Solution

MATH281: QUIZ 1

Name: **ID:**

Question 1(5pts): Let $A = (3, 1, 2)$ and $B = (1, -3, -4)$ be two points in $\mathbb{R}^3$.
(1). Find the cross product of two position vectors of A and B.
(2). Find the equation of the plane parallel to the plane determined by A, B and $O = (0, 0, 0)$ which passes through the point $(1, -6, 7)$.

(1) $\overrightarrow{OA} = \langle 3, 1, 2 \rangle$, $\overrightarrow{OB} = \langle 1, -3, -4 \rangle$ $\quad \overrightarrow{OA} \times \overrightarrow{OB} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 1 & 2 \\ 1 & -3 & -4 \end{vmatrix} = \langle 2, 14, -10 \rangle$

(2) $\overrightarrow{OA} \times \overrightarrow{OB} = \langle 2, 14, -10 \rangle$ is a normal vector

Eq: $2x + 14y - 10z + d = 0$, $(1, -6, 7)$ is on the plane, so:

$2 \cdot (1) + 14(-6) - 10 \cdot 7 + d = 0 \quad d = 152$

Question 2(5pts): Let the vectors $\mathbf{r} = \langle x, y, z \rangle$, $\mathbf{a} = \langle 1, 3, -2 \rangle$ and $\mathbf{b} = \langle -3, 1, 0 \rangle$ satisfy the vector equation $(\mathbf{r} - \mathbf{a}) \cdot (\mathbf{r} - \mathbf{b}) = 0$. Show that this equation represents a sphere. Find the center and radius of the sphere.

$(\vec{r} - \vec{a}) \cdot (\vec{r} - \vec{b}) = \langle x-1, y-3, z+2 \rangle \cdot \langle x+3, y-1, z \rangle$

$= (x-1)(x+3) + (y-3)(y-1) + (z+2)z = 0$

$x^2 + 2x - 3 + y^2 - 4y + 3 + z^2 + 2z = 0$

$x^2 + 2x + y^2 - 4y + z^2 + 2z = 0$, complete square.

$(x^2 + 2x + 1) + (y^2 - 4y + 4) + (z^2 + 2z + 1) = 1 + 4 + 1$

$(x+1)^2 + (y-2)^2 + (z+1)^2 = 6$

So: the equation represents a sphere with

center $(-1, 2, -1)$, Radius $\sqrt{6}$