MATH 444/544, PROBLEMS FOR THE SECOND MIDTERM.

Problem 1. Recall that the alternating group \( A_n \) is the kernel of the homomorphism \( \text{sign} : S_n \rightarrow \{1, -1\} \).

a) Find the number of permutations in \( A_4 \) of order 2.
b) Prove that \( A_4 \) has a normal subgroup \( H \) of order 4.
c) For the subgroup \( H \subset A_4 \) from part b) compute the order of \( A_4/H \). Is the factor group \( A_4/H \) abelian?

Problem 2. Recall that, for a group \( G \), the commutator subgroup \([G, G]\) is the subgroup generated by all elements of the form \( xyx^{-1}y^{-1} \), with \( x, y \in G \).

Find the order of the factor group \( D_4/[D_4, D_4] \). Is the group \( D_4/[D_4, D_4] \) abelian? Cyclic?

Problem 3 Find all the homomorphisms from the group \( S_n \) to \( \mathbb{Z}_3 \).

Problem 4. Recall that a permutation \( f \in S_n \) is called even if \( \text{sign}(f) = 1 \). Prove that every permutation \( \sigma \in S_n \) of an odd order is even.

Problem 5. Let \( p \) be a prime number. Find the number of subgroups of the group \( S_p \) of order \( p \).