Teaching Statement: Ben Elias

We often begin classes with motivating examples. Why not teaching statements?

In the summer of 2007 I taught Ordinary Differential Equations at Columbia. The textbook, typical of textbooks for introductory math courses, inspired some distaste. It contained: instructions on how to use a hammer, how to use a saw, how to use a drill; numerous exercises on hammering, sawing, drilling; no guidance whatsoever on how to be a carpenter. I took a different approach. On both midterm and final exam more than half the grade consisted in answering high level questions separate of any technical proficiency: given a differential equation of a certain type, what can you do with it, and what will those methods yield? The students said they hadn't experienced a test like that before, but the feedback was positive. After all, in an age where computers can perform the technical aspects of the problem efficiently, it is most important to understand which program to run and why.

In days of yore, mathematics (mostly geometry) was taught as part of the trivium, not so much due to its intrinsic virtues but as a fundamental prerequisite for rhetoric. That is, rhetoric is the art of crafting convincing arguments, and math is the one place where arguments are incontrovertible, where one can truly learn rigor. In contrast, for the bulk of the students today, math is seen as a collection of tools and techniques, to be used for physics and engineering. Neither approach aligns with my viewpoint, because neither rigor nor technique is really an accurate description of what makes mathematicians excited (although both are useful skills for other applications).

I am not personally excited about, say, the nitty gritty involved in the method of variation of parameters (to choose just one technique), and it would be disingenuous to get the students excited about it. However, I can get excited about what the method is good for, and what aspects of it are useful. I have a large amount of enthusiasm for big ideas, and judging from feedback this enthusiasm and understanding is effectively shared with the class.

It is equally important to discuss what students should *not* be excited about, so that they do not miss the forest for the trees. This holds true even when teaching classes designed for math majors. The flip side to being bogged down in technique is being mired in rigor. A well-designed lecture will make clear what is a black box, and what is the key theorem. I can not abide long formal presentations that only get to the point when it is time for the theorem to be proven. One should make the key ideas clear with numerous examples and counterexamples, demonstrate the main themes and the subtleties of the object at hand, and then follow through with the formal approach. Rigor is the language used by mathematicians, but it is not what makes them tick.

In spring 2010 I ran the Columbia undergraduate seminar, where the students themselves did most of the lecturing but I designed the curriculum and helped them learn the material independently. The topic was fairly advanced: Coxeter groups, Dynkin diagrams, and flag varieties. It would have been difficult for a student to keep abreast with the full brunt of the course, but we still managed to cover a lot of material because the students understood the key ideas, while missing some of the proofs and finer points. The point of the class was to give the students an opportunity to teach, and I made it clear that I wanted them to teach using arguments that people would understand, not arguments that they would simply follow.

Mathematics is, above all, the subject where people learn how to *think*. This is true at all levels, whether for students pursuing math for its own sake or for those who view it as a means to an end. I organize my teaching around conveying the big ideas, why the techniques work and why we care. I try to make students realize that math is not a toolbox of techniques but is an opportunity to learn about problem solving itself. Making this the focus is not quite enough in today's world, where grades and homework have become so ingrained that people believe that learning the techniques which earn them grades is what learning consists of. I go out of my way to design homework and tests which address the big ideas as well as technical proficiency, so that their desires converge to mine. For the ODE class I provided summary notes which outlined the key concepts. Primarily, it is my contagious enthusiasm for spreading comprehension that makes the method successful.