

Teaching Statement: Ben Elias

The steering wheel is my favorite example of a well-designed tool. Sitting down in the driver's seat, the wheel becomes an extension of one's own body and intentionality. One does not think to turn the wheel in order to cause a series of mechanical reactions, leading to the pivoting of the tires and consequentially a right turn. One does not think about how to move one's arms to rotate the wheel. Simply put, one just turns right. If one had to think things through at every turn, one could never acquire the next set of tools: the rules of the road, or how to maneuver in traffic.

In mathematics every theorem, argument, and paradigm is a tool, through which we can explore more mathematics. Some theories allow us to move forward better than others, using their techniques as stepping stones. Other theories are easier to internalize. Unfortunately, few theories are as accessible as the steering wheel. Though driving school teachers may disagree, I think math teachers have it tough.

I find the concepts of "tools" and "tool acquisition" to be powerful metaphors, when describing how we learn and operate in our world. This holds true at all levels, from the mundane to the mathematical. It takes little introspection to notice the countless things we take for granted in our daily lives, the frameworks we have developed through which we experience the world. The same is true for mathematicians at work, despite the "rigorous" nature of the field. I research flag varieties. When I encounter flag varieties today I scarcely even think of their definition, their basic properties, in a conscious way; the concept itself has become a rich node of connotations, something I just know how to work with and have intuitive feeling for. I doubt any mathematician could think clearly and make progress in their field without internalizing the ideas they work with.

I haven't forgotten the path I took to reach this degree of proficiency, hours spent working examples, staring at the definitions, and facing theorems which at the time seemed daunting. Our primary goal as teachers is to help students internalize and use the tools we share with them, whether the tools be derivatives or derived categories. Most classes assign frequent homework, causing students to repeat by example and extrapolate from definitions until they reach familiarity. Then they measure performance with timed exams, which expect a degree of proficiency and instant recognition which solves the problem quickly, rather than the drawn out calculations or deductions which might appear on homework. This is an effective way to build and then test internalization; I would suggest more frequent quizzes targeted at specific skills, because these skills are often the foundation of more complex skills.

The mistake made most often in introductory courses is underemphasizing high-level goals and tools. A course can provide instructions on how to use a hammer and saw, and give numerous exercises on hammering and sawing, but it can still fail to make the student a carpenter. In my opinion, failure to provide sufficient motivation contributes a great deal to the failure of students to internalize the basic techniques, because this internalization never becomes essential. Multivariable calculus classes often stress that their techniques will eventually be used in other classes; this is not sufficient. A lecture should be titled "how to find the steepest path," not "the gradient." I have found that a few well-placed remarks combined with sheer enthusiasm is often enough; I feel that such motivation is one of my strengths as a teacher. The choice of appropriate metaphors is also important. This is the other reason I love the steering wheel: people immediately grasp how easy it is to use, so that the idea of the steering wheel is an effective metaphor for internalization.

Some of the same issues arise when teaching classes for math majors and beginning graduate students. Students new to math and the formal rigors involved tend to mistake these rigors as the focus of the field itself, and either get bogged down or misguided. It is important as a teacher to stress that theorems are not interesting because of their truth, but because of their applications and the richer math one can build with them. One should be clear which arguments are useful and worth pondering and which ones merely bridge the gap.

The opposite mistake is one I have made in the past and constantly strive to avoid: overemphasizing the high-level approach. I taught Ordinary Differential Equations in the summer of 2007 and

designed the curriculum. In an age where computers perform the technical aspects of a problem efficiently, I believed then that the most significant knowledge was which program to run. I taught the students that knowing what to do with a given differential equation was more important than being able to actually analyze it. I aligned the testing with this goal, giving problems where one had to identify which methods would apply to which problems, and what sorts of results they would yield. Students responded positively to this testing, and it is something I would repeat. Unfortunately, I did not focus enough on mechanical facility with the techniques. As much as one can explain why each tool is correct for its job, without an internal feeling for the tool, the explanation can not fully sink in.

Ultimately, I am not satisfied as a teacher until I hear the click as an idea successfully meshes into a student's brain. It takes both elbow grease and the proper mindset to get it there.