MATH281: QUIZ 2

Name: **ID:**

Question 1 (4pts): find an equation of the tangent plane to the given surface at the specific point.

$$z = y\cos(x-y),\ (2,2,2).$$

$z_x = y\sin(x-y)$

$z_y = \cos(x-y) - y\sin(x-y)$ (product rule)

$z_x(2,2) = 0$, $z_y(2,2) = \cos(0) - 2\sin(0) = 1$

tangent plane: $(z-2) = z_x(2,2)(x-2) + z_y(2,2)(y-2)$

$z - 2 = 0 + y - 2$

$z = y$

Question 2 (6pts): let $z = f(x, y)$ defined by

$$f(x,y) = \frac{xy^2}{2x^2 + y^4}, (x,y) \neq (0,0),$$
$$f(0,0) = 0.$$

(a). Find the limits of $f(x, y)$ when $(x, y) \to (0, 0)$ along the curves $y = x$ and $y^2 = x$. Is $f(x, y)$ a continuous function?

(1) $y = x$, $f(x,y) = \frac{x^3}{2x^2+x^4} = \frac{x}{2+x^2}$ $\lim_{x\to 0} \frac{x}{2+x^2} = 0$

(2) $y^2 = x$, $f(x,y) = \frac{x \cdot x}{2x^2 + x^2} = \frac{x^2}{3x^2} = \frac{1}{3}$

$\lim_{\substack{y^2=x \\ x\to 0}} f(x,y) = \frac{1}{3}$. So, $\lim_{(x,y)\to(0,0)} f(x,y)$ DNE.

$f(x,y)$ is not continuous at $(0,0)$.

(b). Use definition to find partial derivatives of $f_x(0, 0), f_y(0, 0)$, where $f(x, y)$ is the function given above. **Hint:** by definition, you are supposed to find the limits

$$f_x(0,0) = \lim_{h\to 0} \frac{f(h,0) - f(0,0)}{h}; f_y(0,0) = \lim_{h\to 0} \frac{f(0,h) - f(0,0)}{h}.$$

(1) $f(h,0) = \frac{h \cdot 0^2}{2 \cdot h^2 + 0^4} = 0$ $h \neq 0$

$f_x(0,0) = \lim_{h\to 0} \frac{f(h,0) - f(0,0)}{h} = \lim_{h\to 0} \frac{0}{h} = 0$

$f(0,h) = \frac{0 \cdot h^2}{2 0^2 + h^4} = \frac{0}{h^4} = 0$

$f_y(0,0) = \lim_{h\to 0} \frac{f(0,h) - f(0,0)}{h} = \lim_{h\to 0} \frac{0}{h} = 0.$