CAREER: Categorical representation theory of Hecke algebras

Ben Elias

Overview. Kazhdan-Lusztig theory is the study of certain important categories in representation theory and geometry, with the understanding that their Grothendieck group is the (regular representation of the) Iwahori-Hecke algebra. These categories admit an action of a monoidal category whose Grothendieck group is the Hecke algebra itself. As such they are *categorical representations* of the Hecke algebra. Earlier work of the PI and Williamson presented this monoidal category by generators and relations, opening the field to algebraic techniques. The overarching goal of this project is to study the structural properties of categorical representations.

The PI (with Hogancamp) will study general features of categorical representations. The key tool is *categorical diagonalization,* lifting eigenspace decompositions from representation theory. The PI (with Williamson, Juteau, and Young) will study specific categorical representations that are significant in the modular representation theory of algebraic groups and quantum groups. The PI proposes to organize an annual workshop at the University of Oregon, aimed primarily at graduate students, which would continue the successful workshop series organized by Proudfoot and funded by his expiring CAREER grant. The workshop assumes little background, and tries to bring students very deep into a specific field, focusing on a new result in representation theory, algebraic geometry, or related combinatorics. Several of these workshops will be on themes related to categorical representation theory.

Intellectual Merit of the proposed activity. Categorical representations are ubiquitous in classical representation theory and geometry, so their study will shed light on many familiar topics. For example, various knot homology theories are built from categorical representations, and one expects that categorical diagonalization will be a powerful new tool in their computation. It should also help explain the exciting conjectures of Gorsky-Oblomkov-Rasmussen-Shende, which connect HOMFLYPT knot homology to the geometry of flag Hilbert schemes. There are potential applications to TQFTs and low-dimensional topology. Finally, categorical diagonalization will give the first concrete, algebraic approach to Lusztig's character sheaves, making this difficult area more approachable.

In another area of mathematics, almost nothing is known about the characters of rational representations of an algebraic group in finite characteristic. A block of this category is (conjecturally) a categorical representation for the affine Weyl group. Significant progress could be made on this open problem with this new categorical tool. A related construction involving quantum groups at roots of unity could finally provide fresh insight on the longstanding problem of categorifying Hecke algebras attached to complex reflection groups.

Broader impacts of the proposed activity. Categorical and geometric representation theory is a highly technical field with many prerequisites, but the approach taken by the PI is computational, straightforward, and algebraic. This allows for collaborations with computer scientists, and provides nice research projects for undergraduates. It also gives easier, low-tech access for general mathematicians and graduate students to gain intuition while learning the more difficult material. Several successful introductory workshops have already been given by the PI based on this principle.

The proposed workshops aim to serve a similar task in other fields, giving an opening for younger researches to break into a modern research topic, even when that topic is not their area of expertise. With a focus on exercises and engagement, the workshops aim to be an effective educational tool above all else. They will also leave a lasting legacy, with online resources giving future students an easy way into the field. The PI's extensive experience running workshops should translate well into helping other speakers run excellent workshops of their own.