that

$$
dx \wedge dy = r dr \wedge ``d\theta''.
$$