MIDTERM EXAM I—MATH 251-PRACTICE

Time: 3:00pm—3:50pm

Name (print): Weiyong HE

Student ID No.:

Signature: :

Grade:

Problem	Points	Grades
No. 1	24	
No. 2	8	
No. 3	10	
No. 4	8	

Instructions: To receive full credits, all answers must be supported with clear and correct derivations. **No Credit** will be given for the answer without the detailed correct work.

Date: Oct 26th, 2009.

1. Short answer problems.
(a).Find the limit

$$\lim_{t\to 0}\left(\frac{1}{t}-\frac{1}{t^2+t}\right) = \lim_{t\to 0}\frac{(t^2+t)-t}{t(t^2+t)} = \lim_{t\to 0}\frac{t^2}{t^2(t+1)} = \lim_{t\to 0}\frac{1}{1+t} = 1$$

(b).Find the limit

$$\lim_{x\to 1}\frac{x^3-1}{x^2-1} = \lim_{x\to 1}\frac{(x-1)(x^2+x+1)}{(x-1)(x+1)} = \frac{3}{2}$$

(c). Find the horizontal and vertical asymptotes of $g(x)$,

$$g(x)=\frac{1+x^4}{x^2-x^4}$$

(d). Find a and b such that the function

$$f(x)=\begin{cases} x^2+3x+6, x\geq 2\\ ax+b, x<2\end{cases}$$

is differentiable everywhere.
(e). Find the derivative of $g(x)$ and $g'(0)$.

$$g(x)=\frac{\cos x}{\sqrt{e^x+\sin x}}$$

(f). Find an equation of the tangent line at $(1,2)$ to the curve $x^4+y^4=8xy+1$.
(g). Assume that $f(x)$ is continuous everywhere and it is given to you that

$$\lim_{x\to 7}\frac{f(x)+2}{x-7}=-12$$

Find the tangent line to $y=f(x)$ when $x=7$.

(c). $\lim_{x\to\infty} g(x) = \lim_{x\to+\infty}\frac{(1+x^4)/x^4}{(x^2-x^4)/x^4} = \lim_{x\to+\infty}\frac{\frac{1}{x^4}+1}{\frac{1}{x^2}-1} = -1$ $\quad y=-1$ H.A

$\lim_{x\to-\infty} g(x) = \lim_{x\to-\infty}\frac{(1+x^4)/x^4}{(x^2-x^4)/x^4} = \lim_{x\to-\infty}\frac{\frac{1}{x^4}+1}{\frac{1}{x^2}-1} = -1$

$x^2-x^4=0 \quad x=+1,-1,0$

$\lim_{x\to 0} g(x) = +\infty \quad \lim_{x\to 1^-} g(x) = +\infty = \lim_{x\to -1^+} g(x) \quad \lim_{x\to 1^+} g(x) = \lim_{x\to -1^-} g(x) = -\infty$

$x=0,-1,1$ V. A.

(d) $f(2)=\lim_{x\to 2} f(x) \quad 2a+b=4+6+6 \quad f'(2)=(2x+3)\big|_{x=2}=4$

$a=7,\ b=2$

(e). $g'(x) = \left[\cos x \; (e^x + \sin x)^{-\frac{1}{2}}\right]'$

$= -\sin x \,(e^x + \sin x)^{-\frac{1}{2}} - \frac{1}{2}\cos x \cdot (e^x + \sin x)^{-3/2} \cdot (e^x + \cos x)$

$g'(0) = -\sin(0)\,(e^0 + \sin 0)^{-\frac{1}{2}} - \frac{1}{2}\cos 0\,(e^0 + \sin 0)^{-3/2}\,(e^0 + \cos 0)$

$= 0 - \frac{1}{2} \cdot 2 = -1.$

(f) $\frac{d}{dx}(x^4 + y^4) = \frac{d}{dx}(8xy + 1)$

$4x^3 + 4y^3 \frac{dy}{dx} = 8y + 8x\frac{dy}{dx}.$

When $x=1$, $y=2$, we get

$4\cdot + 4\cdot 2^3 \frac{dy}{dx} = 8\cdot 2 + 8\frac{dy}{dx}$

So: $24\frac{dy}{dx} = 12$

$\frac{dy}{dx} = \frac{1}{2}$ when $x=1$, $y=2$.

tangent line: $\frac{y-2}{x-1} = \frac{1}{2}$ $\quad y = \frac{1}{2}x + \frac{3}{2}.$

(g). $\lim\limits_{x\to 7} \frac{f(x)+2}{x-7} = -12$

$$f(x)+2 = \frac{f(x)+2}{x-7}\cdot(x-7)$$

$$\lim_{x\to 7}(f(x)+2) = \lim_{x\to 7}\frac{f(x)+2}{x-7}\cdot(x-7) = \lim_{x\to 7}\frac{f(x)+2}{x-7}\lim_{x\to 7}(x-7) = -12\cdot 0 = 0$$

So. $f(7)+2=0$ $\quad f(7)=-2$

$$f'(7) = \lim_{x\to 7}\frac{f(x)-f(7)}{x-7} = \lim_{x\to 7}\frac{f(x)+2}{x-7} = -12$$

Tangent line: $\frac{y-(-2)}{x-7} = -12$ $\quad \frac{y+2}{x-7} = -12.$

$$y+2 = -12(x-7)$$

$$y = -12x+82.$$

2. Use definition to find the derivative of

$$f(x) = \sqrt{x^2+1}.$$

No points will be given without using definition of derivatives.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\left(\sqrt{x^2+1}\right)' = \lim_{h \to 0} \frac{\sqrt{(x+h)^2+1} - \sqrt{x^2+1}}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{(x+h)^2+1} - \sqrt{x^2+1}}{h} \cdot \frac{\sqrt{(x+h)^2+1} + \sqrt{x^2+1}}{\sqrt{(x+h)^2+1} + \sqrt{x^2+1}}$$

$$= \lim_{x \to 0} \frac{(x+h)^2+1 - (x^2+1)}{h} \cdot \frac{1}{\sqrt{(x+h)^2+1} + \sqrt{x^2+1}}$$

$$= \lim_{x \to 0} \frac{x^2+2xh+h^2+1-x^2-1}{h} \cdot \frac{1}{\sqrt{(x+h)^2+1} + \sqrt{x^2+1}}$$

$$= \lim_{x \to 0} \frac{h(2x+h)}{h} \cdot \frac{1}{\sqrt{(x+h)^2+1} + \sqrt{x^2+1}} = \frac{x}{\sqrt{x^2+1}}.$$

3. (a). Evaluate

$$\lim_{x\to 0} x^{\frac{1}{251}} \cos(x^{-251})$$

or explain if the limit does not exist.

~~When $x\to 0$~~ Since: $|\cos(x^{-251})| \le 1$

when $x \to 0$ $\quad -|x^{\frac{1}{251}}| \le x^{\frac{1}{251}} \cos(x^{-251}) \le |x^{\frac{1}{251}}|$

Take limit $\quad -\lim_{x\to 0} |x^{\frac{1}{251}}| = 0$

$\lim_{x\to 0} |x^{\frac{1}{251}}| = 0$ $\quad$ by Squeeze Theorem $\quad \lim_{x\to 0} x^{\frac{1}{251}} \cos(x^{-251}) = 0$

(b). Let $h(x) = x^2 f(x^2+1)$. Find $h'(x), h''(x)$ in terms of f, f', f''. When $f(-1) = 1$, $f(2) = 3$, $f'(2) = -4$, find $h'(-1)$.

$$h'(x) = 2x f(x^2+1) + x^2 f'(x^2+1)\cdot 2x$$

$$= 2x f(x^2+1) + 2x^3 f'(x^2+1)$$

$$h''(x) = 2 f(x^2+1) + 4x^2 f'(x^2+1) + 6x^2 f'(x^2+1) + 4x^4 f''(x^2+1)$$

$$h'(-1) = 2(-1) f((-1)^2+1) + 2(-1)^3 f'((-1)^2+1)$$

$$= -2 f(2) - 2 f'(2)$$

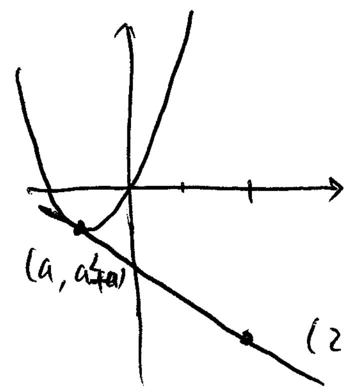
$$= -2\cdot 3 - 2(-4) = 2.$$

4. There are two tangent lines to the parabola $y = x^2 + x$ which pass through $(2, -3)$. Find the equations of two tangent lines.

let (a, a^2+a) at $y=x^2+x$ such that

the tangent line ~~pa~~ through (a, a^2+a) pass through $(2,-3)$

also.



From one hand: the slope is given by

$$y'\Big|_{x=a} = 2x+1\Big|_{x=a} = 2a+1$$

From the other hand, the slope is given by $\frac{a^2+a-(-3)}{a-2} = \frac{a^2+a+3}{a-2}$

So: $2a+1 = \frac{a^2+a+3}{a-2}$

$(2a+1)(a-2) = a^2+a+3$

$2a^2-3a-2 = a^2+a+3$

$a^2-4a-5=0$

$a=5$ or $a=-1$

when $a=5$, $y'=11$

so: $\frac{y+3}{x-2} = 11$ $\quad y = 11x-25$

when $a=-1$, $y'=-1$

$\frac{y+3}{x-2} = -1$ $\quad y = -x-1$