

A SAMPLE MIDTERM

The actual exam has only 5 problems, which could resemble the problems here and the homework problems.

1. Prove that

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \cdots + \frac{1}{n(n+1)} = \frac{n}{n+1}.$$

2. Let

$$S = \left\{ \frac{2n-1}{n^2+1} : n \in \mathbb{Z}_{>0} \right\}.$$

Find $\sup(S)$ and $\inf(S)$.

3. Prove formally that

$$\lim_{n \rightarrow \infty} \frac{n^2 + n}{n^2 + 1} = 1.$$

4. Find

$$\lim_{n \rightarrow \infty} \frac{n + \cos(n)}{2n - \sin(2n)}$$

using the limit laws.

5. Let $s_1 = 1$ and $s_{n+1} = \frac{1}{5}(s_n + 2)$ for $n \in \mathbb{Z}_{>0}$. Prove that sequence $(s_n)_{n \in \mathbb{Z}_{>0}}$ converges and find its limit.

6. Let $(a_n)_{n \in \mathbb{Z}_{>0}}$ and $(b_n)_{n \in \mathbb{Z}_{>0}}$ be sequences in \mathbb{R} . Suppose that $\sum_{n=1}^{\infty} a_n$ converges absolutely and that $(b_n)_{n \in \mathbb{Z}_{>0}}$ is bounded. Prove that $\sum_{n=1}^{\infty} a_n b_n$ converges absolutely.

7. Let $(a_n)_{n \in \mathbb{Z}_{>0}}$ and $(b_n)_{n \in \mathbb{Z}_{>0}}$ be Cauchy sequences. Prove $(a_n b_n)_{n \in \mathbb{Z}_{>0}}$ is a Cauchy sequence by the definition of a Cauchy sequence.

8. (Let $(a_n)_{n \in \mathbb{Z}_{>0}}$ and $(b_n)_{n \in \mathbb{Z}_{>0}}$ be sequences in \mathbb{R} . Suppose that $\lim_{n \rightarrow \infty} a_n = 0$ and $\lim_{n \rightarrow \infty} b_n = 0$. Also suppose that for all $n \in \mathbb{Z}_{>0}$, we have $a_n > 0$ and $b_n > 0$. Prove or disprove:

$$\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = 1.$$

9. Determine, with proof, whether or not the following series converges:

$$\sum_{n=1}^{\infty} \frac{\log(n)}{n}.$$

(In keeping with standard practice in rigorous mathematics, $\log(x)$ means the logarithm to the base e , **not** base 10. You may use the standard facts about the function $x \mapsto \log(x)$ from precalculus and elementary calculus.)

10. Go over all the major theorems we have covered and all the homework problems, in particular, problems

2.2: 8; 2.3: 6, 8; 2.4: 2; 2.5: 5, 6; 2.6: 4; 2.7: 3