MATH 281: Multivariable Calculus

Mar 19, 2010

Final exam—practice

Name:____

ID:____

In order to receive full credit your answer must be **complete**, **legible** and **correct**. Show all of your work, and give adequate explanations. This is a practice exam; the **actual** exam contains **only eight** problems which resemble the problems in the practice exam.

DO NOT WRITE IN THIS BOX!

Problem	Points	Score
1	15 pts	
2	12 pts	
3	10 pts	
4	12 pts	
5	12 pts	
6	12 pts	
7	12 pts	
8	15 pts	
9	15 pts	
Total	115 pts	

1. Consider the curve $\mathbf{r}(t) = (e^t \cos t, e^t \sin t, e^t)$. Compute at $t = \pi$:

(a) The unit tangent vector $\mathbf{T}$, the unit normal vector $\mathbf{N}$ and the unit binormal vector $\mathbf{B}$.

(b) The curvature k.

2. (a) Find equations of the tangent plane and normal line to the surface $z + 2 = xe^y \cos(z)$ at the point $(2, 0, 0)$.

(b) If a plane which passes through the point $(-1, 2, 3)$ is parallel to the given plane above, find the distance of the two planes.

3. Let $z = f(x, y)$ defined by

$$f(x,y) = \frac{x^2 y}{x^4 + y^2}, (x,y) \neq (0,0),$$
$$f(0,0) = 0.$$

Is $f(x, y)$ a continuous function? Why? **Hint:** find if the following limit exists

$$\lim_{(x,y)\to(0,0)} f(x,y).$$

4. The volume of a torus with inner radius r and outer radius R is given by

$$V = \frac{\pi^2}{4}(R-r)^2(R+r).$$

A donut, represented by a torus with $R = 7$ and $r = 3$ is dipped completely in chocolate. If the thickness of the chocolate is 0.2 (that is $dR = 0.2$ and $dr = 0.2$), use differentials to estimate how much chocolate (the volume) was used.

5. Suppose you are climbing a hill whose shape is given by $z = 500 - x^2 - 2y^2$ and you are at the point $(10, 10, 200)$. In which direction should you proceed initially in order to reach the top of the hill in the shortest path? If you climb in that direction, at what angle above horizontal will you be climbing initially?

6. Find and classify the critical points of the function $z = x^3y + 6x^2 - 8y$.

7. (a) Use the chain rule to find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$, where $z = \sqrt{x^2 + 2y^2}, x = s + 2t, y = st^{-1}$.

(b) The radius of a right circular cone is increasing at a rate of 4 inches per minute and its height is decreasing at a rate of 2 inches per minute. At what rate is the volume of the cone changing when the radius is 10 inches and the height is 20 inches?

8. Find the global maximum and the global minimum of the function $f(x, y, z) = 2x^3 + y^4$ subject to the constraint $x^2 + y^2 \leq 16$.

9. Find the volume of the largest rectangular box with edges parallel to the axes that can be inscribed in the ellipsoid

$$\frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{25} = 1.$$

Hint: Week 8 homework, no. 4.