Convergence of Learning Algorithms without a Projection Facility: Corrigendum and Extension

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The paper (Evans and Honkapohja 1998a) studies convergence of the adaptive algorithm

$$\theta_t = \theta_{t-1} + \gamma_t H(\theta_{t-1}, X_t) + \gamma_t^2 \rho_t(\theta_{t-1}, X_t). \tag{1}$$

Here $\theta_t \in \mathbb{R}^d$ is a vector of parameters and $X_t \in \mathbb{R}^k$ is a vector of state variables.[1] For local convergence analysis one fixes an open set $D \subset \mathbb{R}^d$ around the equilibrium point of interest.

Condition (A.3)(i) on the function $H(\theta, x)$ was stated incorrectly. Instead of $|H(\theta, x_1) - H(\theta, x_2)| \leq L_1 |x_1 - x_2|$ the condition should be on the derivatives of H, i.e. $|\partial H(\theta, x_1)/\partial x - \partial H(\theta, x_2)/\partial x| \leq L_1 |x_1 - x_2|$.

However, (A.3) can be generalized and this may prove useful for future applications. We therefore state the general form of (A.3) and indicate how to establish the key affected results with these altered assumptions.

(A.3) For any compact $Q \subset D$ the function $H(\theta, x)$ satisfies $\forall \theta, \theta' \in Q$ and $x_1, x_2 \in \mathbb{R}^k$:

(i) $|\partial H(\theta, x_1)/\partial x - \partial H(\theta, x_2)/\partial x| \leq L_1 |x_1 - x_2| (1 + |x_1|^{p_1} + |x_2|^{p_1})$ for some $p_1 \geq 0$.

(ii) $|H(\theta, 0) - H(\theta', 0)| \leq L_2 |\theta - \theta'|$,

(iii) $|\partial H(\theta, x)/\partial x - \partial H(\theta', x)/\partial x| \leq L_2 |\theta - \theta'| (1 + |x|^{p_2})$ for some $p_2 \geq 0$.

[1](Evans and Honkapohja 1998b) uses the same framework with conditional linear dynamics for state variables.

for some constants L_1, L_2.

This form of (A.3) also necessitates a slight strengthening of (C.3) and (C.4) which are now taken to hold for all $p > 0$.

For the proof of Theorem 1 under the new assumptions we need to establish the Lemma on p. 65 and Step I on pp.77-79.

Proof of Lemma: It can be verified that (A.3) implies

$$|H(\theta, x) - H(\theta', x)| \leq L_2 |\theta - \theta'| (1 + |x|^{p_3})$$

for some $p_3 \geq 0$. This is sufficient for the demonstration that

$$\left|\Pi^n_{\theta_1} H_{\theta_1}(x) - \Pi^n_{\theta_2} H_{\theta_2}(x)\right| \leq |\theta_1 - \theta_2| (K' + K^* + K' |x|^p + K^* |x|^q),$$

which yields the Lemma.

Proof of Step I of Theorem 1: We remark that the step on p. 78 that (A.3) implies $K_3(H'_\theta) < \infty$ is still valid by the arguments of (Benveniste, Metivier, and Priouret 1990), p. 262-3.

We also note that Corollary 2 as proved requires the following boundedness assumption on the state variable: There exists a random variable Z such that $\forall t : |X_t| < Z$ almost surely.

References

BENVENISTE, A., M. METIVIER, AND P. PRIOURET (1990): *Adaptive Algorithms and Stochastic Approximations.* Springer-Verlag, Berlin.

EVANS, G. W., AND S. HONKAPOHJA (1998a): "Convergence of Learning Algorithms without a Projection Facility," *Journal of Mathematical Economics*, 30, 59–86.

——— (1998b): "Economic Dynamics with Learning: New Stability Results," *Review of Economic Studies*, 65, 23–44.

Additional typos in published version of "Convergence of learning algorithms without a projection facility" by George W Evans and Seppo Honkapohja, *Journal of Mathematical Economics,* Vol. 30 (1998), pp. 59-86.

Appendix

p. 78, lines 4 and 5: This should read "... as a result of the inequality $|y_1 - y| (1 + |y_1|^p + |y|^p) \leq K' (1 + |y_1|^q + |y|^q)$, for given $p > 0$, where $K' > 0$ and $q > 0$ are chosen appropriately. Thus for all k: "

p. 78, lines 14 and 15: After the line 13 equation $\Pi_\theta \nu_\theta = \Sigma_{n \geq 1} (\Pi_\theta^n H_\theta - h(\theta))$, in the next two lines $\nu_\theta(y)$ should be $\nu_\theta(x)$, and $\nu_\theta(y) - \nu_{\theta'}(y) =$ should be $\nu_\theta(x) - \nu_{\theta'}(x) =$.

p. 78, last line: $(1 + |x|^S (1 - \rho)^{-1}$ should be $(1 + |x|^S)(1 - \rho)^{-1}$.

p. 79, two lines below (a.3): the left-hand side of the inequality $|\nu_\theta(y) - \nu_{\theta'}(y)|$ should be $|\nu_\theta(x) - \nu_{\theta'}(x)|$.