

Expectations, Learning and Macroeconomic Policy

George W. Evans (Univ. of Oregon and Univ. of St. Andrews)

Lecture 3

(i) Recurrent Hyperinflations and Learning

(ii) Dynamic Predictor Selection and Endogenous Volatility

Recurrent Hyperinflations and Learning Marcet and Nicolini (2003)

The **seigniorage model of inflation extended to open economies** and occasional exchange rate stabilizations explain hyperinflation episodes during the 1980s.

Basic hyperinflation model (seigniorage model of inflation)

- The seigniorage model of inflation with the linear money demand equation

$$M_t^d/P_t = \phi - \phi\gamma(P_{t+1}^e/P_t) \text{ if } 1 - \gamma(P_{t+1}^e/P_t) > 0 \text{ and } 0 \text{ otherwise.}$$

Also exogenous government purchases $d_t > 0$ financed by seigniorage:

$$M_t = M_{t-1} + d_t P_t.$$

- Assuming $d_t = d$ we get

$$\frac{P_t}{P_{t-1}} = \frac{1 - \gamma(P_t^e/P_{t-1})}{1 - \gamma(P_{t+1}^e/P_t) - d/\phi}.$$

- There are two steady states, $\beta_L < \beta_H$, provided $d \geq 0$ is not too large and none if d is above a critical value. Also a continuum of perfect foresight paths converging to β_H .
- Adaptive (steady-state) learning: PLM expectations are

$$\left(\frac{P_{t+1}}{P_t}\right)^e = \beta,$$

and the corresponding ALM is

$$\frac{P_t}{P_{t-1}} = \frac{1 - \gamma\beta}{1 - \gamma\beta - d/\phi} \equiv T(\beta; d).$$

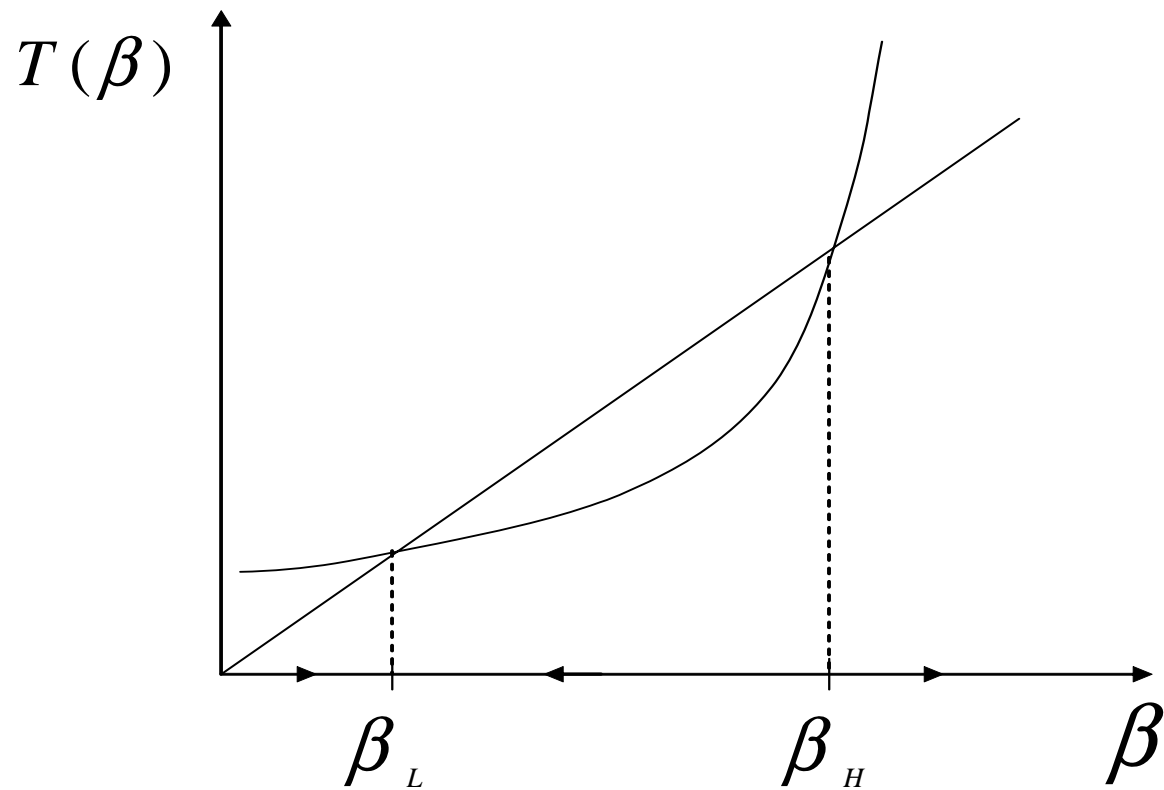
- Under basic decreasing-gain steady-state learning, agents estimate β based on past data, i.e. $P_{t+1}^e/P_t = \beta_{t+1}$, where

$$\beta_t = \beta_{t-1} + t^{-1}(P_{t-1}/P_{t-2} - \beta_{t-1}).$$

- The E-stability differential equation is

$$d\beta/d\tau = T(\beta; d) - \beta,$$

where d is a fixed parameter. β_L is E-stable while β_H is not.



Steady state learning in the hyperinflation model

Since $0 < T'(\beta_L) < 1$ and $T'(\beta_H) > 1$, β_L is E-stable, and therefore locally stable under learning, while β_H is not.

- Empirical Background: four stylized facts about hyperinflation episodes.
 1. Recurrence of hyperinflation episodes.
 2. ERR (exchange rate rules) stop hyperinflations, though eventually new hyperinflations.
 3. During a hyperinflation, seigniorage and inflation are not highly correlated.
 4. Average inflation and seigniorage are strongly positively correlated across countries.
- Marcet-Nicolini Model: an open economy version of the hyperinflation model. Flexible price model with PPP, so that

$$P_t^f e_t = P_t,$$

where P_t^f is the foreign price of goods, assumed exogenous. There is a CA constraint for local currency, government expenditure d_t is *iid*.

- There are floating (like closed economy) and ERR (exchange rate rule) regimes. In ERR e_t is set to satisfy

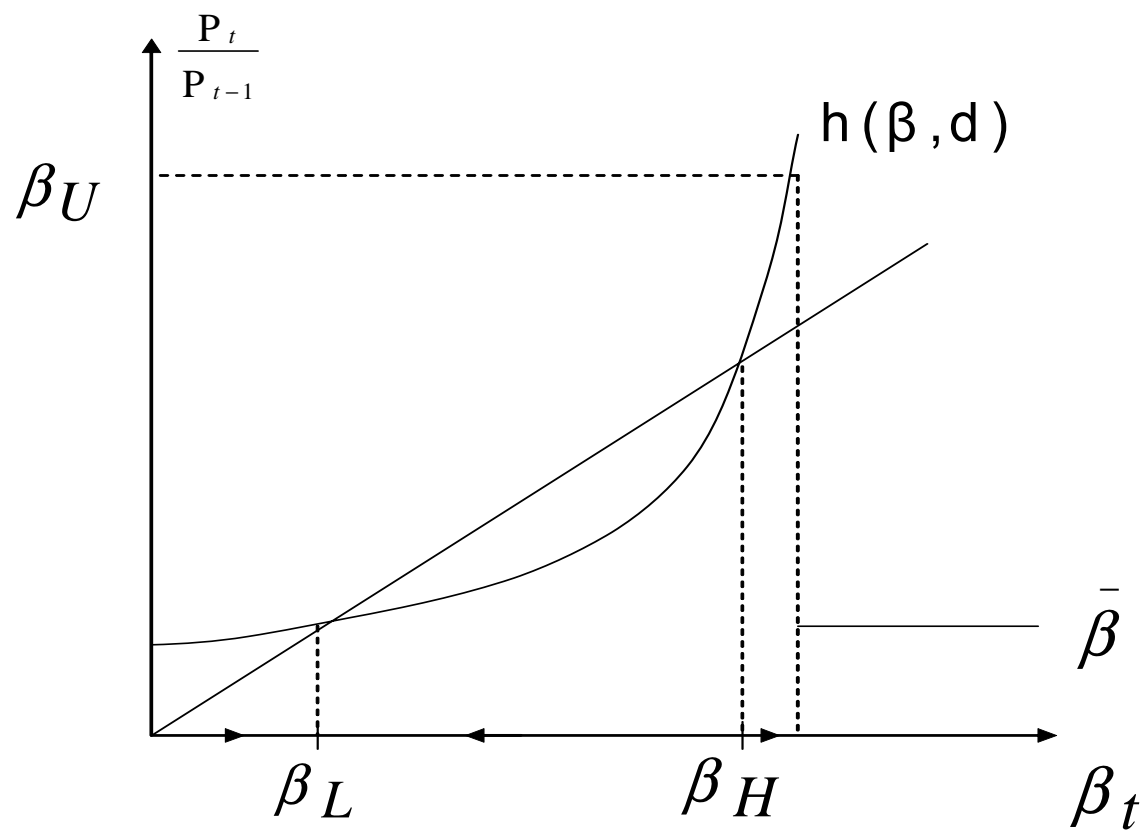
$$\frac{P_t^f}{P_{t-1}^f} \frac{e_t}{e_{t-1}} = \bar{\beta}.$$

Assume a maximum inflation rate tolerated, β_U . ERR is imposed only in periods when inflation would otherwise exceed this bound.

- Learning: simple (decreasing gain) steady-state learning rule, but with the state-contingent gain:

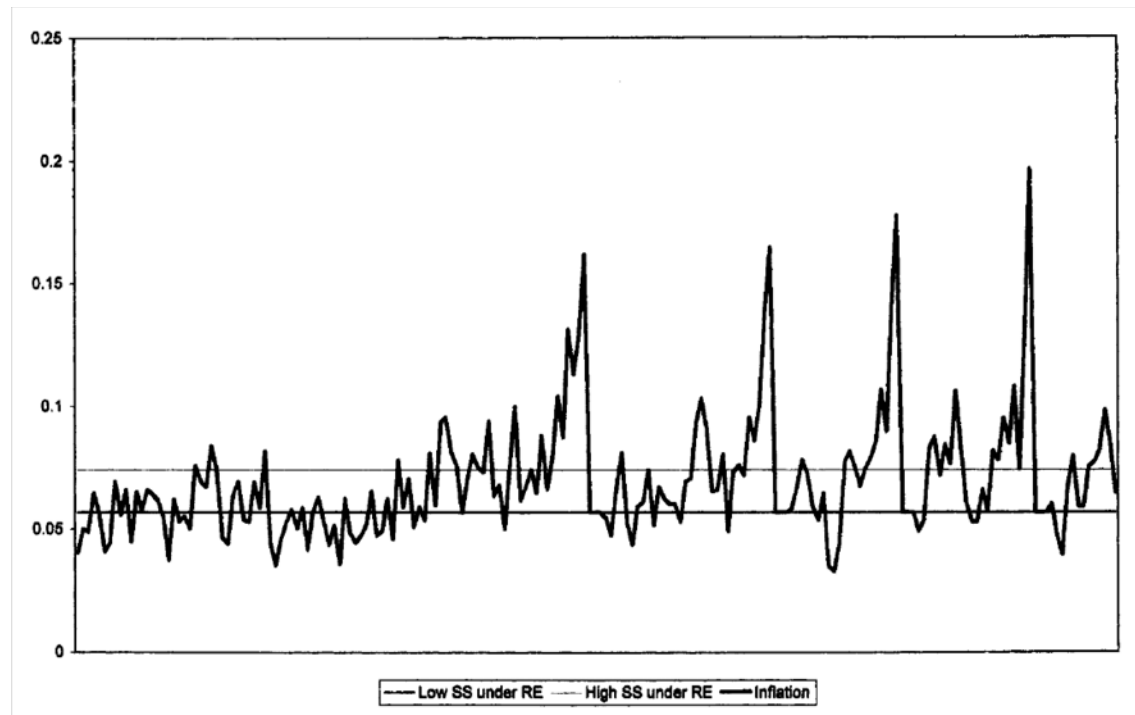
$$\beta_t = \beta_{t-1} + \frac{1}{\alpha_t} \left(\frac{P_{t-1}}{P_{t-2}} - \beta_{t-1} \right),$$

with given β_0 . $\alpha_t = \alpha_{t-1} + 1$ if $\left| \left(\frac{P_{t-1}}{P_{t-2}} - \beta_{t-1} \right) / \beta_{t-1} \right|$ falls below some bound v and otherwise $\alpha_t = \bar{\alpha}$.



Inflation as a function of expected inflation

- The low inflation steady state is locally learnable.
- A sequence of adverse shocks can create explosive inflation. When inflation rises above β^U inflation is stabilized by moving to an ERR.
- The learning dynamics lead to periods of stability alternating with occasional eruptions into hyperinflation.
- All four stylized facts listed above can be matched.



Hyperinflations under learning

- Overall, a very successful application of boundedly rational learning to a major empirical issue.

Dynamic predictor selection & endogenous volatility

Branch & Evans (RED, 2006)

Throughout the lectures we have assumed all agents are using the same econometric model: any **heterogeneity in expectations** has been “mild.”

There are several papers that consider heterogeneity in the sense that different groups of **agents use different forecasting models**.

In this topic we start from the approach introduced by Brock and Hommes (1997) in which agents entertain competing forecasting models – naive cheap models and more costly sophisticated models.

The proportions of agents using the different models at t depends on recent forecasting performance. These **proportions evolve over time**.

Branch and Evans (2007) look at agents choosing between **alternative misspecified models** that are each updated using LS learning, and develop an application to macroeconomics that is able to generate **endogenous volatility**.

EMPIRICAL OVERVIEW

In many countries there is substantial evidence of **stochastic volatility** in output and inflation.

- Cogley and Sargent emphasize **parameter drift**, while
- Sims and Zha emphasize **regime switching**.

Our paper provides a theoretical explanation based on learning and dynamic predictor selection.

THE MODEL

We use a simple Lucas-style AS curve with a “quantity theory” AD curve:

$$AS : q_t = \phi (p_t - p_t^e) + \beta_1' z_t$$

$$AD : q_t = m_t - p_t + \beta_2' z_t + w_t,$$

$$z_t = Az_{t-1} + \varepsilon_t.$$

where w_t, z_t are exogenous and w_t, ε_t are *iid*. This model can be micro-founded along the lines of Woodford (2003). The components of z_t depend on preference, cost and productivity shocks. We assume money supply m_t follows

$$m_t = p_{t-1} + \delta' z_t + u_t,$$

where u_t is *iid*.

Combining equations leads to the reduced form

$$\pi_t = \theta \pi_t^e + \gamma' z_t + \nu_t,$$

where $0 < \theta = (1 + \phi)^{-1} \phi < 1$ and ν_t depends on w_t, u_t .

The unique REE is

$$\pi_t = (1 - \theta)^{-1} \gamma' A z_{t-1} + \gamma' \varepsilon_t + \nu_t.$$

MODEL MISSPECIFICATION

- The world is complex. We think econometricians typically misspecify models.
- By the cognitive consistency principle we therefore believe economic agents misspecify their models.

– To model this simply we assume that z_t is 2×1 and agents choose between two models

$$\pi_t^e = b^1 z_{1,t-1} \text{ and } \pi_t^e = b^2 z_{2,t-1}.$$

If the proportion n_1 uses model 1 then

$$\pi_t^e = n_1 b^1 z_{1,t-1} + (1 - n_1) b^2 z_{2,t-1}.$$

– We impose the RPE (restricted perceptions equilibrium) requirement that, given n , each forecast model satisfies

$$E z_{i,t-1} (\pi_t - b^i z_{i,t-1}) = 0, \text{ for } i = 1, 2.$$

– To close the model we follow Brock-Hommes & assume that n depends on the relative MSE of the two models:

$$n_i = \frac{\exp \{ \alpha E u_i \}}{\sum_{j=1}^2 \exp \{ \alpha E u_j \}} \text{ where } E u = -E (\pi_t - \pi_t^e)^2.$$

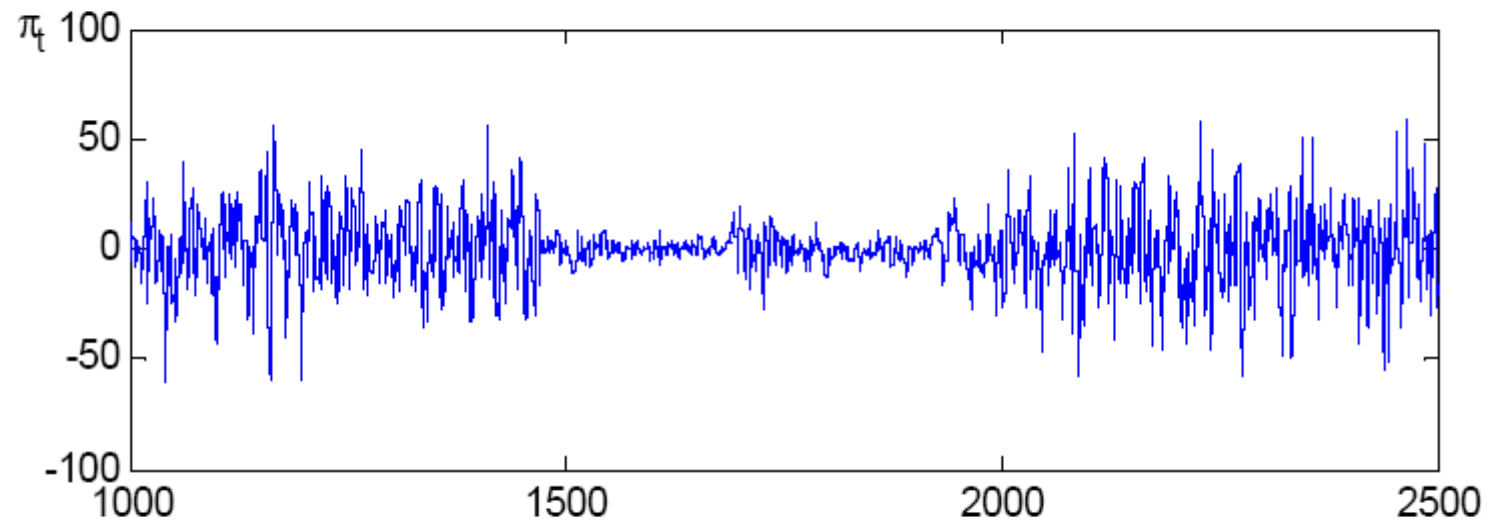
Here $\alpha > 0$ is the BH “intensity of choice” parameter. We pick α large.

– We show that for α large **there can be two ME (Misspecification Equilibria)** for appropriate z_t processes and other parameters. This can happen even though there is a unique RE.

– In one ME n_1 is near 1 and in the other n_1 is near zero.

REAL-TIME LEARNING WITH CONSTANT GAIN

- Now assume agents **update** their **forecasting using constant gain** learning:
 - (i) constant gain learning of parameter values b^1 and b^2 , and
 - (ii) constant gain estimates of $Eu_1 - Eu_2$.
- Simulations exhibit both “**regime-switching**” as n_1 moves quickly between values near 1 and 0 and then stay at these values for an extended period, and **parameter drift** as the estimated coefficients b_t^1 and b_t^2 move around.
- **Simulations** strongly exhibit **endogenous volatility** that is absent under RE.



Simulation under constant gain learning and dynamic predictor selection.