FINAL EXAM—MATH 251—PRACTICE

Time: 3:15pm—5:15pm, Dec 7th

Name (print): ______________________________

Student ID No.: ______________________________

Signature: : ______________________________

Grade: ______________________________

Problem	Points	Grades
No. 1	32	
No. 2	10	
No. 3	10	
No. 4	10	
No. 5	12	
No. 6	12	
No. 7	14	

Instructions: To receive full credits, all answers must be supported with clear and correct derivations. **No Credit** will be given for the answer without the detailed correct work.

Date: Dec 7th, 2009.

1. Short answer problems.
(1).

$$\lim_{s\to\infty} \frac{\sqrt{-s^2+s^4}}{3s^2-s-2}$$

(2). Find the limit of

$$\lim_{x\to 0} \frac{\tan x - \sin x}{x^3}$$

(3). Find the derivative of $y = x^{\sin x}$.

(4). Find the equation of the tangent line at $(1, 2)$ to the curve $\ln(x) + y^2 = e^{2x-y} + 3$.

(5). Let $h(x) = f(g(x))$. If $g(1) = 2, g^{'}(1) = -1,\ f(2) = 2,\ f^{'}(2) = -3$. Find $h(1)$ and $h^{'}(1)$. Then use linear approximation to estimate $h(0.98)$.

(6). Suppose $f(x)$ is a continuous function, and suppose

$$\lim_{x \to 1} \frac{f(x) - 2}{x - 1} = 3.$$

Find $f(1), f^{'}(1)$.

2. Use definition to find the derivative of

$$f(x) = \sqrt{1 + x^2}.$$

No points will be given without using definition of derivatives.

3. Some bones are exhumed from an ancient burial ground. If the ratio of Carbon 14 in the bones is 65% of the ratio in the bones of a living human, then how old are the bones? (The half-life of Carbon 14 is 5730 years and it begins to decay immediately after a person dies).

4. Gravel is dumped from a conveyor belt at a rate of $200 ft^3/min$. It forms a pile in the shape of a right circular cone whose base radius and height are always the same. How fast is the height of the pile increasing when the pile is $20 ft$ high?

5. How many roots does $f(x) = x^4 + 4x - 6$ have? Justify your answer clearly. Then use Newton's method to get the second approximation of the negative root; you can start with $x_0 = -2$.

6. A manufacture has been selling 1700 television sets a week at \$540 each. A market survey indicates that for each 24 rebate offered to a buyer, the number of sets sold will increase by 240 per week.

a) Find the demand function $p = p(x)$, where x is the number of the television sets sold per week.

b) How large rebate should the company offer to a buyer, in order to maximize its revenue?

c) If the weekly cost function is $153000 + 180x$, how should it set the size of the rebate to maximize its profit?

7. Let $f(x) = \frac{\ln(x+1)}{x}$.
(a). Find the domain of $f(x)$ and asymptotes of $f(x)$, if there is any.

(b). Find the first and section derivatives of $f(x)$.

(c). Find the critical points, local maximum, and local minimum of $f(x)$, and the intervals where $f(x)$ is decreasing and increasing.

(d). Find the intervals where $f(x)$ is concave up and down, inflection points. Then use the information $(a) - (d)$ to sketch the graph of $y = f(x)$