

## MIDTERM EXAM I—MATH 251-PRACTICE

Time: 3:00pm—3:50pm

Name (print):

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Student ID No.:

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Signature: :

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Grade:

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Problem	Points	Grades
No. 1	24	
No. 2	8	
No. 3	10	
No. 4	8	

Instructions: To receive full credits, all answers must be supported with clear and correct derivations. **No Credit** will be given for the answer without the detailed correct work.

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*Date:* Oct 26th, 2009.

1. Short answer problems.

(a). Find the limit

$$\lim_{t \rightarrow 0} \left( \frac{1}{t} - \frac{1}{t^2 + t} \right)$$

(b). Find the limit

$$\lim_{x \rightarrow 1} \frac{x^3 - 1}{x^2 - 1}$$

(c). Find the horizontal and vertical asymptotes of  $g(x)$ ,

$$g(x) = \frac{1 + x^4}{x^2 - x^4}$$

(d). Find  $a$  and  $b$  such that the function

$$f(x) = \begin{cases} x^2 + 3x + 6, & x \geq 2 \\ ax + b, & x < 2 \end{cases}$$

is differentiable everywhere.

(e). Find the derivative of  $g(x)$  and  $g'(0)$ .

$$g(x) = \frac{e^{\cos x}}{x^2 + \sin x + 1}$$

(f). Find an equation of the tangent line at  $(1, 2)$  to the curve  $x^4 + y^4 = 8xy + 1$ .

(g). Assume that  $f(x)$  is continuous everywhere and it is given to you that

$$\lim_{x \rightarrow 7} \frac{f(x) + 2}{x - 7} = -12$$

Find the tangent line to  $y = f(x)$  when  $x = 7$ .

2. Use definition to find the derivative of

$$f(x) = \sqrt{x^2 + 1}.$$

No points will be given without using definition of derivatives.

3. (a). Evaluate

$$\lim_{x \rightarrow 0} x^{\frac{1}{251}} \cos(x^{-251})$$

or explain if the limit does not exist.

(b). Let  $h(x) = x^2 f(x^2 + 1)$ . Find  $h'(x), h''(x)$  in terms of  $f, f', f''$ . When  $f(-1) = 1, f(2) = 3, f'(2) = -4$ , find  $h'(-1)$ .

4. There are two tangent lines to the parabola  $y = x^2 + x$  which pass through  $(2, -3)$ . Find the equations of two tangent lines.